Vol 14 No 1, January 2005
1009-1963/2005/14(01)/0137-07

Chinese Physics

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Effect of an external radiation field on the properties of the atoms and cavity field in the two-atom Tavis–Cummings model*

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(Received 26 May 2004; revised manuscript received 20 September 2004)

We study the properties of atoms and cavity field in the two-atom Tavis–Cummings model where the two atoms interact with each other and are also driven by an external classical field. We consider the special case that the cavity is initially in a coherent state. The atomic inversion, the average photons number and the Mandel parameter in the driven Tavis–Cummings model are given and analysed numerically. We pay special attention to the dynamical behaviour of the atoms and the cavity field modified by the external field.

Keywords: Tavis–Cummings model, classical driving field, dipole–dipole interaction, quantum property

PACC: 4250

1. Introduction

The Tavis–Cummings model (TCM)[1] is a theoretical model consisting of many two-level atoms coupled to a single-mode field in a lossless cavity. Since it can exhibit many nonclassical properties, much attention has been paid to it. The effects of the dipole–dipole interaction and phase-interrupting collisions between atoms were studied by Joshi,[2] and the effect of amplitude-phase multistability in the TCM was discussed by Kilin and Krinitskaya.[3] Bogoliubov generalized an exact solution of N two-level atoms interacting with one mode of the quantized electromagnetic field containing an arbitrary number of excitations.[4] Very recently, more profound researches on the TCM have been made.[5−9]

The TCM with two atoms can be defined as a "dressed-atom" with three levels in a cascade configuration.[10] Alsing *et al*[11] have pointed out that early Jaynes–Cummings model (JCM)[12] cannot provide a complete description of an experiment. We think, generally, that the TCM Hamiltonian cannot afford a description of an experiment. To investigate the "dressed-atom", it must be probed. We can do that with a classical external coherent driving field. The experiment can be fully understood by analysing the problem of the "dressed-atom" and the driving field. The vacuum Rabi splitting in the JCM by an external driving field has been observed recently.[13] Alsing *et al* have investigated the dynamic Stark effect for the driven JCM (DJCM) by the external field.[11] The dynamics of the DJCM in the special case of an initial coherent state has been studied by Dutra *et al*[14] and by Zheng and Guo[15] for an initial Fock state and the atom detuning from external field.

In this paper we study the dynamics of the driven two-atom Tavis–Cummings model (DTCM) where the two atoms are driven by an external field and the cavity field is initially in a coherent state. A thorough understanding of the dynamical evolution of the TCM has obvious implications for the performance of quantum information processing, as well as for the understanding of fundamental quantum mechanics.[7] The

*Project supported by the National Fundamental Research Programme (Grant No 2001CB309300), Natural Science Research Fund of the Jiangsu Education Commission of China (Grant No 03KJD140240).
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organization of this paper is as follows. In Section 2 we discuss the DTCM Hamiltonian and the density matrix. In Section 3 we investigate the time evolution of the atomic inversion in DTCM in the case of an initial coherence. In Section 4 the influences of the external radiation field on the properties of the cavity field in the DTCM are studied. Finally, remarks and conclusions are given in Section 5.

2. The density matrix of the DTCM

Consider the DTCM where two identical two-level atoms are in a single-mode high-Q cavity. The dipole–dipole interaction of the atoms can not be neglected when the distance of the atoms is less than the wavelength in the cavity.[2] The atoms in the DTCM are also driven by an external coherent field which is incident on them through the open side of the cavity. Since the coupling between the cavity mode and the atoms is much larger than that between the external field and the atoms, we can ignore spontaneous emission in the external driving field and treat the external coherent field as a classical field over the time period of interest here.[14] The Hamiltonian of the DTCM under the dipole approximation and rotating wave approximation in the Schrödinger picture is

$$H = H_0 + V + V_{\rm e}, \tag{1}$$

where H_0 is the Hamiltonian for the two bare atoms and the cavity field (we assume $\hbar = 1$),

$$H_0 = \omega a^+ a + \frac{1}{2}\omega_0(\sigma_{z1} + \sigma_{z2}), \tag{2}$$

and V is the Hamiltonian for the interaction between the atoms and the cavity field and the dipole–dipole interaction between the atoms,

$$V = g\sum_{i=1}^{2}(a^+\sigma_i^- + \sigma_i^+ a) + \lambda(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-), \tag{3}$$

