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Spin-polarized transport through a double Aharonov–Bohm ring in the presence of magnetic impurity

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The Aharonov–Bohm (AB) interference oscillations manifested through transmission of an electron in a mesoscopic device with a double-ring in the presence of a magnetic impurity serving as the spin-flipper are studied. The spin-flipper is demonstrated to indeed cause the partial decoherence of the AB interference. Moreover, it is found that the spin-up/down transmission coefficients are asymmetric in the flux reversal while the reflection coefficients as well as that the total transmission coefficient are symmetric.

Keywords: spin-polarized transport, Aharonov–Bohm ring, magnetic impurity atom
PACC: 7290, 7210, 0560, 0365

1. Introduction

In recent times, the quantum transport in an open mesoscopic system has become an interesting area due to the progress in nanofabrication. A mesoscopic system is considered as the structures made of metallic or semiconducting materials on the nanometre scale. The scale of length associated with the system dimensions is much smaller than that of the inelastic mean free path or phase breaking length of electrons. One of the main characteristics of a mesoscopic system is the phase coherence of electron throughout the sample which may lead to several interesting and unexpected quantum effects at extremely low temperatures. One of the most interesting mesoscopic devices is the mesoscopic ring. Büttiker studied the properties of a mesoscopic ring coupled to a reservoir theoretically in Ref.[1]. The reservoir acts as a source of electrons, and the carriers emerging from the reservoir do not remember their earlier history. All the scattering processes in the lead and loop are assumed to be elastic, and inelastic process occurs only in the reservoir. Hence there exists a complete spatial separation between the elastic and the inelastic processes. Büttiker and co-workers then predicted the existence of equilibrium persistent-current in an ideal one-dimensional ring in the presence of magnetic flux.[2] Since then, the persistent current in a ring threaded by magnetic flux has been regarded as a phase-sensitive probe of an equilibrium state in a conductor.[3,4] The properties of persistent current change dramatically due to the coherent charge-transfer from mesoscopic ring into other coupled mesoscopic structures, such as wire[5,6] and a quantum dot.[4,7] Recently the study of mesoscopic ring threaded by an AB magnetic flux in the centre of the ring and a magnetic impurity in one arm of the ring shows many properties which contradict the naive expectation.[8–11] The charge transport through a contacted open double-ring structure has been also studied,[12] and it is shown that in a particular case the current behaviour in one arm of the double ring can be the same as in a single ring system. Spin-polarized current in layered materials and in a conducting ferromagnet[13] has been also studied. In the present paper we consider the contacted open double-ring structure with a magnetic impurity located on one arm of the double ring. The magnetic impurity induces spin-polarized transport, which has not been explored.

2. Theoretical treatment

The two connected mesoscopic open rings are threaded by magnetic flux ϕ, and a magnetic impurity atom as a spin-flipper is inserted in the upper arm of the left of the AB ring as depicted in Fig.1

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where the magnetic impurity atom is marked by the symbol $\times$. The electron spin (σ) and the spin of the flipper (S) are coupled via the exchange interaction $-J\sigma \cdot S\delta\,(x-\lambda_1)$.[14] This leads to a scattering of the electron in which the spin states of the electron and the impurity can be changed without exchange of their energies. We consider an impurity capable of flipping M different discrete internal spin states. During passage of the electrons through the double AB ring, the total spin ($\sigma + S$) and its z-component remain conserved. We analyse the spin-up/down and the total transmission (reflection) coefficients with the incident electron spin polarized in the up-direction. We find that spin-up/down and the total reflection coefficients are symmetric in flux reversal whereas spin-up/down transmission coefficients are asymmetric in flux reversal for two cases, i.e. the equal and unequal circumferences of the two AB rings. We also find that the total transmission coefficient which is the sum of spin-up and spin-down transmission coefficients is symmetric in the flux reversal in the above two cases as expected.

