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Eigenstates and eigenergies of seven bosonic modes mixing models*

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We present the explicit analytical results of all the eigenstates and eigenvalues by using a parameter λ without the assumption of Bethe ansatz for three different kinds of seven bosonic modes mixing models. It is shown that the parameter is determined by the roots of a simple polynomial. Besides, we also accurately obtain the explicit analytical expressions of infinite eigenstates and energies without any unknown parameter.

Keywords: spectra, eigenstate, eigenenergy
PACC: 4250, 4265M

1. Introduction

With recent advances in Bose–Einstein condensation (BEC),[1−5] interaction between several bosonic modes, such as the four-wave mixing of atoms, has been studied in depth.[6] Great efforts have been devoted to constructing the nonlinear quantized models to numerate the energy spectra and eigenstates of these nonlinear processes.[7−15] Usually one uses the Bethe ansatz,[14,15] in which several Bethe parameters are utilized to give the shapes of energy eigenstates, to research the spectra. But the reduced equations for Bethe parameters are extremely complicated to solve analytically or numerically. Besides having the significance in spectra, eigenstates also play an important role in the field of quantum information and quantum computation with the operation of state.[16] It is a worthy task to take a novel approach to effectively obtaining both spectra and eigenstates for different completely quantized nonlinear models.

In this paper, we have expanded the effective approach, which was originally developed by Wu and Yang[17−20] and Wu *et al*[21] to deal with eigenstates and eigenenergies of seven bosonic modes. Our approach is an effective method to study the spectra and eigenstates without the assumption of the Bethe ansatz. The following simple forms of eigenstates and spectra display the efficacy of our approach.

2. Model and solution

Here we show that our approach is also powerful in analytically obtaining eigenstates and eigenergies for the fully quantized seven bosonic mode mixing models[17−21] in which $H = H_0 + gH_1$, where $H_0 = \sum_{j=1}^{7} \omega_j a_j^\dagger a_j$, and

$$H_1 = a_1^\dagger a_2 a_3 a_4 a_5 a_6 a_7 + a_7^\dagger a_6^\dagger a_5^\dagger a_4^\dagger a_3^\dagger a_2^\dagger a_1,$$

if

$$\omega_1 = \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 \qquad (1a)$$

$$H_1 = a_1^\dagger a_2 a_3^\dagger a_4 a_5 a_6 a_7 + a_7^\dagger a_6^\dagger a_5^\dagger a_4^\dagger a_3 a_2^\dagger a_1,$$

if

$$\omega_1 + \omega_3 = \omega_2 + \omega_4 + \omega_5 + \omega_6 + \omega_7, \qquad (1b)$$

$$H_1 = a_1^\dagger a_2 a_3^\dagger a_4 a_5^\dagger a_6 a_7^\dagger + a_7 a_6^\dagger a_5 a_4^\dagger a_3 a_2^\dagger a_1,$$

if

$$\omega_1 + \omega_3 + \omega_5 + \omega_7 = \omega_2 + \omega_4 + \omega_6, \qquad (1c)$$

g represents seven bosonic modes coupling, ω_j, a_j and $a_j^\dagger$ denote the frequency, the annihilation and the creation operators for the jth bosonic mode respectively. Clearly, models (1a), (1b) and (1c) can be processed

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with the same procedure. We shall first concentrate on eigenergies and eigenstates for model (1a), and then we will summarize the results of models (1b) and (1c).

First, we note that the eight operators of free Hamiltonian H_0, interacting Hamiltonian H_1 in Eq.(1a), $M = a_1^\dagger a_1 + a_2^\dagger a_2$, $N = a_1^\dagger a_1 + a_3^\dagger a_3$, $L = a_1^\dagger a_1 + a_4^\dagger a_4$, $K = a_1^\dagger a_1 + a_5^\dagger a_5$, $J = a_1^\dagger a_1 + a_6^\dagger a_6$, and $Z = a_1^\dagger a_1 + a_7^\dagger a_7$ commute mutually with each other and $H_0 = \omega_2 M + \omega_3 N + \omega_4 L + \omega_5 K + \omega_6 J + \omega_7 Z$. Consequently, the six operators M, N, L, K, J, and Z are conserved quantities and the operators M, N, L, K, J, Z and H_1 in Eq.(1a) have a complete set of common eigenvectors. It is now obvious that the eigenvalue equation $H|\psi_{M,N,L,K,J,Z;\lambda}\rangle = E_{M,N,L,K,J,Z;\lambda}|\psi_{M,N,L,K,J,Z;\lambda}\rangle$ can be expressed in the following form:

