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Generalized synchronization of hyperchaos and chaos using active backstepping design*

Zhang Hao(张 浩)[a)†], Ma Xi-Kui(马西奎)[a)b)],
Yang Yu(杨 宇)[a)], and Xu Cui-Dong(徐翠东)[a)]

[a)] *School of Electrical Engineering, Xi'an Jiaotong University, Xi'an* 710049, *China*

[b)] *Key Laboratory of High Voltage Engineering and Electrical New Technology under the State Ministry of Education, Chongqing University, Chongqing* 400044, *China*

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A novel active backstepping control method is presented for synchronizing two identical Rössler hyperchaotic systems with each other, and extended to achieve the generalized synchronization of the Chua chaotic system with the Rössler hyperchaotic system. It is a systematic design approach and consists of a recursive procedure interlacing the choice of a Lyapunov function with the design of active control. In particular, this technique gives flexibility in constructing a control law. Numerical experiments verify the feasibility and effectiveness of the proposed control technique.

Keywords: generalized synchronization, hyperchaos, chaos, active backstepping design
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1. Introduction

As a key technique of secret communications, chaos synchronization has attracted increasing interest in the physics, mathematics and engineering community since the early 1990s.[1] Various effective methods[2−12] have been proposed over the last decade to achieve chaos synchronization. However, these methods mainly focused on the study of synchronization of low-dimensional systems with only one positive Lyapunov exponent. This feature restricts the complexity of the chaotic dynamics and suggests the adoption of higher-dimensional chaotic systems for applications to secure communications. In fact, the presence of more than one positive Lyapunov exponent clearly improves the security by generating more complex dynamics.[13,14] Thus, hyperchaos synchronization has become a new subject of active research.

Recently, some interesting results have been reported in Refs.[13–20]. Peng *et al*[15] proposed an approach based on feedback, where the drive signal is a scalar signal that is a linear combination of the variables of the drive system. Duan and Yang[16] presented a parameter control method by use of a scalar signal for implementation of hyperchaos synchronization. Ali and Fang[17] used a scalar signal represented by a linear combination of the original state variables to achieve the synchronization of a chaotic system with a hyperchaotic one. Grassi and Mascolo[13], and Grassi and Miller[14] developed a nonlinear observer design method to synchronize a class of hyperchaotic systems with each other via a scalar signal transmitted, and proved the effectiveness of this method experimentally. More recently, Wang *et al*[18] presented a mutual coupling method to implement the hyperchaotic synchronization in an LC oscillator. Jiang *et al*[19] adopted only one variable unidirectionally coupled approach to achieve hyperchaos synchronization

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†Corresponding author. E-mail: haozhang@mail.xjtu.edu.cn

in an LC oscillator as well. Based on the technique presented by Peng *et al*[15], Duan and Yang[16], and Cheng *et al*[20] proposed a method of recombining systems by use of a linear reversible transform to synchronize hyperchaotic systems with each other. Noticeably, in the above publications regarding hyperchaos synchronization, it is in general assumed that the parameters and structures of the master hyperchaotic systems are the same as the ones of slave hyperchaotic systems. However, in practical applications the master and slave hyperchaotic systems could not be identical and they are always different in some respects. Moreover, in some situations we need achieve the synchronization of one hyperchaotic or chaotic system (receiver) with another different hyperchaotic system (transmitter). In these cases, the above methods will fail to do so.[13−20] How to implement the generalized synchronization of a hyperchaotic system with a chaotic one is now a main research objective here.

In recent years, backstepping design[21−25] and the active control scheme[9−12] have been widely recognized as two powerful design methods to control chaotic systems. Backstepping design can guarantee global stability, tracking, and transient performance for a broad class of strict-feedback nonlinear systems.[26,27] The active control technique gives the flexibility to construct a control law so that it can be used widely to control various nonlinear systems including chaotic systems. Consequently, the main aim here is to use a combination of these two control approaches, i.e. an active backstepping method, to achieve chaotic or hyperchaotic synchronization, especially generalized synchronization.

We propose a novel active controller to synchronize hyperchaos in two Rössler systems with each other, based on the backstepping design. The proposed approach is a systematic design approach and consists of a recursive procedure interlacing the choice of a Lyapunov function with the design of active control.

This paper is organized as follows. In Section 2, a brief description of the Rössler hyperchaotic system is introduced. Section 3 and Section 4 investigate the design of an active backstepping controller for the synchronization of two identical Rössler hyperchaotic systems with each other as well as the generalized synchronization of a Chua chaotic system with a Rössler hyperchaotic system. Numerical simulations are given for illustration and verification in Section 5. Finally, some concluding remarks and comments are given.