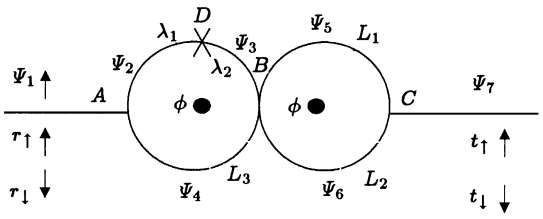


Fig.1. Schematic diagram of an ideal one-dimensional double-ring connected via a point with the impurity located in the upper arm of the left ring threaded by the magnetic flux ϕ.

We adopt the quantum waveguide theory approach,[15,16] and the spin degree of freedom of the electron is treated in the same way as in Ref.[17]. The main problem of the quantum waveguide theory is the boundary conditions both at an intersection crossed by more than two circuits and on the impurity site. The continuity of the wavefunction at the intersection requires

$$\Psi_1 = \Psi_2 = \cdots = \Psi_N, \tag{1}$$

where Ψ_i $(i = 1, 2, \cdots N)$ is the wavefunction in the ith circuit. The conservation of the current (Kirchoff's law) gives rise to

$$\sum_i \frac{\partial \Psi_i}{\partial x_i} = 0 \tag{2}$$

for all the coordinates x's pointing to or pointing back to the intersection. If there is impurity scattering, the continuity of the wavefunction is guaranteed on the impurity site, while the derivative of the wavefunction is discontinuous and has the relation of

$$\frac{\partial \Psi_2}{\partial x}(x = x_i) - \frac{\partial \Psi_1}{\partial x}(x = x_i) = u\,\Psi_1(x = x_i) \tag{3}$$

which is obtained for the δ potential scattering. Here u is the amplitude of the δ potential, x_i is the impurity position and Ψ_1 and Ψ_2 are the incident and outgoing wavefunctions on the impurity site respectively.

We consider the left-incident spin-up electron. An electron spin may or may not be flipped by the spin-flipper. The wavefunctions for the individual segments are seen from Refs.[6–8, 10, 11, 14] to be

$$\begin{aligned}
\Psi_1 &= \left(\mathrm{e}^{\mathrm{i}kx} + r_\uparrow \mathrm{e}^{-\mathrm{i}kx}\right)\chi_m\alpha + r_\downarrow \mathrm{e}^{-\mathrm{i}kx}\chi_{m+1}\beta,\\
\Psi_2 &= \left(A_{1\uparrow}\mathrm{e}^{\mathrm{i}k_1 x} + B_{1\uparrow}\mathrm{e}^{-\mathrm{i}k_2 x}\right)\chi_m\alpha\\
&\quad + \left(A_{1\downarrow}\mathrm{e}^{\mathrm{i}k_1 x} + B_{1\downarrow}\mathrm{e}^{-\mathrm{i}k_2 x}\right)\chi_{m+1}\beta,\\
\Psi_3 &= \left(C_{1\uparrow}\mathrm{e}^{\mathrm{i}k_1 x} + D_{1\uparrow}\mathrm{e}^{-\mathrm{i}k_2 x}\right)\chi_m\alpha\\
&\quad + \left(C_{1\downarrow}\mathrm{e}^{\mathrm{i}k_1 x} + D_{1\downarrow}\mathrm{e}^{-\mathrm{i}k_2 x}\right)\chi_{m+1}\beta,\\
\Psi_4 &= \left(E_{1\uparrow}\mathrm{e}^{\mathrm{i}k_2 x} + F_{1\uparrow}\mathrm{e}^{-\mathrm{i}k_1 x}\right)\chi_m\alpha\\
&\quad + \left(E_{1\downarrow}\mathrm{e}^{\mathrm{i}k_2 x} + F_{1\downarrow}\mathrm{e}^{-\mathrm{i}k_1 x}\right)\chi_{m+1}\beta,\\
\Psi_5 &= \left(A_{2\uparrow}\mathrm{e}^{\mathrm{i}k_1 x} + B_{2\uparrow}\mathrm{e}^{-\mathrm{i}k_2 x}\right)\chi_m\alpha\\
&\quad + \left(A_{2\downarrow}\mathrm{e}^{\mathrm{i}k_1 x} + B_{2\downarrow}\mathrm{e}^{-\mathrm{i}k_2 x}\right)\chi_{m+1}\beta,\\
\Psi_6 &= \left(C_{2\uparrow}\mathrm{e}^{\mathrm{i}k_2 x} + D_{2\uparrow}\mathrm{e}^{-\mathrm{i}k_1 x}\right)\chi_m\alpha\\
&\quad + \left(C_{2\downarrow}\mathrm{e}^{\mathrm{i}k_2 x} + D_{2\downarrow}\mathrm{e}^{-\mathrm{i}k_1 x}\right)\chi_{m+1}\beta,\\
\Psi_7 &= t_\uparrow \mathrm{e}^{\mathrm{i}kx}\chi_m\alpha + t_\downarrow \mathrm{e}^{\mathrm{i}kx}\chi_{m+1}\beta,
\end{aligned} \tag{4}$$