$$\begin{aligned} E_{M,N,L,K,J,Z;\lambda} =& M\omega_2 + N\omega_3 + L\omega_4 \\ &+ K\omega_5 + J\omega_6 + Z\omega_7 + g\lambda \end{aligned}$$

$$H_1|\psi_{M,N,L,K,J,Z;\lambda}\rangle = \lambda|\psi_{M,N,L,K,J,Z;\lambda}\rangle, \quad (2)$$

where $M, N, L, K, J, Z = 0, 1, 2, \cdots$. And by using the method suggested by Wu *et al*,[21] Eq.(2) can be reduced into an operator-differential equation, with energy eigenstates denoted as

$$|\psi_{M,N,L,K,J,Z;\lambda}\rangle = S(a_1^\dagger, a_2^\dagger, a_3^\dagger, a_4^\dagger, a_5^\dagger, a_6^\dagger, a_7^\dagger)|\text{vac}\rangle,$$

$$\begin{aligned} &S(x_1, x_2, x_3, x_4, x_5, x_6, x_7) \\ =&\sum_{j=0}^{q} \frac{\alpha_j}{j!} x_1^j x_2^{M-j} x_3^{N-j} x_4^{L-j} x_5^{K-j} x_6^{J-j} x_7^{Z-j}, \end{aligned} \quad (3)$$

with $|\text{vac}\rangle$ representing the vacuum state and $x_j \equiv a_j^\dagger (j = 1, 2, 3, 4, 5, 6, 7)$, $q = \min(M, N, L, K, J, Z)$. It is easy to find $H_1|\psi\rangle = H_1 S|\text{vac}\rangle = [H_1, S]|\psi\rangle$. We can see from Eq.(3) that the polynomial S is the function of the creation operators and it is independent of the annihilation operators. Noting this, we can acquire the following relation:

$$\begin{aligned} [H_1, S] =& a_1^\dagger [a_2 a_3 a_4 a_5 a_6 a_7, S] + a_7^\dagger a_6^\dagger a_5^\dagger a_4^\dagger a_3^\dagger a_2^\dagger [a_1, S] \\ =& a_1^\dagger \frac{\partial^6}{\partial a_2^\dagger \partial a_3^\dagger \partial a_4^\dagger \partial a_5^\dagger \partial a_6^\dagger \partial a_7^\dagger} + a_7^\dagger a_6^\dagger a_5^\dagger a_4^\dagger a_3^\dagger a_2^\dagger \frac{\partial}{\partial a_1^\dagger}. \end{aligned}$$

The polynomial S in Eq.(3) satisfies the following operator type differential equation:

$$\left(x_1 \frac{\partial^6}{\partial x_2 \partial x_3 \partial x_4 \partial x_5 \partial x_6 \partial x_7} + x_2 x_3 x_4 x_5 x_6 x_7 \frac{\partial}{\partial x_1} \right) S = \lambda S. \quad (4)$$

Equations (2) and (3) result in the following iterating equations:

$$\begin{aligned} a_{j+1} =& \lambda a_j - p_{j-1} a_{j-1}, 0 \le j \le q, \\ q =& \min(M, N, L, K, J, Z), \end{aligned} \quad (5a)$$

where $\alpha_{-1} = \alpha_{q+1} = 0$, and

$$\begin{aligned} p_j =& (j+1)(M-j)(N-j)(L-j) \\ &\times (K-j)(J-j)(Z-j). \end{aligned} \quad (5b)$$