2. System description

Due to its simplicity, the Rössler system[28] has become a standard chaotic system to verify the effectiveness of chaos control strategy. In 1979, Rössler[29] proposed a four-variable oscillator, i.e. the Rössler hyperchaotic system, which contains only one nonlinear term of quadratic type and produces chaos in two directions of hyperbolic instability on a strange attractor. The Rössler hyperchaotic system[29] can be described as follows:

$$\begin{cases} \dot{x}_1 = bx_2 + cx_1, \\ \dot{x}_2 = 3 + x_2x_3, \\ \dot{x}_3 = -x_2 - x_4, \\ \dot{x}_4 = x_1 + x_3 + ax_4, \end{cases} \tag{1}$$

where $a = 0.25$, $b = -0.5$ and $c = 0.05$. This system exhibits a hyperchaotic behaviour starting from proper initial conditions[29], as shown in Fig.1. Numerical evidence indicates that the hyperchaotic attractor has two positive Lyapunov exponents, i.e. $\lambda_1 = 0.109$ and $\lambda_2 = 0.024$.

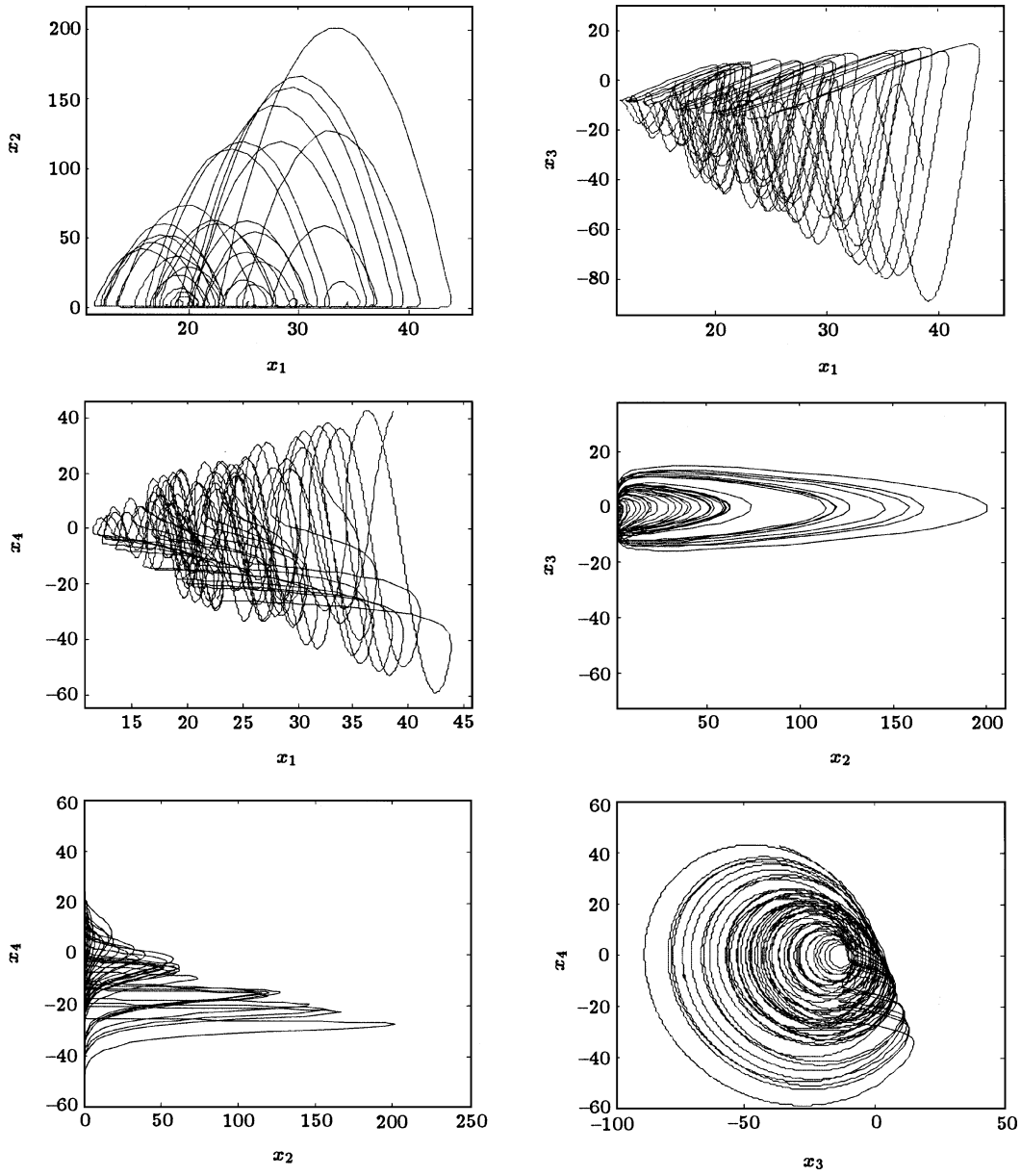


Fig.1. Typical phase portraits of Rössler hyperchaotic system.

