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Photon–magnon interaction process: a mechanism of resonance linewidths of ferromagnet*

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We consider the relaxation processes of parametric excitation of magnons in a microwave resonator. Using the approach of unitary transformation, we calculate the effective amplitude of three-boson terms of Hamiltonian, especially for photon–magnon interaction which is expected to contribute to the resonance linewidth of ferromagnets.

Keywords: magnon, unitary transformation, linewidth
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1. Introduction

Both theory and application of magnetic materials have been attracted much attention in recent years. In particular, the research of magnetic excitations and resonances, such as spin-waves (magnons), is active in the development of various devices involving medical and microwave equipment.[1–7] In the traditional point of view, the mechanism of relaxation processes (resonance linewidths) of ferromagnetic resonance has been considered as the following: three magnons, "bare" four magnons, one magnon–one phonon, two magnons–one phonon, and one magnon–two phonons. However, it should be pointed out that in the earlier parametric spin-wave theories the resonant cavity was only assumed to increase the amplitude of the microwave magnetic field and makes the experimental setup more sensitive to the absorption of microwave power.[8,9] Thus the dynamics of the photons in the cavity was ignored. Bryant *et al* first mentioned that the dynamics of the photon mode in the cavity should be taken into account.[10] Their viewpoint was experimentally verified by Yamazaki *et al*,[11] Safonov[12], Yamazaki and Mino[13] and Safonov and Yamazaki[14] have shown that the nonlinear coupling of photons and parametric magnons leads to an effective nonlinear damping in a parametric system, and given the effective Hamiltonian of a spin-wave system as follows:

$$\begin{aligned} H_e/h =& \omega_R R^+R + \sum_k \omega_k b_k^+ b_k \\ &+ \frac{1}{2}\sum_k \left(G_k R b_k^+ b_{-k}^+ + \text{c.c.}\right) \\ &+ F\left[R^+ \exp\left(-\text{i}\omega_\text{p} t\right) + \text{c.c.}\right] \\ &+ \sum_k T_k^{(R)} R^+ R b_k^+ b_k + H^{(4)}, \end{aligned} \tag{1}$$

$$H^{(4)} = \sum_{k,q} (T_{kq} b_k^+ b_k b_q^+ b_q + \frac{1}{2} S_{kq} b_k^+ b_{-k}^+ b_q b_{-q}), \tag{2}$$

where R^+, R and b_k^+, b_k are the complex variables of the resonant mode (with frequency ω_R) and spin waves, respectively; ω_k is the "bare" spin-wave frequency, G_k the nonlinear coupling parameter between photons and magnons, F the parameter of pumping field. Planck's constant $\hbar$ is used as a dimensional constant in order to have dimensionless complex variables as a classical analogue of Bose operators of creation and annihilation. So the photon–magnon interaction term of the Hamiltonian should be expected to contribute to the resonance linewidth of ferromagnetic materials. Furthermore, it is important to investigate the details of the Hamiltonian in all relaxation processes for studying the resonance linewidth. Here, we focus on the calculation of the amplitude of three-boson terms of the Hamiltonian, especially for photon–magnon interaction, by the approach of uni-

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tary transformation, and we hope that the results of this investigation might be helpful in the interpretation of experimental data obtained from the parallel pumping experiments.

2. The Hamiltonian

Let us start with the spin Hamiltonian $\widehat{H}$ which describes magnetic interactions between atoms in parametrically excited magnetic systems as follows:

$$\widehat{H} = \widehat{H}_{\rm ex} + \widehat{H}_{\rm z} + \widehat{H}_{\rm A} + \widehat{H}_{\rm dd}, \tag{3}$$

where $\widehat{H}_{\rm ex}$ is the energy of exchange spin-spin interaction, $\widehat{H}_{\rm z}$ is the Zeeman energy, $\widehat{H}_{\rm A}$ is the energy of anisotropy, and $\widehat{H}_{\rm dd}$ describes the magnetic dipole-diploe interaction energy. We use HP transformation

$$\begin{aligned} S_j^{\pm} &\Leftrightarrow \sqrt{2S}a_j - \frac{a_j^+ a_j a_j}{\sqrt{8S}}, \\ S_j^{\mp} &\Leftrightarrow \sqrt{2S}a_j^+ - \frac{a_j^+ a_j^+ a_j}{\sqrt{8S}}, \\ S_j^z &= S - a_j^+ a_j, \end{aligned}$$