and $V_{\rm e}$ is that for the interaction with the driving field,

$$V_{\rm e} = g_{\rm e}\sum_{i=1}^{2}(\sigma_i^+\varepsilon {\rm e}^{-{\rm i}\omega_L t} + \sigma_i^-\varepsilon^* {\rm e}^{{\rm i}\omega_L t}), \tag{4}$$

a^+ and a are creation and annihilation operators for the cavity mode; σ_{zi}, σ_i^+, σ_i^- are atomic pseudospin operators, ε is the amplitude of the driving field; g is the atom–cavity-field coupling coefficient, and $g_{\rm e}$ is the atom–external-field coupling coefficient, λ is the dipole–dipole interaction coefficient; ω and ω_L are the frequencies of the cavity mode and the driving field, respectively; ω_0 is the atomic transition frequency. We discuss the situation, $\omega_L = \omega_0 = \omega$. The interaction picture Hamiltonian is

$$\begin{aligned} V^{\rm I} =& {\rm e}^{{\rm i}H_0 t}(V + V_{\rm e}){\rm e}^{-{\rm i}H_0 t} \\ =& g\sum_{i=1}^{2}(a^+\sigma_i^- + \sigma_i^+ a) + \lambda(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-) \\ &+ g_{\rm e}\sum_{i=1}^{2}(\sigma_i^+\varepsilon + \sigma_i^-\varepsilon^*). \end{aligned} \tag{5}$$

We can perform the unitary transformation:[14]

$$\begin{aligned} & D(\beta)V^{\rm I}D^+(\beta) \\ =& g\sum_{i=1}^{2}(a^+\sigma_i^- + \sigma_i^+ a) + \lambda(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-) = V, \end{aligned} \tag{6}$$

where $D(\beta) = \exp(\beta a^+ - \beta^* a)$ is the displace operator,[10] and $\beta = \varepsilon g_{\rm e}/g$. The Schrödinger equation in the interaction picture for the DTCM is

$${\rm i}\frac{\partial\,|\Phi(t)\rangle}{\partial t} = V^{\rm I}\,|\Phi(t)\rangle\,. \tag{7}$$

Because $V^{\rm I}$ is time independent, the state of the system at time t is

$$|\Phi(t)\rangle = U_{\rm D}(t)\,|\Phi(0)\rangle\,, \tag{8}$$

where $U_{\rm D}(t) = {\rm e}^{-{\rm i}V^{\rm I}t}$ is the time evolution operator for the DTCM in the interaction picture. Using Eq.(6) we can obtain $U_{\rm D}(t) = D^+(\beta)U(t)D(\beta)$, where

$$U(t) = {\rm e}^{-{\rm i}Vt} \tag{9}$$

is the time evolution operator for the TCM in the interaction picture. Equation (8) can be rewritten as

$$|\Phi(t)\rangle = D^+(\beta)U(t)D(\beta)\,|\Phi(0)\rangle\,. \tag{10}$$

The density matrix at time t is

$$\rho(t) = |\Phi(t)\rangle\langle\Phi(t)| = D^+(\beta)\rho_{\rm D}(t)D(\beta), \tag{11}$$

where $\rho_{\rm D}(t)$ is the displaced TCM density matrix,

$$\rho_{\rm D}(t) = U(t)D(\beta)\rho(0)D^+(\beta)U^+(t), \tag{12}$$

and the density matrix at time t=0 is

$$\rho(0) = |\Phi(0)\rangle\langle\Phi(0)|\,. \tag{13}$$

3. The time evolution of the atomic inversion in the DTCM

The atomic inversion in the DTCM is the difference between the probabilities of the two atoms being all in the excited state $|e_1,e_2\rangle$ and being all in the ground state $|g_1,g_2\rangle$,

$$W(t)=\frac{1}{2}\langle\sigma_z\rangle=\frac{1}{2}\mathrm{Tr}[\rho(t)\sigma_z]=\frac{1}{2}\mathrm{Tr}[\rho_{\mathrm{D}}(t)\sigma_z], \quad (14)$$

where we have used the equation $\mathrm{Tr}(AB)=\mathrm{Tr}(BA)$; and the pseudospin operator of the two atoms is

$$\sigma_z=\sigma_{z1}+\sigma_{z2}. \quad (15)$$

Considering the two identical atoms, we can use three swap-symmetry wavefunctions of the atoms as base vectors,