where $k_1 = k + \dfrac{2\pi\phi}{\phi_0 L}$ and $k_2 = k - \dfrac{2\pi\phi}{\phi_0 L}$ with k being the wavevector of incident electron. $r_{\uparrow(\downarrow)}$ and $t_{\uparrow(\downarrow)}$ denote the spin-up (down) reflection and transmission amplitudes. α, β are the corresponding spinors of the up and down spin states of the electron respectively, i.e. $\sigma_z\alpha = \dfrac{1}{2}\alpha$ and $\sigma_z\beta = -\dfrac{1}{2}\beta$. χ_m represents the spinor state of the impurity[17] i.e. $S_z\chi_m = m\chi_m$.

The boundary conditions, i.e. Eqs.(1)–(3) at the points A, B, C and D (see Fig.1) for the wavefunctions (4) can be written as

$$\begin{aligned}
\Psi_1|_{x=0} &= \Psi_2|_{x=0} = \Psi_3|_{x=0},\\
\Psi_1'|_{x=0} &= \Psi_2'|_{x=0} + \Psi_3'|_{x=0},\\
\Psi_2|_{x=\lambda_1} &= \Psi_3|_{x=0},\\
\Psi_3'|_{x=0} - \Psi_2'|_{x=\lambda_1} &= \Omega\,(\sigma \cdot S)\,\Psi_3|_{x=0},\\
\Psi_3|_{x=\lambda_2} &= \Psi_4|_{x=L_3} = \Psi_5|_{x=0} = \Psi_6|_{x=0},\\
\Psi_3'|_{x=\lambda_2} + \Psi_4'|_{x=L_3} &= \Psi_5'|_{x=0} + \Psi_6'|_{x=0},\\
\Psi_5|_{x=L_1} &= \Psi_7|_{x=0} = \Psi_6|_{x=L_2},\\
\Psi_5'|_{x=L_1} + \Psi_6'|_{x=L_2} &= \Psi_7'|_{x=0}.
\end{aligned} \tag{5}$$

Here we set $\hbar = 2m^* = 1$ and thus $\Omega = 2m^*J/\hbar^2 = J$ is the coupling constant indicating the

strength of the spin-exchange interaction. The primes in Eq.(5) denote the spatial derivatives of the wavefunctions. Equation (4) along with the boundary conditions (5) can be solved to obtain the amplitudes $r_{\uparrow(\downarrow)}$ and $t_{\uparrow(\downarrow)}$. Since the analytical formulae are tedious, in the following we confine ourselves to the graphical interpretation of the results. We have taken the flipper to be a spin-half object ($M = 2$) situated in the upper arm of the left ring. When the initial state of the spin-flipper is the same as that of the incident electron $\left(\sigma_z = \frac{1}{2}, m = \frac{1}{2}\right)$ there is no spin-flipper scattering, otherwise we have the possibility of spin-flip scattering. In the case of no spin-flip scattering the problem reduces to that of simple potential scattering from the impurity. The parameters used for the numerical analysis are indicated in the figure captions.