for Eq.(3), which in turn are related to spin wave operators by

$$\begin{aligned} a_j &= \frac{1}{\sqrt{N}} \sum_k a_k \exp({\rm i}k \cdot r), \\ a_j^+ &= \frac{1}{\sqrt{N}} \sum_k a_k^+ \exp(-{\rm i}k \cdot r), \end{aligned}$$

where N is the number of elementary cells in the crystal. Considering the dynamics of the cavity mode in the form of $e_k c_k^+ c_k$, we have the general expression of Hamiltonian including photon–magnon interaction as follows:

$$\begin{aligned} \widehat{H} =& \sum_k (\epsilon_k b_k^+ b_k + e_k c_k^+ c_k) \\ &+ \sum_{k_1,k_2,k_3} \{\psi_1(k_1,k_2,k_3) b_{k_1}^+ b_{k_2}^+ b_{k_3} + {\rm h.c.}\} \\ &\times \Delta(k_1 + k_2 - k_3) \\ &+ \sum_{k_1,k_2,k_3} \{\psi_2(k_1,k_2,k_3) b_{k_1}^+ b_{k_2}^+ c_{k_3} + {\rm h.c.}\} \\ &\times \Delta(k_1 + k_2 - k_3) \\ &+ \frac{1}{2} \sum_{k_1,k_2,k_3,k_4} \Phi(k_1,k_2,k_3,k_4) b_{k_1}^+ b_{k_2}^+ b_{k_3} b_{k_4} \\ &\times \Delta(k_1 + k_2 - k_3 - k_4), \end{aligned} \tag{4}$$

with

$$e_k = \hbar\omega_R,$$

$$\begin{aligned} \psi_1(k_1,k_2,k_3) =& \frac{3}{2} g^2 \mu_{\rm B}^2 \frac{\sqrt{2SN}}{V_s} \Big\{ \beta_{xz} + {\rm i}\beta_{yz} \\ &- 4\pi \frac{k_{1x}k_{1y} + {\rm i}k_{1y}k_{1z}}{k_1^2} \Big\}, \end{aligned}$$

$$\psi_2(k_1,k_2,k_3) = \frac{1}{2} G(k) = {\rm i} V_k \sqrt{\frac{\pi\hbar\omega_p}{2V_R}},$$

$$\begin{aligned} \varepsilon_k =& \{[g\mu_{\rm B}(H + H_{\rm A}) + S(J_0 - J_k)] \\ &\times [g\mu_{\rm B}(H + H_{\rm A}) + S(J_0 - J_k) \\ &+ g\mu_{\rm B} 4\pi M_0 \sin^2\theta_k]\}^{\frac{1}{2}}. \end{aligned}$$

Here all terms with five and more operators are omitted. The h.c. denotes Hermitian conjugate, $\Delta(k)$ is the Kronecker delta function, $b(b^+)$ denotes the operator of magnon and $c(c^+)$ is the operator of photon. ψ_1 is the three-magnon process, ψ_2 represents the photon–magnon process, and Φ is the "bare" amplitude of four-magnon interaction. According to the S theory in which the wave pairs with equal and oppositely directed wavevectors play a definite role in the parametric excitation of waves,[8] Eq.(4) can be rewritten as

$$\begin{aligned} \widehat{H} =& \sum_k (\epsilon_k b_k^+ b_k + e_k c_k^+ c_k) \\ &+ \sum_{k_1,k_2,k_3} \{\psi_1(k_1,k_2,k_3) b_{k_1}^+ b_{k_2}^+ b_{k_3} + {\rm h.c.}\} \\ &\times \Delta(k_1 + k_2 - k_3) \\ &+ \sum_{k_1,k_2,k_3} \{\psi_2(k_1,k_2,k_3) b_{k_1}^+ b_{k_2}^+ c_{k_3} + {\rm h.c.}\} \\ &\times \Delta(k_1 + k_2 - k_3) \\ &+ \frac{1}{2} \sum_{k,q} \Phi'(k,-k,q,-q) b_k b_{-k} b_q^+ b_{-q}^+ \\ &+ \frac{1}{2} \sum_{k,q} \Phi''(k,q,k,q) b_k^+ b_q^+ b_k b_q, \end{aligned} \tag{5}$$