$$\begin{aligned}|u_1\rangle&=|e_1,e_2\rangle,\\ |u_2\rangle&=\frac{1}{\sqrt{2}}(|g_1,e_2\rangle+|e_1,g_2\rangle),\\ |u_3\rangle&=|g_1,g_2\rangle,\end{aligned} \quad (16)$$

where $|e_i\rangle$ and $|g_i\rangle$ (i=1, 2) are the excited and ground states of the ith atom, respectively. If the atoms are initially in the excited state and the cavity field in the coherent state, $|\Phi(0)\rangle=|u_1\rangle\otimes|\alpha\rangle$, then Eq.(12) can be rewritten as

$$\rho_{\mathrm{D}}(t)=U(t)D(\beta)|u_1\rangle|\alpha\rangle\langle\alpha|\langle u_1|D^+(\beta)U^+(t).$$

Using the equations

$$D(\beta)|\alpha\rangle=\exp[-(\alpha\beta^*-\alpha^*\beta)/2]|\alpha+\beta\rangle,$$

and

$$\rho_{\mathrm{D}}(t)=U(t)|u_1\rangle|\alpha+\beta\rangle\langle\alpha+\beta|\langle u_1|U^+(t), \quad (17)$$

we have

$$\begin{aligned}W(t)=&\frac{1}{2}\sum_{n=0}^{\infty}\sum_{j=1}^{3}\langle n|\langle u_j|\rho_{\mathrm{D}}(t)\sigma_z|u_j\rangle|n\rangle\\ =&\sum_{n=0}^{\infty}[\langle n|\langle u_1|\rho_{\mathrm{D}}(t)|u_1\rangle|n\rangle\\ &-\langle n|\langle u_3|\rho_{\mathrm{D}}(t)|u_3\rangle|n\rangle],\end{aligned} \quad (18)$$

where we have used the equations $\sigma_z|u_1\rangle=2|u_1\rangle$, $\sigma_z|u_2\rangle=0$, $\sigma_z|u_3\rangle=-2|u_3\rangle$, and

$$\langle n|\langle u_1|\rho_{\mathrm{D}}(t)|u_1\rangle|n\rangle=|\langle n|\langle u_1|U(t)|u_1\rangle|\alpha+\beta\rangle|^2, \quad (19)$$

$$\langle n|\langle u_3|\rho_{\mathrm{D}}(t)|u_3\rangle|n\rangle=|\langle n|\langle u_3|U(t)|u_1\rangle|\alpha+\beta\rangle|^2. \quad (20)$$

Expanding the coherent state $|\alpha+\beta\rangle$ in terms of the Fock state $|m\rangle$, we can obtain

$$\begin{aligned}&U(t)|u_1\rangle|\alpha+\beta\rangle\\ =&\mathrm{e}^{-|\alpha+\beta|^2/2}\sum_{m=0}^{\infty}\frac{(\alpha+\beta)^m}{\sqrt{m!}}U(t)|u_1\rangle|m\rangle.\end{aligned} \quad (21)$$

$U(t)|u_1\rangle|m\rangle$ is the state of the normal TCM at time t in the case of an initial state $|u_1\rangle|m\rangle$. It can be given by the Schrödinger equation in the interaction picture,[16]

$$\begin{aligned}U(t)|u_1\rangle|m\rangle=&D_1^m|u_1\rangle|m\rangle+D_2^m|u_2\rangle|m+1\rangle\\ &+D_3^m|u_3\rangle|m+2\rangle,\end{aligned} \quad (22)$$

where

$$\begin{aligned}D_1^m=&\frac{m+2}{2m+3}-\frac{m+1}{2(2m+3)\delta_m}\\ &\times(E_3^m\mathrm{e}^{-\mathrm{i}E_2^mt}-E_2^m\mathrm{e}^{-\mathrm{i}E_3^mt}),\end{aligned} \quad (23)$$

$$D_2^m=\frac{\sqrt{m+1}g}{\sqrt{2}\delta_m}(\mathrm{e}^{-\mathrm{i}E_2^mt}-\mathrm{e}^{-\mathrm{i}E_3^mt}), \quad (24)$$

$$\begin{aligned}D_3^m=&-\frac{\sqrt{m^2+3m+2}}{2m+3}\\ &\times\left[1+\frac{1}{2\delta_m}(E_3^m\mathrm{e}^{-\mathrm{i}E_2^mt}-E_2^m\mathrm{e}^{-\mathrm{i}E_3^mt})\right],\end{aligned} \quad (25)$$

$$\begin{aligned}E_{2,3}^m=&\frac{\lambda}{2}\pm\delta_m,\\ \delta_m=&\frac{1}{2}\sqrt{\lambda^2+8(2m+3)g^2},\quad m=0,1,2,\cdots.\end{aligned} \quad (26)$$