3. Results and discussions

We assume that the flipper is situated symmetrically at the centre of the upper arm of the left of the AB ring, i.e. $\lambda_1 = \lambda_2$. It is worth noting that owing to the presence of spin degree of freedom the problem under consideration although it is one-dimensional becomes a multichannel one. In Fig.2 we plot the spin-up reflection coefficient $R_\uparrow = |r_\uparrow|^2$, spin-down reflection coefficient $R_\downarrow = |r_\downarrow|^2$ and total reflection coefficient $R = R_\uparrow + R_\downarrow$ as a function of the magnetic flux parameter $\eta = \frac{\phi}{\phi_0}$ for the equal circumference of the left and right AB rings. We find the AB oscillations with a flux periodicity of $2\pi\phi_0$. The three reflection amplitudes are all symmetric in the flux reversal as expected. Figure 3 shows the spin-up transmission coefficient $T_\uparrow = |t_\uparrow|^2$, spin-down transmission coefficient $T_\downarrow = |t_\downarrow|^2$ and total transmission coefficient $T = T_\uparrow + T_\downarrow$ versus $\frac{\phi}{\phi_0}$ for the equal circumference of two AB rings. These transmission coefficients also show AB oscillations with a flux periodicity of $2\pi\phi_0$. We see that total transmission coefficient is symmetric in the flux reversal while spin-up/down transmission coefficient is asymmetric. We have also verified these behaviours of the reflection and transmission coefficients for different values of the incident wave vector k of the electron and impurity strength Ω. In Fig.4 and Fig.6, we have plotted the spin-up reflection coefficient $R_\uparrow = |r_\uparrow|^2$, spin-down reflection coefficient $R_\downarrow = |r_\downarrow|^2$ and total reflection coefficient $R = R_\uparrow + R_\downarrow$ as a function of the magnetic flux $\frac{\phi}{\phi_0}$ for both cases that the circumference of the left ring is longer and shorter than that of the right ring respectively. The plots of the variations of the spin-up transmission coefficient $T_\uparrow$, spin-down transmission coefficient $T_\downarrow$ and total transmission coefficient $T = T_\uparrow + T_\downarrow$ with the flux $\frac{\phi}{\phi_0}$ are shown in Fig.5 and Fig.7 when the circumference of the left ring is longer and shorter than that of the right ring respectively. All reflection coefficients and total transmission coefficient are symmetric while spin-up/down transmission coefficients are asymmetric about the flux, but in the case of unequal circumference of two rings the period of the AB oscillations becomes bigger than that of the AB oscillations with equal circumference.

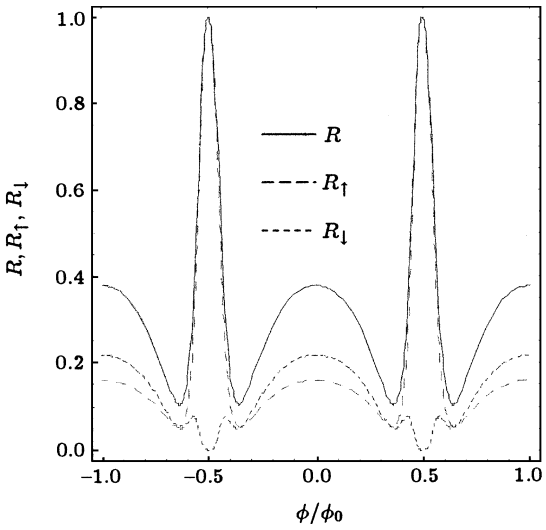


Fig.2. R, $R_\uparrow$ and $R_\downarrow$ for the spin-flip scattering case with parameters $kL_1 = kL_2 = kL_3 = 0.5\pi$, $k\lambda_1 = k\lambda_2 = 0.25\pi$, $\frac{\Omega}{k} = 3$.

