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Generation of the nonlocal quantum entanglement of three three-level particles by local operations*

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We propose a scheme to realize the nonlocal quantum entanglement of three three-level particles by using a three-particle entangled state of three levels as a quantum channel with the aid of some local unitary transformations. This scheme can be directly generalized to the nonlocal quantum entanglement of N three-level particles.

Keywords: quantum entanglement, three-level particle, unitary transformation
PACC: 0367, 0365

1. Introduction

Nonlocal quantum entanglement is not only a key ingredient for the tests of quantum nonlocality,[1,2] but also a basic resource in quantum information processing, such as quantum cryptography,[3] quantum dense coding,[4] quantum teleportation,[5−13] quantum error correction[14] and the quantum computer.[15] So the research on entangled states has attracted more and more attention. Most of the research on quantum information processing is based on the entanglement generation of two-level quantum systems.[16,17] Recently, Zou *et al*[18] and Jin *et al*[19] proposed two schemes to generate the entangled states of three-level atoms in cavity quantum electrodynamics (QED). Dai *et al*[20,21] proposed a scheme to teleport multi-level particle state. Moreover, Zhan *et al*[22] presented a scheme to generate the entangled state of multi-level atoms. Dür *et al*[23] presented a scheme of purification by using nonlocal quantum operations. Lately, Zheng *et al*[24] proposed a scheme to probabilistically generate an unknown nonlocal two-particle entangled state of two levels by using a partial two-particle entangled state of two levels and performing some nonlocal CNOT operations.

This paper is organized as follows. In Section 2, we propose a scheme to probabilistically generate an unknown nonlocal three-particle entangled state of three levels by using a three-particle entangled state of three levels as the quantum channel. In Section 3, we generalize this scheme to probabilistically generate an unknown nonlocal N-particle entangled state of three levels.

2. How to realize an unknown local quantum entanglement of three three-level particles by local operations

Suppose that the three unknown states of the three-level particles can be expressed as

$$\begin{aligned}|\psi\rangle_1 &= \sum_{j=0}^{2} a_j|j\rangle_1,\\ |\psi\rangle_2 &= \sum_{n=0}^{2} b_n|n\rangle_2,\\ |\psi\rangle_3 &= \sum_{m=0}^{2} c_m|m\rangle_3,\end{aligned} \tag{1}$$

where $\sum_{j=0}^{2}|a_j|^2=1$, $\sum_{n=0}^{2}|b_n|^2=1$, and $\sum_{m=0}^{2}|c_m|^2=1$. Therefore, the joint state of particles (1, 2, 3) is given by

$$|\psi\rangle_{123}=|\psi\rangle_1\otimes|\psi\rangle_2\otimes|\psi\rangle_3. \tag{2}$$

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We perform a unitary transformation $U_0 = U_{12} \otimes U_{13}$ on particles (1, 2) and particles (1, 3) under the basis $\{|00\rangle, |01\rangle, |02\rangle, |10\rangle, |11\rangle, |12\rangle, |20\rangle, |21\rangle, |22\rangle\}_{12(13)}$, respectively. The unitary transformations U_{12} and U_{13} may take the form of the following 9×9 matrices:

$$U_{12} = U_{13} = \begin{pmatrix} I & 0 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & A_2 \end{pmatrix}, \tag{3}$$

where I is the 3×3 identity matrix and 0 is the 3×3 zero matrix. A_1 and A_2 are the 3×3 matrices and may be written as

$$A_1 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \tag{4}$$

After performing the unitary transformation U_0, the state in Eq.(2) becomes

$$|\psi\rangle_{123} = \sum_{r=0}^{2} |R_r\rangle, \tag{5}$$

where $|R_r\rangle$ is defined as

$$\begin{aligned} |R_r\rangle =& a_j |j\rangle_1 \otimes \sum_{n=0}^{2} b_n |(n+j) \bmod 3\rangle_2 \\ & \otimes \sum_{m=0}^{2} c_m |(m+j) \bmod 3\rangle_3, \\ & (r = j \in \{0,1,2\}). \end{aligned} \tag{6}$$