Now the problem is to solve Eq.(5) as mentioned in Refs.[17–21]. We will first give a solution to Eq.(5) and then prove it. The solution is

$$\alpha_j = \alpha_0 \det A^{(j)}(\lambda), \quad j = 0, 1, 2, \cdots, q,$$

$$q = \min(M, N, L, K, J, Z), \quad (6)$$

with the parameter λ determined by

$$\det A^{(q+1)}(\lambda) = 0, \quad q = \min(M, N, L, K, J, Z), \quad (7)$$

where $\det A^{(0)}(\lambda) \equiv 1$, $A^{(1)}(\lambda) \equiv \lambda$. $A^{(k)}(\lambda)$ $(k \ge 2)$ is a $k \times k$ trigonal matrix with the matrix elements $A_{ij}^{(n)}(\lambda) = \lambda\delta_{i,j} - \delta_{i+1,j} - p_j \delta_{i,j+1}$, $(i, j = 0, 1, \cdots, k-1)$. Expanding Eq.(7), we can obtain the explicit form of $\det A^{(n)}$ as follows:

$$\begin{aligned} \det A^{(n)}(\lambda) =& \lambda^n + \sum_{k=1}^{\text{int}(n/2)} (-1)^k \lambda^{n-2k} \\ &\times \Bigg(\sum_{i_1=2(k-1)}^{n-2} \cdots \sum_{i_1=2(k-m)}^{i_{m-1}-2} \cdots \\ &\sum_{i_k=0}^{i_{k-1}-2} p_{i_1} \cdots p_{i_m} \cdots p_{i_k} \Bigg) \end{aligned} \quad (8)$$

with $n \ge 2$, $\text{int}(z)$ denoting the z integer part, and p_j referring to the one in Eq.(5b).

Then we can obtain the results of eigenenergies $E \equiv E_{M,N,L,K,J,Z}$ and eigenstates $|\psi\rangle \equiv |\psi_{M,N,L,K,J,Z;\lambda}\rangle$ by using Eqs.(2), (3) and (6) and noting the Fock states $|n, m, k, s, l, v, u\rangle \equiv (\sqrt{n!m!k!s!l!v!u!})^{-1} a_1^{\dagger n} a_2^{\dagger m} a_3^{\dagger k} a_4^{\dagger s} a_5^{\dagger l} a_6^{\dagger v} a_7^{\dagger u}|\text{vac}\rangle$ as

$$E = \omega_2 M + \omega_3 N + \omega_4 L + \omega_5 K + \omega_6 J + \omega_7 Z + g\lambda, \quad (9a)$$

$$|\psi\rangle = \sum_{j=0}^{\min(M,N,K,L,J,Z)} c_j |j, M-j, N-j, L-j, K-j, J-j, Z-j\rangle,$$

$$c_j = c_0\sqrt{\frac{(M-j)!(N-j)!(L-j)!(K-j)!(J-j)!(Z-j)!}{j!M!N!L!K!J!Z!}}\det A^{(j)}(\lambda), \tag{9b}$$

where $c_0 = \sqrt{M!N!L!K!J!Z!}\alpha_0$ is a normalization constant and $M, N, L, K, J, Z = 0, 1, 2, \cdots$.

Equations (8) and (9) give explicitly the analytical expressions of all the energy eigenvalues and eigenstates by utilizing the parameter λ for model (1a). It is also simple to determine λ from Eq.(7). Comparing Eq.(7) with Eq.(8), we conclude that λ is determined by the equation $W_k(\lambda^2) = 0 (\lambda G_k(\lambda^2) = 0)$ for an odd(even)-integer number M with $k = \text{int}[(q+1)/2]$. Here $W_k(x)$ and $G_k(x)$ are polynomials of order k for variable x. The last step towards obtaining λ is to solve the equation $G_k(x) = 0$ with $\lambda = 0, \pm\sqrt{x}$ for an even-integer number q or solve $W_k(x) = 0$ with $\lambda = \pm\sqrt{x}$ for an odd-integer number q. This implies that we only need solve a polynomial of order $\text{int}[(q+1)/2]$. By using λ and Eq.(9), it is easy to obtain the explicit analytical expressions of all the energy eigenvalues and eigenstates for model (1a).