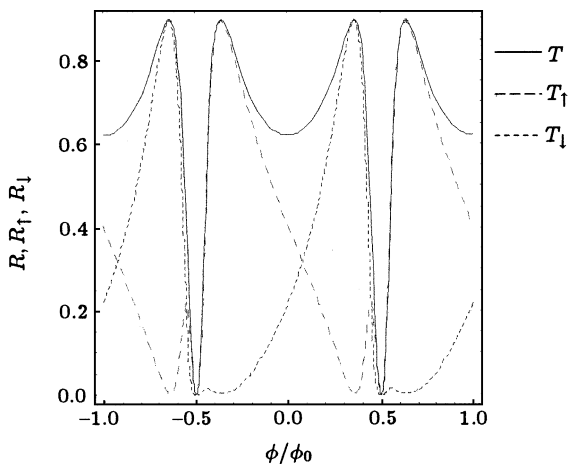


Fig.3. T, $T_\uparrow$ and $T_\downarrow$ for the spin-flip scattering case with parameters $kL_1 = kL_2 = kL_3 = 0.5\pi$, $k\lambda_1 = k\lambda_2 = 0.25\pi$, $\frac{\Omega}{k} = 3$.

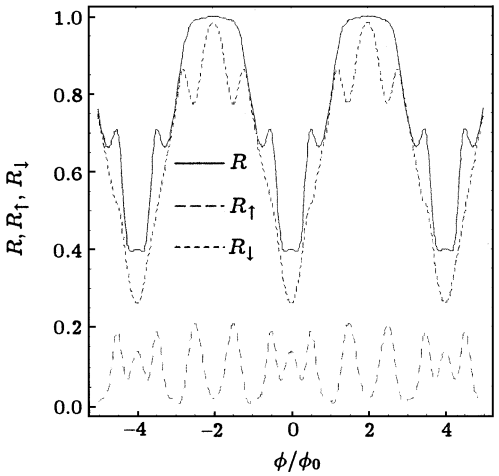


Fig.4. The variations of R, $R_\uparrow$ and $R_\downarrow$ with the flux $\frac{\phi}{\phi_0}$ for the spin-flip scattering case with $kL_1 = kL_2 = 0.25$, $kL_3 = 0.5\pi$, $k\lambda_1 = k\lambda_2 = 0.25\pi$, $\frac{\Omega}{k} = 3$.

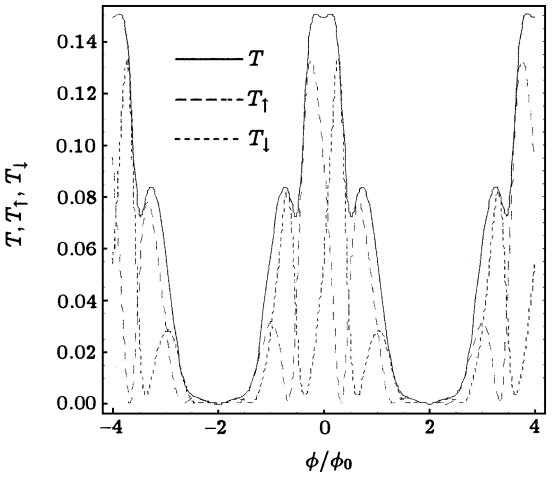


Fig.5. The variations T, $T_\uparrow$ and $T_\downarrow$ with the flux $\frac{\phi}{\phi_0}$ for the spin-flip scattering case with $kL_1 = kL_2 = 0.25$, $kL_3 = 0.5\pi$, $k\lambda_1 = k\lambda_2 = 0.25\pi$, $\frac{\Omega}{k} = 3$.