So the local quantum entanglement of the three three-level particles is determined by the coefficients a_j, b_n, and $c_m (j, n, m = 0, 1, 2)$. If the coefficients $a_0 = a_1 = a_2 = 1/\sqrt{3}$ and $b_1 = b_2 = c_1 = c_2 = 0$, we can obtain a local three-particle maximally entangled state of three levels, which can be expressed as

$$|\psi\rangle_{123} = \frac{1}{\sqrt{3}} (|000\rangle + |111\rangle + |222\rangle)_{123}. \tag{7}$$

3. How to realize an unknown nonlocal quantum entanglement of three three-level particles by local operations

A scheme is proposed to probabilistically generate an unknown nonlocal three-particle entangled state of three levels by using a three-particle entangled state of three levels as the quantum channel. Without loss of generality, the quantum channel of the three-particle entangled state of three levels is given by

$$|\psi\rangle_{456} = \sum_{k=0}^{2} \lambda_k |kkk\rangle_{456}, \tag{8}$$

where $\sum_{k=0}^{2} |\lambda_k|^2 = 1$, and $|\lambda_2|$ is the smallest among these coefficients' absolute values. Particles (1, 4) belong to Alice. Particles (2, 5) and particles (3, 6) belong to Bob and Charlie, respectively. Furthermore, Alice, Bob, and Charlie are spatially separated. Therefore, the state of the whole system is composed of three unknown quantum states, and the quantum channel is given by

$$|\psi\rangle_{123456} = |\psi\rangle_1 \otimes |\psi\rangle_2 \otimes |\psi\rangle_3 \otimes |\psi\rangle_{456}. \tag{9}$$

Firstly, Alice performs a unitary transformation U_1 on particles (1, 4) under the basis $\{|00\rangle, |01\rangle, |02\rangle, |10\rangle, |11\rangle, |12\rangle, |20\rangle, |21\rangle, |22\rangle\}_{14}$. The unitary transformation U_1 may take the form of the following 9×9 matrix

$$U_1 = \begin{pmatrix} I & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_1 \end{pmatrix}. \tag{10}$$

The unitary transformation U_1 will transform the state in Eq.(9) into

$$\begin{aligned} |\psi\rangle_{123456} = & \left[\sum_{x,j=0}^{2} a_j \lambda_k |jkk\rangle_{156} \otimes |x\rangle_4 \right] \\ & \otimes |\psi\rangle_2 \otimes |\psi\rangle_3, (k = (j+x) \bmod 3). \end{aligned} \tag{11}$$

Alice measures the state of particle 4. If the measured result is $|1\rangle_4$, the state in Eq.(11) will become

$$\begin{aligned} |\psi\rangle_{12356} = & \sum_{(j,k)} a_j \lambda_k |jkk\rangle_{156} \otimes \sum_{n=0}^{2} b_n |n\rangle_2 \otimes |\psi\rangle_3, \\ & ((j,k) \in \{(0,1),(1,2),(2,0)\}). \end{aligned} \tag{12}$$

Then Alice informs Bob of her measured result via classical communication.

Secondly, Bob performs a unitary transformation U_2 on particles (5, 2) under the basis $\{|00\rangle, |01\rangle, |02\rangle, |10\rangle, |11\rangle, |12\rangle, |20\rangle, |21\rangle, |22\rangle\}_{52}$. The unitary transformation U_2 may take the form of the following 9×9 matrix:

$$U_2 = \begin{pmatrix} A_2 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & A_1 \end{pmatrix}. \tag{13}$$

After performing the unitary transformation U_2, the state in Eq.(12) will be transformed into

$$|\psi\rangle_{12356} = \sum_{(j,k)} a_j\lambda_k|jkk\rangle_{156} \otimes \sum_{n=0}^{2} b_n|(n+j)\text{mod } 3\rangle_2 \otimes |\psi\rangle_3, ((j,k) \in \{(0,1),(1,2),(2,0)\}). \quad (14)$$