By substituting Eqs.(22) and (21) into Eqs.(19) and (20), we have

$$\begin{aligned}\langle n|\langle u_1|\rho_{\mathrm{D}}(t)|u_1\rangle|n\rangle=&\mathrm{e}^{-|\alpha+\beta|^2}\frac{|\alpha+\beta|^{2n}}{n!}|D_1^n|^2,\\ &n\geq 0;\end{aligned} \quad (27)$$

$$\begin{aligned}\langle n|\langle u_3|\rho_{\mathrm{D}}(t)|u_3\rangle|n\rangle=&\mathrm{e}^{-|\alpha+\beta|^2}\frac{|\alpha+\beta|^{2(n-2)}}{(n-2)!}|D_3^{n-2}|^2,\\ &n\geq 2;\end{aligned} \quad (28)$$

$$\langle n|\langle u_3|\rho_{\mathrm{D}}(t)|u_3\rangle|n\rangle=0,\quad 0\leq n<2;$$

then we have

$$\begin{aligned}W(t)=&\mathrm{e}^{-|\alpha+\beta|^2}\left\{\sum_{n=0}^{\infty}\left(\frac{|\alpha+\beta|^{2n}}{n!}|D_1^n|^2\right)\right.\\ &\left.-\sum_{n=2}^{\infty}\left[\frac{|\alpha+\beta|^{2(n-2)}}{(n-2)!}|D_3^{n-2}|^2\right]\right\}\\ =&\mathrm{e}^{-|\alpha+\beta|^2}\sum_{n=0}^{\infty}\frac{|\alpha+\beta|^{2n}}{n!}(|D_1^n|^2-|D_3^n|^2).\end{aligned} \quad (29)$$

If the external field vanishes ($\beta=0$), we can obtain the atomic inversion in the normal TCM in the case of an initial coherent state. We can see that the effect of the external field on the atomic inversion in

the DTCM is the initial coherent state which is displaced from α to $\alpha+\beta$.[14] Figure 1 shows the time evolution of the atomic inversion in the DTCM under different external fields or different initial coherent states of the cavity. From the plot we can see that with increasing $\alpha+\beta$, the collapse time decreases, thus the Rabi oscillations become fast and the amplitude of the super-revival decreases.

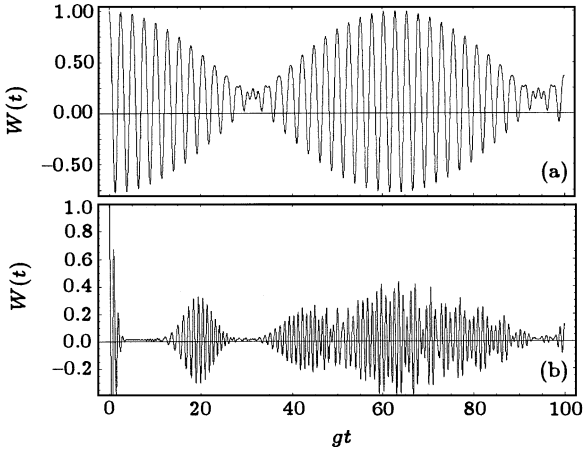


Fig.1. The evolution of the atomic inversion in the driven TCM for the two atoms initially in the exited state and cavity in the coherent state. The parameters are (λ/g=0.1): (a) $\alpha+\beta$=0 and (b) $\alpha+\beta$=3.