3. Synchronization of two identical hyperchaotic systems with each other

Assume that there are two identical Rössler hyperchaotic systems playing the roles of the master system and the slave system, respectively. Here the master system is described as Eq.(1), and the slave system is given by

$$\begin{cases} \dot{y}_1 = by_2 + cy_1 + u_1, \\ \dot{y}_2 = 3 + y_2y_3 + u_2, \\ \dot{y}_3 = -y_2 - y_4 + u_3, \\ \dot{y}_4 = y_1 + y_3 + ay_4 + u_4. \end{cases} \tag{2}$$

For brevity, $x = (x_1, x_2, x_3, x_4)^{\mathrm{T}}$ denotes the state vector of the master system, $y = (y_1, y_2, y_3, y_4)^{\mathrm{T}}$ denotes the state vector of the slave system and $u(t) = (u_1, u_2, u_3, u_4)^{\mathrm{T}}$ is the control input vector.

Our goal is to design an appropriate active backstepping controller $u(x)$ such that the trajectory of the slave system asymptotically approaches the one of the master system, and finally the two systems implement their synchronization with each other in the sense that

$$\lim_{t\to\infty} \|y - x\| = 0, \tag{3}$$

where $\|\cdot\|$ is the Euclidean norm.

Referring to the active control theory,[24−27] the

subtraction of Eq.(1) from Eq.(2) now gives

$$\begin{cases} \dot{e}_1 = be_2 + ce_1 + u_1, \\ \dot{e}_2 = e_2e_3 + e_2x_3 + x_2e_3 + u_2, \\ \dot{e}_3 = -e_2 - e_4 + u_3, \\ \dot{e}_4 = e_1 + e_3 + ae_4 + u_4, \end{cases} \tag{4}$$

where $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$ and $e_4 = y_4 - x_4$. Equation (4) can be considered as a control problem with the control input $u(t)$, which is a function of the error vector e. As long as these feedbacks stabilize the system, the error vector e converges to zero as time t goes to infinity. This implies that the master system and the slave system are synchronized with each other.

The backstepping design procedure is recursive. At the ith step, the ith-order subsystem is stabilized with respect to a Lyapunov function V_i by the design of a virtual control α_i and a control input function u_i. Now we begin to design the active controller based on the backstepping design method as follows.

Step 1 Let $z_1 = e_1$, then we can obtain its derivative

$$\dot{z}_1 = \dot{e}_1 = be_2 + cz_1 + u_1, \tag{5}$$

where $e_2 = \alpha_1(z_1)$ can be regarded as a virtual controller.

For the design of $\alpha_1(z_1)$ to stabilize z_1-subsystem (5), we can choose the following Lyapunov function:

$$V_1 = \frac{1}{2}z_1^2. \tag{6}$$

Then the derivative of V_1 is

$$\dot{V}_1 = z_1\dot{z}_1 = z_1[b\alpha_1(z_1) + cz_1 + u_1]. \tag{7}$$

If we choose $\alpha_1(z_1) = -\dfrac{c+1}{b}z_1$ and $u_1 = 0$, then $\dot{V}_1$ is negative and definite. This implies that the z_1-subsystem (5) is asymptotically stable.

Since the virtual control function $\alpha_1(z_1)$ is estimative, the error between e_2 and $\alpha_1(z_1)$ is

$$z_2 = e_2 - \alpha_1(z_1). \tag{8}$$

Then, we can obtain the following (z_1, z_2)-subsystem:

$$\begin{cases} \dot{z}_1 = bz_2 - z_1, \\ \dot{z}_2 = \left(z_2 - \dfrac{c+1}{b}z_1 + x_2\right)e_3 \\ \qquad +z_2x_3 - \dfrac{c+1}{b}x_3z_1 + (c+1)z_2 \\ \qquad -\dfrac{c+1}{b}z_1 + u_2, \end{cases} \tag{9}$$

where $e_3 = \alpha_2(z_1, z_2)$ can be considered as virtual controller.