Then Bob performs a unitary transformation U on particle 5 under the basis $\{|0\rangle, |1\rangle, |2\rangle\}_5$. The unitary transformation U is the following 3×3 matrix:

$$U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \mathrm{e}^{2\pi \mathrm{i}/3} & \mathrm{e}^{4\pi \mathrm{i}/3} \\ 1 & \mathrm{e}^{4\pi \mathrm{i}/3} & \mathrm{e}^{2\pi \mathrm{i}/3} \end{pmatrix}. \quad (15)$$

The unitary transformation U will transform the state in Eq.(14) into

$$|\psi\rangle_{12356} = \frac{1}{\sqrt{3}}\Bigg[\sum_{(j,k)} a_j\lambda_k|jk\rangle_{16} \otimes \sum_{n=0}^{2} b_n|(n+j)\text{mod } 3\rangle_2 \otimes |0\rangle_5 + \sum_{(j,k)} \mathrm{e}^{2\pi \mathrm{i}k/3} a_j\lambda_k|jk\rangle_{16} \otimes \sum_{n=0}^{2} b_n|(n+j)\text{mod } 3\rangle_2 \otimes |1\rangle_5 + \sum_{(j,k)} \mathrm{e}^{4\pi \mathrm{i}k/3} a_j\lambda_k|jk\rangle_{16} \otimes \sum_{n=0}^{2} b_n|(n+j)\text{mod } 3\rangle_2 \otimes |2\rangle_5\Bigg] \otimes |\psi\rangle_3, ((j,k) \in \{(0,1),(1,2),(2,0)\}). \quad (16)$$

Bob measures the state of particle 5. If the measured result is $|1\rangle_5$, the state in Eq.(16) will become

$$|\psi\rangle_{1236} = \frac{1}{\sqrt{3}} \sum_{(j,k)} \mathrm{e}^{2\pi \mathrm{i}k/3} a_j\lambda_k|jk\rangle_{16} \otimes \sum_{n=0}^{2} b_n|(n+j)\text{mod } 3\rangle_2 \otimes \sum_{m=0}^{2} c_m|m\rangle_3, ((j,k) \in \{(0,1),(1,2),(2,0)\}). \quad (17)$$

Then Bob informs Charlie of his measured result via classical communication.

Thirdly, Charlie performs a unitary transformation U_2 in Eq.(13) on particles (6, 3) under the basis $\{|00\rangle, |01\rangle, |02\rangle, |10\rangle, |11\rangle, |12\rangle, |20\rangle, |21\rangle, |22\rangle\}_{63}$. The unitary transformation U_2 will transform the state in Eq.(17) into

$$|\psi\rangle_{1236} = \frac{1}{\sqrt{3}}[\mathrm{e}^{2\pi \mathrm{i}/3}\lambda_1|R_0\rangle \otimes |1\rangle_6 + \mathrm{e}^{4\pi \mathrm{i}/3}\lambda_2|R_1\rangle \otimes |2\rangle_6 + \lambda_0|R_2\rangle \otimes |0\rangle_6]. \quad (18)$$

Then Charlie performs a unitary transformation U in Eq.(15) on particle 6 under the basis $\{|0\rangle, |1\rangle, |2\rangle\}_6$. The unitary transformation U will transform the state in Eq.(18) into

$$|\psi\rangle_{1236} = \frac{1}{3}\Bigg[\sum_{(k,r)} \mathrm{e}^{2\pi \mathrm{i}k/3}\lambda_k|R_r\rangle \otimes |0\rangle_6 + \sum_{(k,r)} \mathrm{e}^{4\pi \mathrm{i}k/3}\lambda_k|R_r\rangle \otimes |1\rangle_6 + \sum_{(k,r)} \lambda_k|R_r\rangle \otimes |2\rangle_6\Bigg], ((k,r) \in \{(1,0),(2,1),(0,2)\}). \quad (19)$$

Charlie measures the state of particle 6. If the measured result is $|1\rangle_6$, the state in Eq.(19) will become

$$|\psi\rangle_{123} = \frac{1}{3} \sum_{(k,r)} \mathrm{e}^{4\pi \mathrm{i}k/3}\lambda_k|R_r\rangle, ((k,r) \in \{(1,0),(2,1),(0,2)\}). \quad (20)$$

Next, Charlie informs Alice of his measured result via classical communication.