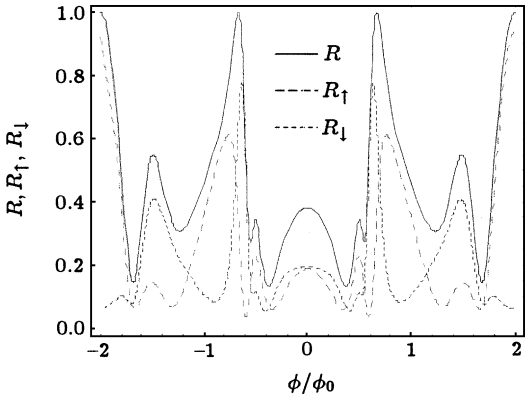


Fig.6. R, $R_\uparrow$ and $R_\downarrow$ for the spin-flip scattering case. The parameters are $kL_1 = kL_2 = 0.75$, $kL_3 = 0.5\pi$, $k\lambda_1 = k\lambda_2 = 0.25\pi$, $\frac{\Omega}{k} = 3$.

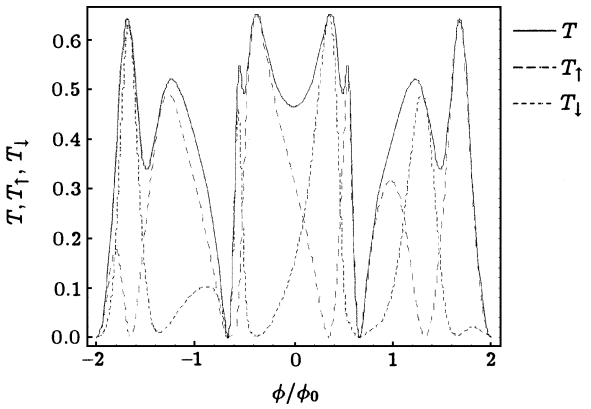


Fig.7. T, $T_\uparrow$ and $T_\downarrow$ for the spin-flip scattering case. The parameters are $kL_1 = kL_2 = 0.75$, $kL_3 = 0.5\pi$, $k\lambda_1 = k\lambda_2 = 0.25\pi$, $\frac{\Omega}{k} = 3$.

As discussed in Ref.[10], due to multiple reflections the presence of a magnetic impurity does not lead to "which-path" information that would imply the complete blocking of spin-down transmission. We find the partial loss of interference due to the spin-flipper. In Fig.8 we show the relationships of the total transmission coefficients $T = T_\uparrow + T_\downarrow$ for the spin-flip scattering (SFS) case and $T = T_\uparrow$ ($T_\downarrow = 0$) for the non-spin-flip scattering (NSFS) case versus the interaction strength Ω with the flux $\frac{\phi}{\phi_0} = 2$. It is interesting to note that the amplitude of AB oscillations for the spin-flip scattering is always smaller than that for the non-spin-flip case for all non-zero values of coupling strength Ω. This means that the partial decoherence exists.

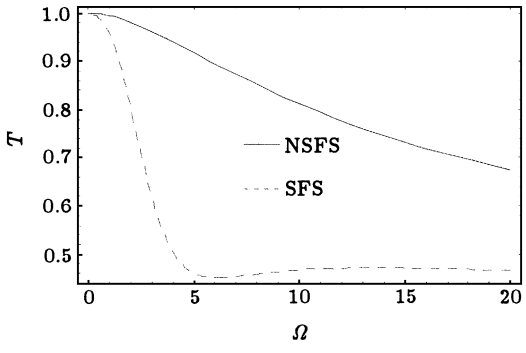


Fig.8. Variations of oscillation amplitude of T with Ω for the two cases of SPS and NSPS with parameters $kL_1 = kL_2 = kL_3 = 0.5\pi$, $k\lambda_1 = k\lambda_2 = 0.25\pi$, $\phi = 2\phi_0$.

4. Conclusion

We have studied the effect of magnetic impurity scattering on the quantum transport through a double

AB ring in the presence of spin-flip with the equal and unequal circumferences of the two rings respectively. The spin-flipper makes spin-up/down transmission coefficients asymmetric in flux reversal whereas all the reflection coefficients and the total transmission coefficient symmetric. Moreover, the magnetic impurity indeed leads to the partial decoherence. Our research may have a potential application in the design of mesoscopic devices.

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