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Electron tunnelling phase time and dwell time through an associated delta potential barrier*

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The electron tunnelling phase time $\tau^{\rm p}$ and dwell time $\tau_{\rm D}$ through an associated delta potential barrier $U(x)=\xi\delta(x)$ are calculated and both are in the order of $10^{-17}-10^{-16}$s. The results show that the dependence of the phase time on the delta barrier parameter ξ can be described by the characteristic length $l_{\rm c}=\hbar^2/m_{\rm e}\xi$ and the characteristic energy $E_{\rm c}=m_{\rm e}\xi^2/\hbar^2$ of the delta barrier, where $m_{\rm e}$ is the electron mass, $l_{\rm c}$ and $E_{\rm c}$ are assumed to be the effective width and height of the delta barrier with $l_{\rm c}E_{\rm c}=\xi$, respectively. It is found that $\tau_{\rm D}$ reaches its maximum and $\tau_{\rm D}=\tau^{\rm p}$ as the energy of the tunnelling electron is equal to $E_{\rm c}/2$, i.e. as $l_{\rm c}=\lambda_{\rm DB}$, $\lambda_{\rm DB}$ is de Broglie wave length of the electron.

Keywords: associated delta potential barrier, tunnelling phase times, tunnelling dwell times characteristic length, characteristic energy

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The study of tunnelling time is one of most attractive and controversial problems in quantum mechanics. Many theoretical and experimental attempts have been made to solve this problem.[1−7] There are several theoretical models to calculate the tunnelling time. One of them is the phase time ($\tau^{\rm p}$) mode, in which the wave packets are incident on the barrier and go through the barrier, and a phase delay for the peak of the wave packet is calculated.[1,2] The phase of the quantum system has the physical significance, and the phase time provides a clearer physical picture for the tunnelling time, though its physical causality is still controversial.[5] Next is $\tau_{\rm D}$, the dwell time mode, which is defined as the ratio of the number of particles within the barrier to the incident flux.[7] The delta function is frequently invoked to describe charge density for a point charge and very short range potential such as the potential produced by nuclear forces. In this paper, we calculate the phase time $\tau^{\rm p}$ and dwell time $\tau_{\rm D}$ for the electrons tunnelling through the associated delta potential barrier, and reveal the correlation of the associated delta barrier parameter ξ with the tunnelling time.

$$U(x)=\xi\delta(x), \tag{1}$$

where $\xi>0$ in 10^{-1}eV·nm). The parameter ξ describes how fast the delta potential $U(x)$ approaches ∞ as $x\to 0$, thus ξ is critical for the shape of the delta potentials. If the electrons are incident on the left of the barrier, as shown in Fig.1, its Schrödinger equation is

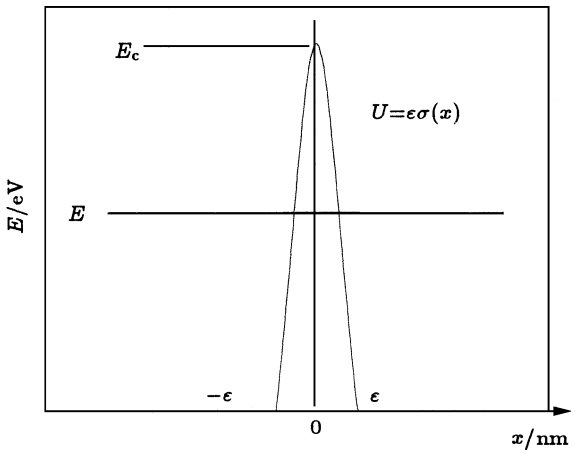


Fig.1. The associated delta barrier $\xi\delta(x)$.

$$-\frac{\hbar^2}{2m_{\rm e}}\frac{{\rm d}^2\psi(x)}{{\rm d}x^2}+\psi(x)\xi\delta(x)=E_{\rm e}\psi(x), \tag{2}$$