Finally, Alice performs a unitary transformation H on particle 1 under the basis $\{|0\rangle, |1\rangle, |2\rangle\}_1$. The unitary transformation H may take the form of the following 3×3 matrix:

$$H = \begin{pmatrix} \mathrm{e}^{-4\pi \mathrm{i}/3} & 0 & 0 \\ 0 & \mathrm{e}^{-2\pi \mathrm{i}/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (21)$$

After performing the unitary transformation H, the

state in Eq.(20) will become

$$|\psi\rangle_{123} = \frac{1}{3}\sum_{(k,r)} \lambda_k |R_r\rangle, \quad (k,r) \in \{(1,0),(2,1),(0,2)\}). \qquad (22)$$

Then Alice introduces an auxiliary particle A with its initial state being $|0\rangle_A$, so the joint state of particles 1, 2, 3, and A is

$$|\psi\rangle_{123A} = \frac{1}{3}\sum_{(k,r)} \lambda_k |R_r\rangle \otimes |0\rangle_A, \quad ((k,r) \in \{(1,0),(2,1),(0,2)\}). \qquad (23)$$

Alice performs a unitary transformation U_{1A} on particles (1,A) under the basis $\{|00\rangle, |01\rangle, |02\rangle, |10\rangle, |11\rangle, |12\rangle, |20\rangle, |21\rangle, |22\rangle\}_{1A}$. The unitary transformation U_{1A} is

$$U_{1A} = \begin{pmatrix} B_1 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & B_2 \end{pmatrix}, \qquad (24)$$

where I is the 3×3 identity matrix and 0 is the 3×3 zero matrix. B_1 and B_2 are the 3×3 matrices and may be written as

$$B_1 = \begin{pmatrix} \frac{\lambda_2}{\lambda_1} & 0 & \sqrt{1-\left(\frac{\lambda_2}{\lambda_1}\right)^2} \\ 0 & 1 & 0 \\ \sqrt{1-\left(\frac{\lambda_2}{\lambda_1}\right)^2} & 0 & -\frac{\lambda_2}{\lambda_1} \end{pmatrix},$$

$$B_2 = \begin{pmatrix} \frac{\lambda_2}{\lambda_0} & 0 & \sqrt{1-\left(\frac{\lambda_2}{\lambda_0}\right)^2} \\ 0 & 1 & 0 \\ \sqrt{1-\left(\frac{\lambda_2}{\lambda_0}\right)^2} & 0 & -\frac{\lambda_2}{\lambda_0} \end{pmatrix}. \qquad (25)$$

After performing the unitary transformation U_{1A}, the state in Eq.(23) becomes

$$|\psi\rangle_{123A} = \frac{\lambda_2}{3}\left(\sum_{r=0}^{2}|R_r\rangle\right)\otimes|0\rangle_A + \frac{1}{3}\left(\sqrt{\lambda_1^2-\lambda_2^2}|R_0\rangle + \sqrt{\lambda_0^2-\lambda_2^2}|R_2\rangle\right)\otimes|2\rangle_A. \qquad (26)$$

Alice measures the state of the auxiliary particle A. If the measured result is $|0\rangle_A$, the nonlocal quantum entanglement of three three-level particles is successfully realized with the probability of $|\lambda_2|^2/9$. Otherwise, it fails. It can be easily proved that 27 kinds of states have the same probability $|\lambda_2|^2/9$. Therefore, the total successful probability is $3|\lambda_2|^2$. If the coefficients $a_0 = a_1 = a_2 = 1/\sqrt{3}$ and $b_1 = b_2 = c_1 = c_2 = 0$, we obtain a nonlocal three-particle maximally entangled state of three levels, which is the same as that in Eq.(7). The total successful probability is $3|\lambda_2|^2$. If the coefficients $\lambda_0 = \lambda_1 = \lambda_2 = 1/\sqrt{3}$, the quantum channel is composed of a three-particle maximally entangled state of three levels. The total successful probability equals 1. All possible unitary transformations are given in Table 1.