Equations (5)–(7) demonstrate that $\lambda = 0$ is a solution to Eq.(6) only when $q = \min(M, N, L, K, J, Z)$ is a non-negative even-integer number, and $\lambda = 0$ leads to $\alpha_{2m-1} = 0$ and $\alpha_{2m} = (-1)^m p_{2(m-1)} p_{2(m-2)} \cdots p_0 \alpha_0$, with $m = 1, 2, \cdots$. Consequently, Eq.(6) under the condition $\lambda = 0$ becomes

$$E = \omega_2 M + \omega_3 N + \omega_4 L + \omega_5 K + \omega_6 J + \omega_7 Z, \tag{10a}$$

$$|\psi\rangle = \sum_{n=0}^{q/2} (-1)^n \eta_n |2n, M-2n, N-2n, L-2n, K-2n, J-2n, Z-2n\rangle,$$

$$\eta_n = C\sqrt{\frac{(2n)!(M-2n)!(N-2n)!(L-2n)!(K-2n)!(J-2n)!(Z-2n)!}{(M-2n)!!(N-2n)!!(L-2n)!!(K-2n)!!(J-2n)!!(Z-2n)!!}}, \tag{10b}$$

where $M, N, L, K, J, Z = q, q+1, \cdots$, with $q = \min(M, N, L, K, J, Z)$ being a non-negative even-integer number, C being a normalization constant, $(2m+1)!! = (2m+1)(2m-1)\cdots 1$ and $(2m)!! = 2m(2m-2)\cdots 2 \equiv 2^m m!$. It is pointed out that Eq.(9b) gives all the possible eigenstates with $\lambda = 0$ corresponding to the situation where there exist no interactions among the seven modes.

The corresponding results for models (1b) and (1c) are given below without detailed derivation. The eigenenergies and eigenstates for model (1b) respectively are

$$E_{M,N,L,K,J,Z;\lambda} = M\omega_2 + N\omega_3 + L\omega_4 + K\omega_5 + J\omega_6 + Z\omega_7 + g\lambda, \tag{11a}$$

$$|\psi_{M,N,L,K,J,Z;\lambda}\rangle = S(a_1^\dagger, a_2^\dagger, a_3^\dagger, a_4^\dagger, a_5^\dagger, a_6^\dagger, a_7^\dagger)|\text{vac}\rangle, \tag{11b}$$

$$S(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = \sum_{j=0}^{q} \frac{\alpha_j}{j!} x_1^j x_2^{M-j} x_3^{N+j} x_4^{L-j} x_5^{K-j} x_6^{J-j} x_7^{Z-j}, \tag{11c}$$

where $S = S(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$ satisfies the differential equation

$$\left(x_1 x_3 \frac{\partial^5}{\partial x_2 \partial x_4 \partial x_5 \partial x_6 \partial x_7} + x_2 x_4 x_5 x_6 x_7 \frac{\partial^2}{\partial x_1 \partial x_3}\right) S = \lambda S, \tag{12}$$

and $M, N, L, K, J, Z = 0, 1, 2, \cdots$ are the eigenvalues of the operators $M = a_1^\dagger a_1 + a_2^\dagger a_2$, $N = a_3^\dagger a_3 - a_1^\dagger a_1$, $L = a_1^\dagger a_1 + a_4^\dagger a_4$, $K = a_1^\dagger a_1 + a_5^\dagger a_5$, $J = a_1^\dagger a_1 + a_6^\dagger a_6$, $Z = a_1^\dagger a_1 + a_7^\dagger a_7$ respectively, $x_j \equiv a_j^\dagger (j = 1, 2, 3, 4, 5, 6, 7)$, and $q = \min(M, L, K, J, Z)$. Here we have chosen $n_3 \geq n_1 (n_j$ denotes the eigenvalue of $a_j^\dagger a_j$ or the occupying number of mode $j)$ without loss of any generality, because model (1b) is symmetric in changing the modes' labelling of modes 1 and 3. For model (1b), we can obtain Eqs.(4)–(9) if we use $q = \min(M, L, K, J, Z)$ and