4. The effect of external field on properties of the cavity field in the DTCM

4.1. Average photon number

From Eqs.(11) and (12), the average photon number in the cavity field in the DTCM can be evaluated,

$$\begin{aligned}\langle n(t)\rangle =&\mathrm{Tr}[\rho(t)a^{+}a]\\ =&\langle n(t)\rangle_{\mathrm{D}}+|\beta|^{2}-[\beta^{*}\langle a(t)\rangle_{\mathrm{D}}+\mathrm{c.c.}],\end{aligned}\quad(30)$$

where $\langle n(t)\rangle_{\mathrm{D}}$ is the average photon number in the TCM with a displaced initial state $|\alpha+\beta\rangle$,

$$\langle n(t)\rangle_{\mathrm{D}}=\mathrm{Tr}[\rho_{\mathrm{D}}(t)a^{+}a],\quad \langle a(t)\rangle_{\mathrm{D}}=\mathrm{Tr}[\rho_{\mathrm{D}}(t)a].\quad(31)$$

Because of the $\langle a(t)\rangle_{\mathrm{D}}$ term, the dynamics of $\langle n(t)\rangle$ is strikingly different from its counterpart in the TCM. The average photon number in the cavity field in the TCM is

$$\langle n(t)\rangle_{\mathrm{T}}=\mathrm{Tr}[\rho_{\mathrm{T}}(t)a^{+}a],\quad(32)$$

where $\rho_{\mathrm{T}}(t)$ is the density matrix in the TCM,

$$\rho_{\mathrm{T}}(t)=U(t)\rho(0)U^{+}(t)=U(t)\,|u_1\rangle\,|\alpha\rangle\,\langle\alpha|\,\langle u_1|\,U^{+}(t).\quad(33)$$

Evaluating Eq.(32), we obtain

$$\begin{aligned}\langle n(t)\rangle_{\mathrm{T}}=&\mathrm{e}^{-|\alpha|^{2}}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}(n\,|D_1^n|^{2}+(n+1)\,|D_2^n|^{2}\\ &+(n+2)\,|D_3^n|^{2}).\end{aligned}\quad(34)$$

So the mean value of a in the TCM is

$$\begin{aligned}\langle a(t)\rangle_{\mathrm{T}}=&\mathrm{Tr}[\rho_{\mathrm{T}}(t)a]\\ =&\mathrm{e}^{-|\alpha|^{2}}\sum_{n=0}^{\infty}\frac{\alpha\,|\alpha|^{2n}}{n!}\\ &\times\left[D_1^{n+1}(D_1^n)^{*}+\sqrt{\frac{n+2}{n+1}}D_2^{n+1}(D_2^n)^{*}\right.\\ &\left.+\sqrt{\frac{n+3}{n+1}}D_3^{n+1}(D_3^n)^{*}\right].\end{aligned}\quad(35)$$

Substituting α for $\alpha+\beta$ in Eqs.(34) and (35) yields the expression of $\langle n(t)\rangle_{\mathrm{D}}$ and $\langle a(t)\rangle_{\mathrm{D}}$, and then the expression of $\langle n(t)\rangle$ from Eq.(30).

We compare the plot of $\langle n(t)\rangle_{\mathrm{T}}$ with $\langle n(t)\rangle_{\mathrm{D}}$ in Fig.2. It is shown that the value and the fast oscillation frequency of the $\langle n(t)\rangle_{\mathrm{D}}$ are bigger than those of $\langle n(t)\rangle_{\mathrm{T}}$, but the amplitudes of the super-revival of $\langle n(t)\rangle_{\mathrm{D}}$ are smaller than those of $\langle n(t)\rangle_{\mathrm{T}}$.

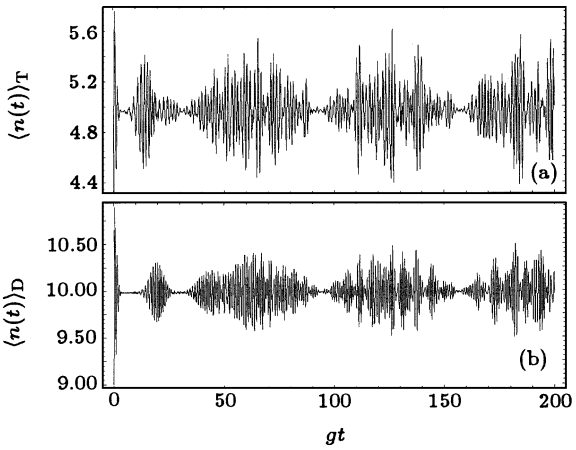


Fig.2. (a) The mean value of n in the TCM for λ/g=0.1 and α=2, and (b) the mean value of n in the TCM with a displaced initial state $|\alpha+\beta\rangle$ for λ/g=0.1, α= 2, β=1.