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where m_e is the electron mass and E_e is the electron energy. The solution of Eq.(2) must be in the from of

$$\psi_1(x) = e^{ikx} + Re^{-ikx} \quad x < 0, \tag{3}$$

$$\psi_2(x) = Te^{ikx} \quad x > 0, \tag{4}$$

where $E_e = \hbar^2 k^2/2m_e$, R and T are the amplitudes of the reflected and the transmitted waves, respectively. We assume that the amplitude of the incident wave, i.e. the first term on the right-hand side of Eq.(3), is unity. Note that the point x=0 is singularity; we integrate Eq.(2) between $-\varepsilon$ and $+\varepsilon$ (ε is a small positive number), then take the limit as $\varepsilon \to 0^+$ (i.e., ε approaches zero when kept positive) and thus have[8]

$$\psi_2'(0^+) - \psi_1'(0^-) = \frac{2m_e\xi}{\hbar^2}\psi(0). \tag{5a}$$

Also, at the point $x = 0$,

$$\psi_2(0^+) = \psi_1'(0^-). \tag{5b}$$

Using Eqs.(3) and (4), Eq.(5b) can be rewritten as

$$1 + R = T, \tag{6}$$

because at the point x= 0 the wavefunctions are not continuous. Equation (5a) shows that at the point x= 0 the first derivative of the wavefunction is not continuous. Using the boundary conditions Eq.(5a) and (5b), we obtain

$$R = |R|e^{i\beta} = -\frac{1 + il_c k}{1 + l_c^2 k^2}, \quad \beta = \arctan(l_c k), \tag{7}$$

$$T = |T|\,e^{i\alpha} = \frac{l_c k(l_c k - i)}{1 + l_c^2 k^2}, \quad \alpha = \arctan(-1/l_c k), \tag{8}$$

and

$$|R|^2 = \frac{1}{1 + l_c^2 k^2},$$
$$|T|^2 = \frac{l_c^2 k^2}{1 + l_c^2 k^2} = 1\Big/\left(1 + \frac{1}{(l_c k)^2}\right), \tag{9}$$

where $l_c = \hbar^2/(m_e\xi)$, which has the length dimension and can be defined as the characteristic length of the delta potential barrier, and $E_c = m_e\xi^2/\hbar^2$ is defined as the characteristic energy of the delta potential barrier.[9] Note that $|R|^2 + |T|^2$=1.

Since the characteristic length l_c depends on the electron mass m_e and the parameter ξ, $l_c = \hbar^2/m_e\xi$ can be considered as the effective width of an associated delta barrier $\xi\delta(x)$. Obviously, $l_c E_c \equiv \xi$ for the delta barrier with the given parameters ξ represents the area of the associated delta barrier: in other words, its height is E_c and its width is l_c. Figure 2 shows the curves of the tunnelling probability $|\,T\,|^2$ versus the wavenumber k. It is shown that as $k \to 0$, i.e. $\lambda_{DB} \to \infty$ (or $E_e \to 0$), the tunnelling probability $|\,T\,|^2$ through the delta barrier approaches zero. Note that though the tunnelling probability $|\,T\,|^2$ increases with the wavenumber k, $|T|^2 \to 1$ as $k \to \infty$, i.e. $\lambda_{DB} \to 0$ or $l_c/k \to 0$.

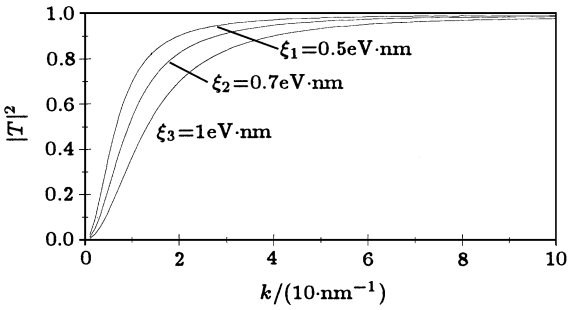


Fig.2. Electron tunnelling probability $|T|^2$ versus k for ξ=0.5 eV·nm, 0.7eV·nm and 1eV·nm, respectively.

For the associated delta potential barrier, the electron tunnelling phase time of the reflected wave is[1,2]

$$\tau_R^p = \frac{1}{\nu}\frac{d\beta}{dk}, \tag{10}$$

and the phase time of the transmitted wave is[1,2]

$$\tau_T^p = \frac{1}{\nu}\frac{d\alpha}{dk}, \tag{11}$$

where $\nu = \hbar^{-1}(\,dE/dk) = \hbar k/m_e$ is the group velocity and $k = \sqrt{2m_eE_e}/\hbar$. l_c is in 10^{-1}nm and k is in 10^{-1}nm^{-1}. Substituting Eqs.(7) and (8) into Eqs.(9) and (10), we obtain