$$p_j = (j+1)(j+1+N)(M-j)(L-j) \times (K-j)(J-j)(Z-j)$$

for these equations and with the replacements $N - j \to N + j$ in Eqs.(8b) and (9b). And then the eigenenergies and eigenstates for model (1c) are

$$E_{M,N,L,K,J,Z;\lambda} = M\omega_2 + N\omega_3 + L\omega_4 + K\omega_5 + J\omega_6 + Z\omega_7 + g\lambda, \tag{13a}$$

$$|\psi_{M,N,L,K,J,Z;\lambda}\rangle = S(a_1^\dagger, a_2^\dagger, a_3^\dagger, a_4^\dagger, a_5^\dagger, a_6^\dagger, a_7^\dagger)|\text{vac}\rangle, \tag{13b}$$

$$S(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = \sum_{j=0}^{q} \frac{\alpha_j}{j!} x_1^j x_2^{M-j} x_3^{N+j} x_4^{L-j} x_5^{K+j} x_6^{J-j} x_7^{Z+j}, \tag{13c}$$

where $S = S(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$ satisfies the differential equation

$$\left(x_1x_3x_5x_7\frac{\partial^3}{\partial x_2\partial x_4\partial x_6} + x_2x_4x_6\frac{\partial^4}{\partial x_1\partial x_3\partial x_5\partial x_7}\right)S = \lambda S, \qquad (14)$$

and $M, N, L, K, J, Z = 0, 1, 2, \cdots$ are the eigenvalues of the operators $M = a_1^\dagger a_1 + a_2^\dagger a_2$, $N = a_3^\dagger a_3 - a_1^\dagger a_1$, $L = a_1^\dagger a_1 + a_4^\dagger a_4$, $K = a_5^\dagger a_5 - a_1^\dagger a_1$, $J = a_1^\dagger a_1 + a_6^\dagger a_6$, $Z = a_1^\dagger a_1 - a_7^\dagger a_7$ respectively, $x_j \equiv a_j^\dagger (j = 1, 2, 3, 4, 5, 6, 7)$, and $q = \min(M, L, J)$. Here we have chosen $n_3 \geq n_1$, $n_5 \geq n_1$, $n_7 \geq n_1$ (n_j denotes the eigenvalue of $a_j^\dagger a_j$ or the occupying number of mode j). For model (1c), Eqs.(4)–(9) still hold true if we use $q = \min(M, L, J)$ and $p_j = (j+1)(j+1+N)(j+1+K)(j+1+Z)(M-j)(L-j)(J-j)$, for these equations and with the replacements $N - j \to N + j$, $K - j \to K + j$ and $Z - j \to Z + j$ in Eqs.(8b) and (9b).

To determine the unknown parameter λ by finding the roots of a polynomial of order int$[(q+1)/2]$ implies that λ's explicit analytical expressions for $q = \min(M, N, L, K, J, Z) \leq 9$ can readily be obtained by a simple Mathematica code for models (1a), (1b) and (1c). Here we only list the explicit analytical expressions of model (1a) for $q \leq 2$ (there are $(q+1)\lambda$ values for a given q. Q, R, S, T and Y represent five largest members among six non-negative integers M, N, L, K, J, Z):

$$q = 0 \Rightarrow \lambda = 0; q = 1 \Rightarrow \lambda = \pm\sqrt{QRSTY}, \qquad (15a)$$

$$q = 2 \Rightarrow \lambda = 0, \pm\sqrt{2(Q-1)(R-1)(S-1)(T-1)(Y-1) + 2QRSTY}. \qquad (15b)$$

We can obtain analytically and explicitly all the energy eigenvalues and eigenstates of model (1a) without any unknown parameter. It is emphasized that Q, R, S, T and Y (the five largest members among six non-negative integers M, N, L, K, J, Z) can take infinite values $Q, R, S, T, Y = q, q+1, q+2, \cdots$ in these analytical results, and hence they actually represent infinite eigenstates and eigenenergies even for $q \leq 9$. For $q > 9$, the parameter λ can be determined numerically.