where

$$\begin{aligned} &\Phi'(k,-k,q,-q) \\ =& \Big\{ \frac{1}{N} J_{-k+q} + \frac{1}{4N}(J_{-q} + J_q + J_k + J_{-k}) \\ &+ g^2\mu_{\rm B}^2 \frac{1}{V_s} \Big[-\beta_{zz} + 4\pi \Big(\frac{-k_x + q_x}{-k+q} \Big)^2 - \frac{4\pi}{3} \Big] \\ &- \frac{1}{2} g^2 \mu_{\rm B}^2 \frac{1}{V_s} \Big[-\beta_{xx} - \beta_{yy} + 2\pi \Big(\frac{q_x^2 + q_y^2}{-q} \Big) \\ &+ 2\pi \Big(\frac{k_x^2 + k_y^2}{k} \Big) - \frac{8\pi}{3} \Big] + \frac{2}{N}\beta_{\rm A} \Big\}, \\ &\Phi''(k,q,k,q) \\ =& \Big\{ \frac{1}{N} J_0 + \frac{1}{4N}(J_{-q} + J_q + J_k + J_{-k}) \\ &+ g^2\mu_{\rm B}^2 \frac{1}{V_s} \Big[-\beta_{zz} + 4\pi - \frac{4\pi}{3} \Big] \\ &- \frac{1}{2} g^2 \mu_{\rm B}^2 \frac{1}{V_s} \Big[-\beta_{xx} - \beta_{yy} + 2\pi \Big(\frac{q_x^2 + q_y^2}{q} \Big) \\ &+ 2\pi \Big(\frac{k_x^2 + k_y^2}{k} \Big) - \frac{8\pi}{3} \Big] + \frac{2}{N}\beta_{\rm A} \Big\}. \end{aligned}$$

Here $H_{\rm A}$ is the field of anisotropy, $V_{\rm s}$ the volume of the sample, β the factor of depolar anisotropy, $\mu_{\rm B}$ the Bohr magneton; the angle $\theta_{\rm k}$ describes the deviation of the wavevector k from the direction of the static magnetic field, and M_0 is the saturation magnetization.

Similar to the BCS approximation in the superconductivity theory,[15] the processes of the even interaction terms can be conveniently studied after diagonalization of the Hamiltonian in general magnetic theory. So, we remove the three-boson interaction terms in Eq.(5) by means of unitary transformation and obtain the corresponding effective four-boson interaction amplitudes as follows.

3. Unitary transformation to on-site interaction

The general form of unitary transformation can be written as

$$\widetilde{H}(\theta) = \exp(\theta\check{R})H\exp(-\theta\check{R}), \tag{6}$$

where $\check{R}$ is an anti-Hermitian operator and θ a formal parameter. This expression is the solution of the following equation:

$$\frac{{\rm d}\widetilde{H}(\theta)}{{\rm d}\theta} = \left[\check{R}, \widetilde{H}(\theta)\right], \tag{7}$$

with the initial condition $\widetilde{H}(0) = H$.

We first write the most general form of $\widetilde{H}(\theta)$ as an expansion in terms of operator combinations with unknown θ-dependent coefficients, and the most general form of $\check{R}$ as the anti-Hermitian operator combinations,[16] and then substitute these expressions into Eq.(7), and thereby obtain a set of linear differential equations by comparing coefficients in analogous operator combinations. Solving these equations under the initial conditions, the transformed Hamiltonian $\widetilde{H}(\theta)$ is deduced. In order to eliminate the three-boson terms, we need to put their coefficients (for example, for θ=1) equal to zero. This condition defines the choice of $\check{R}$. Also, we omit all terms containing five and more Bose operators in $\widetilde{H}(\theta)$. Because of $\psi_1(\psi_2) \ll \epsilon_k$, the order of creation or annihilation operator in the interaction terms is not so important. For example, we change the order of operators

$$b^+_{k_1} b^+_{k_2} b_{k_3} b_{k_4} = b^+_{k_1} b_{k_3} b^+_{k_2} b_{k_4} - b^+_{k_1} b_{k_4} \Delta(k_2 - k_3),$$

and then the role of additional terms will be negligibly small compared to the "bare" quadratic terms.