Table 1. The unitary transformation U, U_2, H, U_{1A}, and state of measurement particle.

4	U_2	U	5	U_2	U	6	H	U_{1A}
$\|0\rangle_4$	$\mathrm{diag}(I, A_1, A_2)$	U	$\|0\rangle_5$	$\mathrm{diag}(I, A_1, A_2)$	U	$\|0\rangle_6$	$\mathrm{diag}(1,1,1)$	$\mathrm{diag}(B_2, B_1, I)$
						$\|1\rangle_6$	$\mathrm{diag}(1, e^{-2\pi i/3}, e^{-4\pi i/3})$	
						$\|2\rangle_6$	$\mathrm{diag}(1, e^{-4\pi i/3}, e^{-2\pi i/3})$	
			$\|1\rangle_5$			$\|0\rangle_6$	$\mathrm{diag}(1, e^{-4\pi i/3}, e^{-2\pi i/3})$	
						$\|1\rangle_6$	$\mathrm{diag}(1, e^{-2\pi i/3}, e^{-4\pi i/3})$	
						$\|2\rangle_6$	$\mathrm{diag}(1,1,1)$	
			$\|2\rangle_5$			$\|0\rangle_6$	$\mathrm{diag}(1, e^{-4\pi i/3}, e^{-2\pi i/3})$	
						$\|1\rangle_6$	$\mathrm{diag}(1,1,1)$	
						$\|2\rangle_6$	$\mathrm{diag}(1, e^{-2\pi i/3}, e^{-4\pi i/3})$	
$\|1\rangle_4$	$\mathrm{diag}(A_2, I, A_1)$	U	$\|0\rangle_5$	$\mathrm{diag}(A_2, I, A_1)$	U	$\|0\rangle_6$	$\mathrm{diag}(1,1,1)$	$\mathrm{diag}(B_1, I, B_2)$
						$\|1\rangle_6$	$\mathrm{diag}(e^{-2\pi i/3}, e^{-4\pi i/3}, 1)$	
						$\|2\rangle_6$	$\mathrm{diag}(e^{-4\pi i/3}, e^{-2\pi i/3}, 1)$	
			$\|1\rangle_5$			$\|0\rangle_6$	$\mathrm{diag}(e^{-2\pi i/3}, e^{-4\pi i/3}, 1)$	
						$\|1\rangle_6$	$\mathrm{diag}(e^{-4\pi i/3}, e^{-2\pi i/3}, 1)$	
						$\|2\rangle_6$	$\mathrm{diag}(1,1,1)$	
			$\|2\rangle_5$			$\|0\rangle_6$	$\mathrm{diag}(e^{-4\pi i/3}, e^{-2\pi i/3}, 1)$	
						$\|1\rangle_6$	$\mathrm{diag}(1,1,1)$	
						$\|2\rangle_6$	$\mathrm{diag}(e^{-2\pi i/3}, e^{-4\pi i/3}, 1)$	

Table 1. (*Continued*)