$$\tau_T^p = \tau_R^p = \frac{m_e\hbar^2\xi}{\nu(m_e^2\xi^2 + \hbar^4k^2)} = \frac{l_c}{\nu} \times \frac{1}{1 + l_c^2k^2}$$
$$= \frac{m_e l_c}{\hbar k}\frac{1}{1 + l_c^2k^2} = \frac{c_1}{\sqrt{E_e}}\left(\frac{\xi}{\xi^2 + c_2E_e}\right), \tag{12}$$

where for electron c_1=1.286×10^{-15}, c_2=15.28, ξ is the delta barrier parameter in 10^{-1}eV·nm and E_e is electron energy in eV, τ_T^p and τ_R^p are phase times in s. This is illustrated in Fig.3. The curves a, b and c show that electron phase time τ^p versus k for ξ_1 =0.5eV·nm, ξ_2=0.7eV·nm and ξ_3=1eV·nm, respectively. There are intersection points A, B and C for the curves with different ξ. For example, the curves a and b intersect at the point A when $k = \frac{m_e\sqrt{\xi_1\xi_2}}{\hbar^2}$=0.7766. In this kind of circumstance, τ^p=0.706×10^{-16}s.

Note that the phase time of the reflected wave is equal to that of the transmitted wave. As can be seen in Fig.3, the phase time $\tau_{\mathrm{T}}^{\mathrm{p}}$ and $\tau_{\mathrm{R}}^{\mathrm{p}}$ are close to zero as $k \to \infty$, (or $E_{\mathrm{e}} \to \infty$). On the other hand, as $k \to 0$ (or $E_{\mathrm{e}} \to 0$), both $\tau_{\mathrm{T}}^{\mathrm{p}}$ and τ_{R}^{p} are close to infinity.

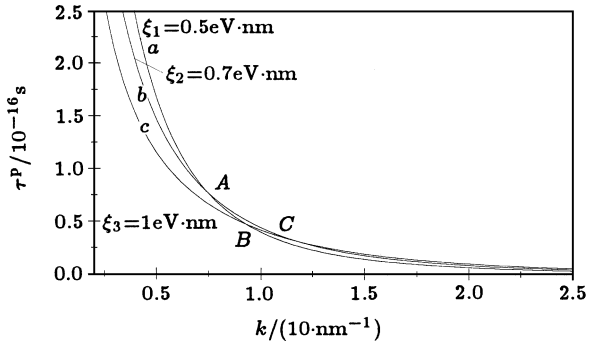


Fig.3. Electron tunnelling phase time τ^{p} versus k for ξ=0.5eV·nm, 0.7eV·nm and 1eV·nm, respectively.

Next, let us consider the dwell time of tunnelling electrons through the delta barrier $\xi\delta(x)$. The dwell time, τ_{D}, is defined as[7,10,11]

$$\tau_{\mathrm{D}} = \frac{1}{\nu}\int_{\mathrm{B}} |\psi|^2 \,\mathrm{d}x, \tag{13}$$

where ψ is the electron wavefunction, B denotes the barrier region $(-l_{\mathrm{c}}/2, l_{\mathrm{c}}/2)$.

Thus Eq.(14) is performed:

$$\begin{aligned}\tau_{D} =& \frac{1}{\nu}\int_{-l_{\mathrm{c}}/2}^{+l_{\mathrm{c}}/2} |\psi_1|^2 \,\mathrm{d}x \\ =& \frac{1}{\nu}\left(\int_{-l_{\mathrm{c}}/2}^{0} |\psi_1|^2_{x=0^-} \,\mathrm{d}x \right. \\ &\left. + \int_{0}^{+l_{\mathrm{c}}/2} |\psi_2|^2_{x=0^+} \,\mathrm{d}x\right).\end{aligned} \tag{14}$$

Here, ψ_1 is the electron wavefunction at $x \to 0^-$ in the barrier region of $x < 0$ and $|\psi_1|^2_{x=0^-}$ is the probability density for electron to be found at $x \to 0^-$ on the left-hand side of the delta barrier; ψ_2 is the electron transmitting wavefunction at $x \to 0^+$ on the right-hand side in the barrier region of $x > 0$, $|\psi_2|^2_{x\to 0^+}$ is the probability density of tunnelling electron at $x \to 0^+$; $\nu = \hbar k/m_{\mathrm{e}}$ is the group velocity.