Figure 3 illustrates the average photon number in the cavity field in the DTCM for β=0.2, 0.5 and 1. Equation (30) shows that the average photon number in the DTCM increases with increasing external field for the $|\beta|^{2}$ term. The numerical analysis of Eq.(30) shows that, when the driving field is weak (i.e. β is small), the influence of the $\langle a(t)\rangle_{\mathrm{D}}$ term on $\langle n(t)\rangle$ is weak, and $\langle n(t)\rangle$ also shows the collapses and revivals

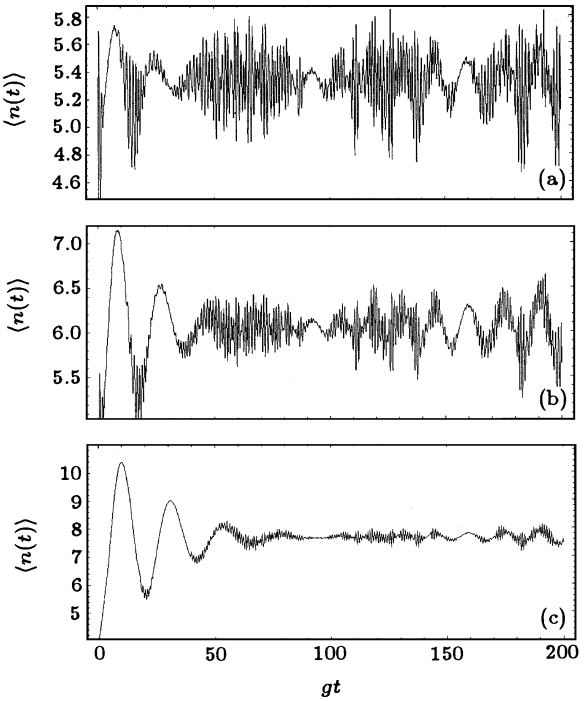


Fig.3. The mean value of n in the driven TCM for the two atoms initially in the exited state and the cavity in the coherent state.The parameters are (λ/g=0.1, α=2): (a) β=0.2, (b) β=0.5 and (c) β=1.

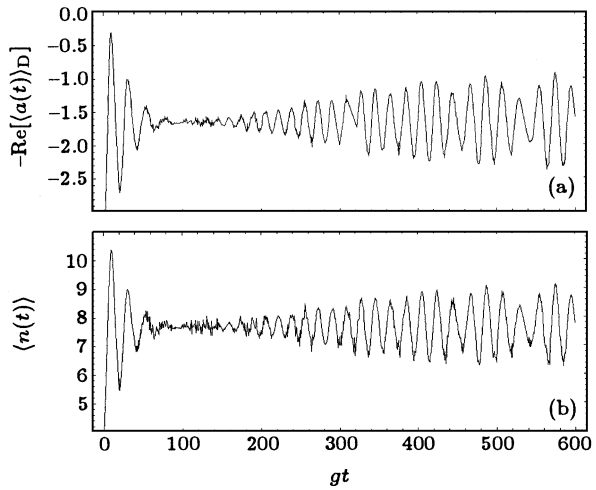


Fig.4. The plots of $-\mathrm{Re}[\langle a(t)\rangle_{\mathrm{D}}]$ (a) and $\langle n(t)\rangle$ (b). The collapses and revivals occur in a much longer time when the external field is strong. The parameters are λ/g=0.1, α=2, β=1.

at a short time scale like the TCM, except for the origin time (see Figs.3(a) and 3(b)). But the time mean value of $\langle n(t)\rangle$ is modulated by the $\langle a(t)\rangle_{\mathrm{D}}$ term. Since the amplitude of –Re[$\langle a(t)\rangle_{\mathrm{D}}$] is large at the beginning (see Fig.4(a)), the plot shows a prodigious difference between Figs.3(a) and 2(a) at the beginning. We also see that the oscillation frequency of $\langle n(t)\rangle$ decreases gradually with the increase of external field. The super-revival disappears in a short time when β=1 (see Fig.3(c)). It exhibits only in a much longer time. Figure 4(b) shows that the first super-revival happens at about gt=600, and the outlines of $\langle n(t)\rangle$ and $-\mathrm{Re}[\langle a(t)\rangle_{\mathrm{D}}]$ are alike. The external field is the main factor for the fluctuation of the photon number in the case of β=1.