It is easy to show that the result of commutation of combinations of Bose operators,

$$\left[\underbrace{b^+ \cdots b^+}_{n_1}\underbrace{b \cdots b}_{m_1}, \underbrace{b^+ \cdots b^+}_{n_2}\underbrace{b \cdots b}_{m_2}\right],$$

leads to a sum of compositions containing $n_1 + m_1 + n_2 + m_2 - 2$ Bose operators: $n_1 + n_2 - 1$ creation and $m_1 + m_2 - 1$ annihilation operators. For $n_2 = m_2 = 1$ the result of commutation contains compositions with the same numbers of creation (n_1) and annihilation (m_1) operators. This means that if we want to remove a term

$$\psi \underbrace{b^+ \cdots b^+}_{n_1}\underbrace{b \cdots b}_{m_1} + {\rm h.c.}$$

from the initial Hamiltonian, we should use the anti-Hermitian operator of the form

$$\check{R}_\psi \underbrace{b^+ \cdots b^+}_{n_1}\underbrace{b \cdots b}_{m_1} - {\rm h.c.}$$

Thus, in order to remove the three-boson term of on-site interaction $b^+_{k_1} b^+_{k_2} c_{k_3}$, we can use the anti-Hermitian operator

$$\begin{aligned}\check{R}_{\rm P-M} = \sum_{q_1,q_2,q_3} &[R_{\rm P-M}(q_1,q_2,q_3) b^+_{k_1} b^+_{k_2} c_{k_3} - {\rm h.c.}] \\ &\times \Delta(q_1 + q_2 - q_3),\end{aligned} \tag{8}$$

and inserting it into Eq.(6), we obtain

$$\begin{aligned}\widetilde{H}(\theta)_{\rm P-M} = &\sum_{k_1,k_2,k_3} \{\psi_\theta(k_1,k_2,k_3,\theta) b^+_{k_1} b^+_{k_2} c_{k_3} + {\rm h.c.}\} \\ &\times \Delta(k_1 + k_2 - k_3) \\ &+ \frac{1}{2} \sum_{k_1,k_2,k_3,k_4} \Phi_\theta(k_1,k_2,k_3,k_4,\theta) \\ &\times b^+_{k_1} b^+_{k_2} b_{k_3} b_{k_4} \\ &\times \Delta(k_1 + k_2 - k_3 - k_4).\end{aligned} \tag{9}$$

Substituting Eq.(9) into Eq.(7), we can obtain the following equations:

$$\begin{aligned}&\frac{\rm d}{{\rm d}\theta}\psi_\theta(k_1,k_2,k_3,\theta) \\ &= -(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3}) R_{\rm P-M}(k_1,k_2,k_3),\end{aligned} \tag{10}$$

$$\begin{aligned}&\frac{\rm d}{{\rm d}\theta}\Phi_\theta(k_1,k_2,k_3,k_4,\theta) \\ &= -2R_{\rm P-M}(k_1,k_2,-k_1-k_2)\psi^*_\theta(k_3,k_4,-k_3-k_4,\theta) \\ &\quad -2R_{\rm P-M}(k_3,k_4,-k_3-k_4)\psi_\theta(k_1,k_2,-k_1-k_2,\theta).\end{aligned} \tag{11}$$

Solving these equations under the initial condition $\psi_\theta(\theta = 0) = \psi_2$, $\Phi_\theta(\theta = 0) = 0$, and letting $\psi_\theta(\theta = 1) = 0$, we have

$$R_{\mathrm{P-M}}(q_1, q_2, q_3) = \frac{\psi_2(k_1, k_2, k_3)}{\epsilon_{q_1} + \epsilon_{q_2} - \epsilon_{q_3}},$$

and

$$\begin{aligned}\Phi_{\mathrm{P-M}} =& \psi_2(k_1, k_2, k_1 + k_2)\psi_2^*(k_3, k_4, k_3 + k_4) \\ &\times \left(\frac{1}{\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_1+k_2}} \right. \\ &\left. + \frac{1}{\epsilon_{k_3} + \epsilon_{k_4} - \epsilon_{k_3+k_4}} \right). \end{aligned} \tag{12}$$