3. Summary and conclusions

In summary, we have numerated the explicit analytical expressions of all the energy eigenvalues and eigenstates in terms of a parameter λ without the assumption of Bethe ansatz for three different kinds of seven bosonic mode mixing models. It is shown that the parameter λ is determined by finding, either analytically or numerically, the roots of a polynomial of order int$[(q+1)/2]$ while the analytical expression of the polynomial has been explicitly obtained. Obviously this procedure of determining the parameter λ is simple compared with solving the very complicated reduced system of Bethe equations for the unknown Bethe parameters by use of the same model in the previous studies.[14] Besides, we have obtained the explicit analytical expressions for all the eigenstates under the condition $\lambda = 0$ and have presented λ's explicit analytical expressions corresponding to infinite number of energy eigenvalues and eigenstates for $q \leq 9$. Therefore, our approach has not only overcome all the serious drawbacks of the method with the algebraic Bethe ansatz but also obtained the above-mentioned new analytical results for the energy eigenvalues and eigenstates.

Our results generally describe models with seven bosonic modes mixed. They might be valuable for the investigation of maximum entangled pairs, quantum information and quantum calculation. They are expected to play a role in validating the applicable range of approximations such as number-conserving Bogoliubov theory. Moreover, the present effective approach is suitable for other nonlinear models of multi-wave mixing.

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References

[1] Anderson M H *et al* 1995 *Science* **269** 198
[2] Davis B *et al* 1995 *Phys. Rev. Lett.* **75** 3969
[3] Bradley C C *et al* 1995 *Phys. Rev. Lett.* **75** 1687
[4] Hu Z F, Du C G and Li S Q 2003 *Chin. Phys.* **12** 403
[5] Yin J P, Gao W J, Wang H F, Long Q and Wang Y Z 2000 *Chin. Phys.* **11** 1157
[6] Leggett A J 2001 *Rev. Mod. Phys.* **73** 307
[7] Wu S D, Zhan Z M and Jin L X 2002 *Chin. Phys.* **11** 1272

Wu S D, Chen A X and Jin L X 2003 *Acta Phys. Sin.* **52** 1630 (in Chinese)
Wu S D, Qu Z J, Zhan Z M and Jin L X 2001 *Acta Phys. Sin.* **50** 1925 (in Chinese)
Chen A X, Wu S D, Jin L X and Zhan Z M 2003 *Acta Phys. Sin.* **52** 2466 (in Chinese)

[8] Deng L, Hageley E W, Wen J, Trippenbach M, Band Y, Julienne P S, Simsarian J E, Helmerson K, Rolston S L and Philips W D 1999 *Nature* (London) **398** 218

[9] Goldstein E V and Meystre P 1999 *Phys. Rev.* A **59** 1509

[10] Wu Y, Yang X, Sun C P, Zhou X J and Wang Y Q 2000 *Phys. Rev.* A **61** 043604

[11] Villain P, Ohberg P, Santos L, Sanpera A and Lewenstein M 2001 *Phys. Rev.* A **64** 023606

[12] Heurich J, Pu H, Moore M G and Meystre P 2001 *Phys. Rev.* A **63** 033605

[13] Yang Q, Seo J T, Greekmore S, Temple D A, Ye P, Bonner C, Namkung M, Jung S S and Kim J H 2003 *Phys. Rev.* A **67** 013603

[14] Andreev V A and Ivanova O A 2001 *J. Phys.* A **35** 8587

[15] Zhou H Q, Links J, Mckenzie R H and Guan X W 2003 *J. Phys.* A **36** L113

[16] Nielsen M and Chuang I 2002 *Quantum and Quantum Communication* (Cambridge: Cambridge University Press) p101

[17] Wu Y and Yang X 2003 *Opt. Lett.* **28** 1793
Wu Y and Yang X 2004 *Opt. Lett.* **29** 15

[18] Wu Y and Yang X 2004 *J. Opt. Soc. Am.* B **21** 73

[19] Wu Y and Yang X 2003 *J. Opt.* B: *Quantum Semiclass. Opt.* **5** 414

[20] Wu Y and Yang X 2003 *Phys. Rev.* A **68** 013608

[21] Wu Y, Yang X and Xiao Y 2001 *Phys. Rev. Lett.* **86** 2200