						$\|0\rangle_6$	$\mathrm{diag}(1,1,1)$	
			$\|0\rangle_5$			$\|1\rangle_6$	$\mathrm{diag}(\mathrm{e}^{-4\pi\mathrm{i}/3},1,\mathrm{e}^{-2\pi\mathrm{i}/3})$	
						$\|2\rangle_6$	$\mathrm{diag}(\mathrm{e}^{-2\pi\mathrm{i}/3},1,\mathrm{e}^{-4\pi\mathrm{i}/3})$	
						$\|0\rangle_6$	$\mathrm{diag}(\mathrm{e}^{-4\pi\mathrm{i}/3},1,\mathrm{e}^{-2\pi\mathrm{i}/3})$	
$\|2\rangle_4$	$\mathrm{diag}(A_1,A_2,I)$	U	$\|1\rangle_5$	$\mathrm{diag}(A_1,A_2,I)$	U	$\|1\rangle_6$	$\mathrm{diag}(\mathrm{e}^{-2\pi\mathrm{i}/3},1,\mathrm{e}^{-4\pi\mathrm{i}/3})$	$\mathrm{diag}(I,B_2,B_1)$
						$\|2\rangle_6$	$\mathrm{diag}(1,1,1)$	
						$\|0\rangle_6$	$\mathrm{diag}(\mathrm{e}^{-2\pi\mathrm{i}/3},1,\mathrm{e}^{-4\pi\mathrm{i}/3})$	
			$\|2\rangle_5$			$\|1\rangle_6$	$\mathrm{diag}(1,1,1)$	
						$\|2\rangle_6$	$\mathrm{diag}(\mathrm{e}^{-4\pi\mathrm{i}/3},1,\mathrm{e}^{-2\pi\mathrm{i}/3})$	

This scheme can be directly generalized to an unknown nonlocal quantum entanglement of N three-level particles. We suppose that the N unknown three-level particle states can be expressed as

$$|\psi\rangle_1 = \sum_{j=0}^{2} a_j|j\rangle_1,$$
$$|\psi\rangle_2 = \sum_{n=0}^{2} b_n|n\rangle_2, \cdots$$
$$|\psi\rangle_N = \sum_{m=0}^{2} c_m|m\rangle_N. \qquad (27)$$

The quantum channel of an N-particle entangled state of three levels is

$$|\psi\rangle_{1'2'\ldots N'} = \sum_{k=0}^{2} \lambda_k|kk\ldots k\rangle_{1'2'\ldots N'}. \qquad (28)$$

We suppose each one of the N persons has two particles, say, the first person has particles $(1,1')$; the second person has particles $(2,2')$;...; the Nth person has particles (N,N'). In order to successfully realize a nonlocal quantum entanglement of N unknown three-level particles, the first person has to perform unitary transformation U_1 on his two particles $(1,1')$ and makes a measurement on particle $1'$ of the quantum channel. The second person performs the unitary transformation U_2 on his two particles $(2,2')$ and the unitary transformation U on particle $2'$ of the quantum channel, then makes measurement on particle $2'$ of the quantum channel. And so on. That is to say, each one of the N persons except the first one has to perform unitary transformation U_2 on his two particles and the unitary transformation U on one particle of the quantum channel, and makes measurement on that particle of the quantum channel. Afterwards, the first person continues to perform a unitary transformation H' on particle 1 and another unitary transformation U_{1A} on particle 1 and an auxiliary particle A. Then the first person measures the state of the auxiliary particle A. If the measured result is $|0\rangle_A$, the nonlocal quantum entanglement of N three-level particles is successfully realized with the probability of $|\lambda_2|^2/(\sqrt{3})^{N-1}$. It should be noted that the unitary transformation H' is related to the particle number N.

Similarly, we can directly generalized this scheme to probabilistically generate an unknown nonlocal quantum entanglement of N d-level particles by using an N-particle entangled state of d-level as quantum channel with the aid of appropriate unitary transformations U_1'', U_2'', U'', H'', and U_{1A}''. We must emphasize that the unitary transformation U'' is

$$U'' = \frac{1}{\sqrt{d}} \sum_{\beta=0}^{d-1} \mathrm{e}^{2\pi i\alpha\beta/d}|\beta\rangle\langle\alpha|,$$
$$(\alpha = 0,1,\ldots,d-1). \qquad (29)$$

When d=3, it turns to the scheme we have discussed in detail.

In summary, we have proposed a scheme to probabilistically realize an unknown nonlocal quantum entanglement of three three-level particles by using a three-particle entangled state of three levels as the quantum channel with the aid of some local unitary transformations. We also generalize this scheme to generate the nonlocal quantum entanglement of N d-level particles.

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