So the part contributing to $\Phi'(k, -k, q, -q)$ is

$$\begin{aligned}&\Phi'_{\mathrm{P-M}}(k, -k, q, -q) \\ =& \psi_2(k, k, 0)\psi_2^*(q, -q, 0) \\ &\times \left(\frac{1}{\epsilon_k + \epsilon_k - \epsilon_0} + \frac{1}{\epsilon_q + \epsilon_q - \epsilon_0} \right), \end{aligned} \tag{13}$$

and the part contributing to $\Phi''(k, q, k, q)$ is

$$\begin{aligned}&\Phi''_{\mathrm{P-M}}(k, q, k, q) \\ =& \psi_2(k, q, k + q)\psi_2^*(k, q, k + q) \\ &\times \left(\frac{1}{\epsilon_k + \epsilon_q - \epsilon_{k+q}} + \frac{1}{\epsilon_k + \epsilon_q - \epsilon_{k+q}} \right). \end{aligned} \tag{14}$$

It can be seen from Eqs.(12) and (13) that the on-site interaction contributes to the amplitude of four-magnon scattering.

In order to compare the effective Hamiltonian (1) with the theoretical treatment (4), we have to remove the three-magnon term from Eq.(4) and calculate the corresponding effective four-magnon interaction amplitude with the same procedure as the above. Using the anti-Hermitian operator

$$\begin{aligned}\check{R}_{3\Rightarrow 4} =& \sum_{q_1,q_2,q_3} [R_{3\Rightarrow 4}(q_1, q_2, q_3) b_{k_1}^+ b_{k_2}^+ b_{k_3} - \mathrm{h.c.}] \\ &\times \Delta(q_1 + q_2 - q_3),\end{aligned}$$

the part contributing to the $\Phi'(k, -k, q, -q)$ is obtained as

$$\begin{aligned}&\Phi'_{3\Rightarrow 4}(k, -k, q, -q) \\ =& \psi_1(k, -k, 0)\psi_1^*(q, -q, 0) \\ &\times \left(\frac{1}{\epsilon_k + \epsilon_{-k} - \epsilon_0} + \frac{1}{\epsilon_q + \epsilon_{-q} - \epsilon_{q-q}} \right) \\ &\psi_1(k, q - k, q)\psi_1^*(-q, -k + q, -k) \\ &\times \left(\frac{1}{\epsilon_k + \epsilon_{q-k} - \epsilon_q} + \frac{1}{\epsilon_{-q} + \epsilon_{-k+q} - \epsilon_{-k}} \right) \\ &- \psi_1(k, -q - k, -q)\psi_1^*(q, -k - q, -k) \\ &\times \left(\frac{1}{\epsilon_k + \epsilon_{-q-k} - \epsilon_{-q}} + \frac{1}{\epsilon_q + \epsilon_{-k-q} - \epsilon_{-k}} \right) \\ &- \psi_1(-k, q + k, q)\psi_1^*(-q, k + q, k) \\ &\times \left(\frac{1}{\epsilon_{-k} + \epsilon_{q+k} - \epsilon_q} + \frac{1}{\epsilon_{-q} + \epsilon_{k+q} - \epsilon_k} \right) \\ &- \psi_1(-k, -q + k, -q)\psi_1^*(q, k - q, k) \\ &\times \left(\frac{1}{\epsilon_{-k} + \epsilon_{-q+k} - \epsilon_{-q}} + \frac{1}{\epsilon_q + \epsilon_{k-q} - \epsilon_k} \right), \end{aligned} \tag{15}$$