4.2. The Mandel Q parameter

The normalized variance of the photon distribution is given by the Mandel Q parameter for the cavity field

$$Q(t)=\frac{\left\langle(\Delta n)^2\right\rangle-\langle n\rangle}{\langle n\rangle}=\frac{\left\langle n^2\right\rangle-\langle n\rangle^2}{\langle n\rangle}-1. \tag{36}$$

If $Q(t)>0$, the cavity field is super-Poissonian; if Q(t)=0, it is Poissonian; if $Q(t)<0$, it is sub-Poissonian. For a single-mode field, the super-Poissonian distribution is equivalent to a bunching effect, and the sub-Poissonian distribution to an anti-bunching effect. We write the expression as

$$\begin{aligned}\left\langle n^2\right\rangle =&\mathrm{Tr}[\rho n^2]=\mathrm{Tr}[\rho_{\mathrm{D}}D(\beta)n^2D^+(\beta)]\\ =&\mathrm{Tr}[\rho_{\mathrm{D}}(n-\beta^*a-\beta a^+ +|\beta|^2)^2]\\ =&\left\langle n^2\right\rangle_{\mathrm{D}}+4|\beta|^2\langle n\rangle_{\mathrm{D}}+|\beta|^4+|\beta|^2\\ &+[(\beta^{*2}\left\langle a^2\right\rangle_{\mathrm{D}}\\ &-\beta^*\langle na+an+2|\beta|^2a\rangle_{\mathrm{D}})+\mathrm{c.c.}],\end{aligned} \tag{37}$$

where $\langle M\rangle_{\mathrm{D}}=\mathrm{Tr}[\rho_{\mathrm{D}}M]$. Using $\gamma=\alpha+\beta$, we obtain

$$\begin{aligned}\left\langle n^2\right\rangle_{\mathrm{D}} =&\mathrm{e}^{-|\gamma|^2}\sum_{n=0}^{\infty}\frac{|\gamma|^{2n}}{n!}(n^2|D_1^n|^2+(n+1)^2|D_2^n|^2\\ &+(n+2)^2|D_3^n|^2),\end{aligned} \tag{38}$$

$$\begin{aligned}\langle na\rangle_{\mathrm{D}} =&\mathrm{e}^{-|\gamma|^2}\sum_{n=0}^{\infty}\frac{\gamma|\gamma|^{2n}}{n!}\Big[nD_1^{n+1}(D_1^n)^*\\ &+\sqrt{(n+2)(n+1)}D_2^{n+1}(D_2^n)^*\\ &+(n+2)\sqrt{\frac{n+3}{n+1}}D_3^{n+1}(D_3^n)^*\Big],\end{aligned} \tag{39}$$

$$\begin{aligned}\left\langle an\right\rangle_{\mathrm{D}} =&\mathrm{e}^{-|\gamma|^2}\sum_{n=0}^{\infty}\frac{\gamma|\gamma|^{2n}}{n!}\Big[(n+1)D_1^{n+1}(D_1^n)^*\\ &+(n+2)\sqrt{\frac{n+2}{n+1}}D_2^{n+1}(D_2^n)^*\\ &+(n+3)\sqrt{\frac{n+3}{n+1}}D_3^{n+1}(D_3^n)^*\Big],\end{aligned} \tag{40}$$

$$\begin{aligned}\left\langle a^2\right\rangle_{\mathrm{D}} =&\mathrm{e}^{-|\gamma|^2}\sum_{n=0}^{\infty}\frac{\gamma^2|\gamma|^{2n}}{n!}\Big[D_1^{n+2}(D_1^n)^*\\ &+\sqrt{\frac{n+3}{n+1}}D_2^{n+2}(D_2^n)^*\\ &+\sqrt{\frac{(n+4)(n+3)}{(n+2)(n+1)}}D_3^{n+2}(D_3^n)^*\Big].\end{aligned} \tag{41}$$

Thus $\langle n^2 \rangle$ is worked out, and so is $Q(t)$.