Substituting Eqs.(3), (4), (6) and (7)–(9) into Eq.(14), we can than rewrite the integral in Eq.(14) as

$$\begin{aligned}\tau_{\mathrm{D}} =& \frac{1}{\nu}\left(\int_{-\frac{l_{\mathrm{c}}}{2}}^{0} |1+R|^2_{x=0^-}\,\mathrm{d}x + \int_{0}^{+\frac{l_{\mathrm{c}}}{2}} |T|^2_{x=0^+}\,\mathrm{d}x\right) = \frac{l_{\mathrm{c}}^3 k^2}{\nu(1+l_{\mathrm{c}}^2k^2)} \\ =& \frac{m_{\mathrm{e}} l_{\mathrm{c}}}{\hbar k}\frac{l_{\mathrm{c}}^2k^2}{(1+l_{\mathrm{c}}^2k^2)} = \frac{c_1}{\xi}\left(\frac{c_2\sqrt{E_{\mathrm{e}}}}{\xi^2 + c_2E_{\mathrm{e}}}\right).\end{aligned} \tag{15}$$

As shown in Eq.(15), the dwell time τ_{D} is close to zero as $k \to \infty$, (or $E_{\mathrm{e}} \to \infty$). In contrast, as $k \to 0$, (or $E_{\mathrm{e}} \to 0$), τ_{D} is also close to zero. In this case there exist no tunnelling phenomena actually.

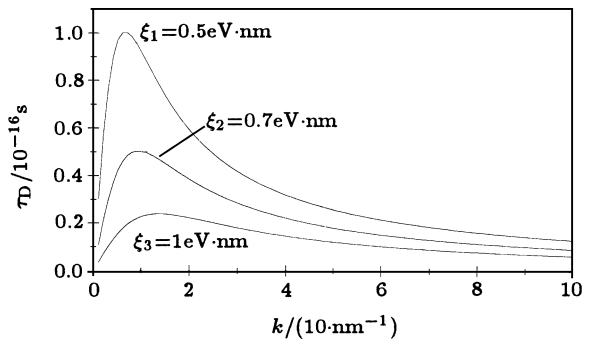


Fig.4. Electron dwell time τ_{D} versus k for ξ=0.5eV·nm, 0.7eV·nm and 1eV·nm, respectively.

Figure 4 shows the curves of the dwell time τ_{D} versus k for the given delta barrier parameter ξ. Note that τ_{D} has the maximum $\frac{m_{\mathrm{e}}}{2\hbar}l_{\mathrm{c}}^2$ at $1/k = l_{\mathrm{c}}$ and $1/k$ is the de Broglie wavelength $\lambda\!\!\!^{-}_{\mathrm{DB}}$. We can see the maximum value decreases and its position shifts to the side of small k as the delta barrier parameter ξ increases. For example, τ_{D} reaches the maxima 1.005×10^{-16}s, 0.5130×10^{-16}s and 0.2513×10^{-16}s as k is equal to $1/l_{\mathrm{c}}$ at ξ=0.5, 0.7 and 1eV·nm, respectively.

The results of the analysis on τ^{p} and τ_{D} prompt three conclusions:

First, τ^{p} approaches infinity as $k \to 0$ (or $E_{\mathrm{e}} \to 0$). In this case, the delta barrier is close to being opaque. In contrast, τ^{p} is close to zero as $k \to \infty$ (or $E_{\mathrm{e}} \to \infty$). In other words, the delta barrier is close to being transparent.

Second, the dwell time τ_{D} takes a maximum value as $\lambda\!\!\!^{-}_{\mathrm{DB}} = l_{\mathrm{c}}$. Possibly this is the reason that the resonance absorption of incident electron wave takes place within the delta barrier as $\lambda\!\!\!^{-}_{\mathrm{DB}} = l_{\mathrm{c}}$. And τ_{D}=0 as $k \to 0(E_{\mathrm{e}} = 0)$ or $k \to \infty(E_{\mathrm{e}} = \infty)$ from Eq.(15).

Third, the phase time τ^{p} of the reflected wave is equal to the dwell time τ_{D} of the incident wave through the delta barrier as $\lambda_{\mathrm{DB}}=1/k = l_{\mathrm{c}}$. In that time, $E_{\mathrm{e}} = E_{\mathrm{c}}/2$.

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