and the part to $\Phi''(k, q, k, q)$ is

$$\begin{aligned}&\Phi''_{3\Rightarrow 4}(k, q, k, q) \\ =& \psi_1(k, q, k + q)\psi_1^*(k, q, k + q) \\ &\times \left(\frac{1}{\epsilon_k + \epsilon_q - \epsilon_{k+q}} + \frac{1}{\epsilon_k + \epsilon_q - \epsilon_{k+q}} \right) \\ &- \psi_1(k, 0, k)\psi_1^*(q, 0, q) \\ &\times \left(\frac{1}{\epsilon_k + \epsilon_0 - \epsilon_k} + \frac{1}{\epsilon_q + \epsilon_0 - \epsilon_q} \right) \\ &- \psi_1(k, q - k, q)\psi_1^*(k, q - k, q) \\ &\times \left(\frac{1}{\epsilon_k + \epsilon_{q-k} - \epsilon_k} + \frac{1}{\epsilon_k + \epsilon_{q-k} - \epsilon_q} \right) \\ &- \psi_1(q, k - q, k)\psi_1^*(q, k - q, k) \\ &\times \left(\frac{1}{\epsilon_q + \epsilon_{k-q} - \epsilon_k} + \frac{1}{\epsilon_q + \epsilon_{k-q} - \epsilon_k} \right) \\ &- \psi_1(q, 0, q)\psi_1^*(k, 0, k) \\ &\times \left(\frac{1}{\epsilon_q + \epsilon_0 - \epsilon_q} + \frac{1}{\epsilon_k + \epsilon_0 - \epsilon_k} \right). \end{aligned} \tag{16}$$

Now we can rewrite the amplitude of the four-magnon term of Hamiltonian of parametrically excited spin wave system as follows:

$$\begin{aligned}T_{kq} =& \frac{1}{2}\Phi''(k, q, k, q) + \Phi''_{3\Rightarrow 4}(k, q, k, q) \\ &+ \Phi''_{\mathrm{P-M}}(k, q, k, q), \\ S_{kq} =& \Phi'(k, -k, q, -q) + 2\Phi''_{3\Rightarrow 4}(k, -k, q, -q) \\ &+ \Phi'_{\mathrm{P-M}}(k, -k, q, -q).\end{aligned}$$

Thus, the linewidth contribution to the four-magnon interaction consists of three parts of Hamiltonian: on-site interaction, three-magnon process and "bare" four-magnon scattering. Because the explicit amplitude of all four-boson terms has been obtained, the linewidth contribution could be calculated based on spin wave theory.

4. Conclusion

We have analysed the relaxation processes of parametric excitations of magnons in a microwave resonator, and constructed the explicit amplitude of three-boson terms of the Hamiltonian, especially for

on-site interaction, by the unitary transformation. The "bare" four-magnon scattering is not the only cause for the linewidth contribution of four-magnon, the on-site interaction is also expected to play an important role in the resonant linewidth contribution of ferromagnets.

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References

[1] Hou B H, Xu Y X, Han S Y, Yi S and Shen B G 1999 *Acta Phys. Sin.* **48** 527 (in Chinese)
[2] Liu G Q and Tsai C S 1998 *Acta Phys. Sin.* **47** 997 (in Chinese)
[3] Wang L 2000 *Chin. Phys.* **9** 685
[4] Wigen P E 1984 *Thin Solid Films* **114** 135
[5] Kivimaki A, Naves de Brito A, Aksele S, Aksele H and Sairanen O P 1993 *Phys. Rev. Lett.* **71** 4307
[6] Montag B and Reinhard P G 1995 *Phys. Rev.* B **51** 14686
[7] Tranquada J M, Lee C H, Yamada K, Lee Y S, Regnault L P and Ronnow H M 2004 *Phys. Rev.* B **69** 174507
[8] Zakharov V E, L'vov V S and Starobinets S S 1975 *Sov. Phys. Usp.* **17** 896
[9] L'vov V S 1994 *Wave Turbulence under Parametric Excition* (Berlin: Springer) p10
[10] Bryant P, Jeffries C and Nakamula K 1988 *Phys. Rev.* A **38** 4223
[11] Yamazaki H, Yunoki Y, Mino M, Mitsudo S and Safonov V L 1992 *J. Magn. Magn. Mater.* **104–107** 1059
[12] Safonov V L 1991 *J. Magn. Magn. Mater.* **97** L1
[13] Yamazaki H and Mino M 1989 *Prog. Theor. Phys.* Suppl. 400
[14] Safonov V L and Yamazaki H 1996 *J. Magn. Magn. Mater.* **161** 275
[15] Bardeen J, Cooper L N and Schrieffer J R 1957 *Phys. Rev.* **103** 1175
[16] Shi Q, Safonov V L, Mino M and Yamazaki H 1998 *Phys. Lett.* A **238** 258