The time evolution of $Q(t)$ in the TCM is plotted in Fig.5 for λ/g=0.1. It shows that the cavity field exhibits periodically super-Poissonian, Poissonian and sub-Poissonian, i.e. it oscillates between the bunching and antibunching effects. We notice that the time evolution of the cavity field is from Poissonian to sub-Poissonian at first for the initial vacuum state (see Fig.5 (a)), but from Poissonian to super-Poissonian at first for the initial coherent state when α increases (see Fig.5 (b)). With increasing $|\alpha|$, the oscillation of $Q(t)$ becomes fast and the equilibrium position of it moves up. When $|\alpha|$ is big enough (for example, $|\alpha|$=10), the cavity field becomes strongly super-Poissonian all the time.

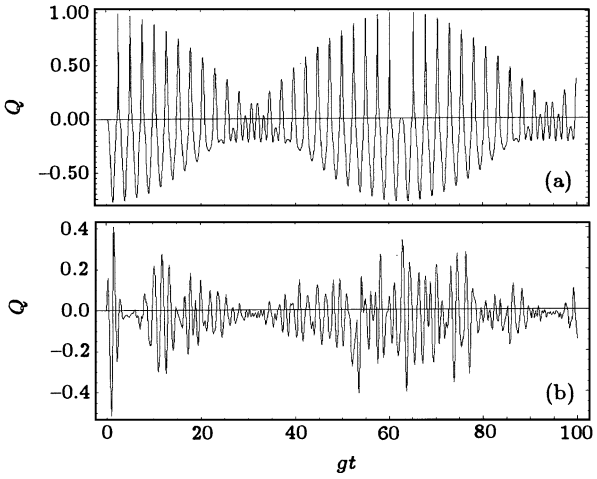


Fig.5. The Mandel Q parameter in the TCM (β=0) for the two atoms initially in the exited state. The parameters are (λ/g=0.1): (a) α=0 and (b) α=2.

In Fig.6, we plot $Q(t)$ in the DTCM for the different external fields. We notice that the influence of the external field on the photon distribution is remarkable. This makes the photon antibunching effect weak obviously (compare Fig.6 (a) with Fig.5 (b)). When β=1, the photons of the cavity field exhibit only three times weak antibunching at the beginning, then appears the bunching effect for the rest of its time evolution (see Fig.6 (b)).

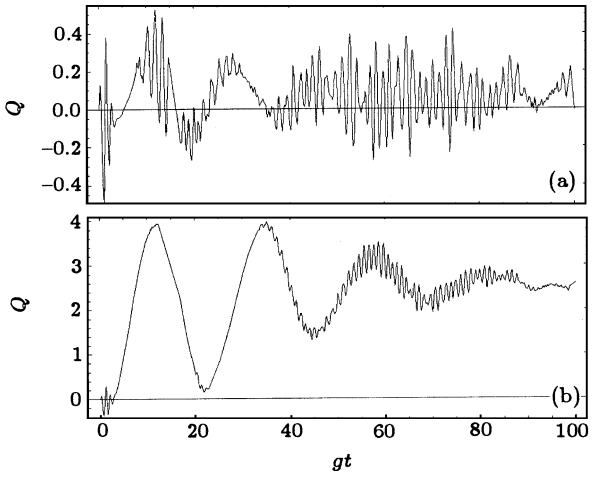


Fig.6. The Mandel Q parameter in the driven TCM for the two atoms initially in the exited state and the cavity in the coherent state. The parameters are (λ/g=0.1, α=2): (a) β=0.2 and (b) β=1.

5. Conclusions

We have investigated the dynamic features of the driven TCM where the two atoms are driven by the external field. The motivation for this study was to investigate, in particular, the influence of an external coherent field on the properties of the TCM. We restricted our analysis to the case when the cavity field is initially in the coherent state and the two atoms are both in the excited state. We have shown the time evolution of the atomic inversion in the DTCM and found that, with the external field added to the atoms, the collapse time decreases, the Rabi oscillations become fast and the amplitude of the super-revival decreases. We have noticed that, when the driving field is weak, the average photon number in the cavity field in the DTCM still shows the collapses and revivals in a short time like the TCM, except for the origin time. With increasing external field, the oscillation frequency of the average photon number decreases gradually, and the super-revival can exhibit only at a much larger time. We have also noticed that the influence of the external field on the photon distribution is remarkable. It makes the photon antibunching effect weak obviously. When the external field is strong enough, the photons exhibit only several times weaker antibunching at the beginning, then the bunching effect appears for the rest of its time evolution.

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