Step 2 In order to stabilize the (z_1, z_2)-subsystem (9), we can choose a Lyapunov function V_2 defined by

$$V_2 = V_1 + \frac{1}{2}z_2^2. \tag{10}$$

Its derivative is given by

$$\begin{aligned} \dot{V}_2 =& \dot{V}_1 + z_2\dot{z}_2 = -z_1^2 + bz_1z_2 + z_2\dot{z}_2 \\ =& -z_1^2 + z_2\left(z_1 - \frac{c+1}{b}z_1 + x_2\right)\alpha_2(z_1, z_2) \\ &+ z_2\Big[bz_1 + x_3z_2 + (c+1)z_2 - \frac{c+1}{b}z_1 \\ &- \frac{c+1}{b}x_3z_1 + u_2\Big]. \end{aligned} \tag{11}$$

If $\alpha_2(z_1, z_2) = 0$ and $u_2 = -\left(b - \dfrac{c+1}{b} - \dfrac{c+1}{b}x_3\right)z_1 - (c + 2 + x_3)z_2$, then $\dot{V}_2 = -z_1^2 - z_2^2 < 0$, which makes the (z_1, z_2)-subsystem (9) asymptotically stable.

Similarly, assume that $z_3 = e_3 - \alpha_2(z_1, z_2)$, then we can derive the following (z_1, z_2, z_3)-subsystem:

$$\begin{cases} \dot{z}_1 = bz_2 - z_1, \\ \dot{z}_2 = \left(z_2 - \dfrac{c+1}{b}z_1 + x_2\right)z_3 - z_2 - bz_1, \\ \dot{z}_3 = -z_2 + \dfrac{c+1}{b}z_1 - e_4 + u_3, \end{cases} \tag{12}$$

where $e_4 = \alpha_3(z_1, z_2, z_3)$ can be regarded as a virtual controller.

Step 3 We can choose a Lyapunov function V_3 defined by

$$V_3 = V_2 + \frac{1}{2}z_3^2, \tag{13}$$

in order to make the (z_1, z_2, z_3)-subsystem (12) stable.

The derivative of V_3 is

$$\begin{aligned} \dot{V}_3 =& \dot{V}_2 + z_3\dot{z}_3 \\ =& -z_1^2 - z_2^2 - z_3\alpha_3(z_1, z_2, z_3) \\ &+ z_3\left(z_1z_2 + x_2z_2 - z_2 + \frac{c+1}{b}z_1 \right. \\ &\left. - \frac{c+1}{b}z_1z_2 + u_3\right). \end{aligned} \tag{14}$$

We can choose $a_3(z_1, z_2, z_3) = 0$ and $u_3 = -\left(1 - \dfrac{c+1}{b}\right)z_1z_2 - (x_2 - 1)z_2 - \dfrac{c+1}{b}z_1 - z_3$, so that $\dot{V}_3 = -z_1^2 - z_2^2 - z_3^2 < 0$, which makes the (z_1, z_2, z_3)-subsystem (12) asymptotically stable.

Let $z_4 = e_4 - \alpha_3(z_1, z_2, z_3)$, then we can have the following (z_1, z_2, z_3, z_4)-system:

$$\begin{cases} \dot{z}_1 = bz_2 - z_1, \\ \dot{z}_2 = \left(z_2 - \dfrac{c+1}{b}z_1 + x_2\right)z_3 - z_2 - bz_1, \\ \dot{z}_3 = -x_2 z_2 - \left(1 - \dfrac{c+1}{b}\right)z_1 z_2 - z_4 - z_3, \\ \dot{z}_4 = z_1 + z_3 + az_4 + u_4. \end{cases} \tag{15}$$

Step 4 In order to stabilize the (z_1, z_2, z_3, z_4)-system (15), we can choose a Lyapunov function V_4 defined by

$$V_4 = V_3 + \frac{1}{2}z_4^2. \tag{16}$$

The derivative of V_4 is expressed as

$$\begin{aligned} \dot{V}_4 =& \dot{V}_3 + z_4\dot{z}_4 \\ =& -z_1^2 - z_2^2 - z_3^2 + z_4(az_4 + z_1 + u_4). \end{aligned} \tag{17}$$

If $u_4 = -z_1 - (a+1)z_4$, then $\dot{V}_3 = -z_1^2 - z_2^2 - z_3^2 - z_4^2 < 0$, making the (z_1, z_2, z_3, z_4)-system (15) asymptotically stable.

Finally, the fill dimension (z_1, z_2, z_3, z_4)-system is

$$\begin{cases} \dot{z}_1 = bz_2 - z_1, \\ \dot{z}_2 = \left(z_2 - \dfrac{c+1}{b}z_1 + x_2\right)z_3 - z_2 - bz_1, \\ \dot{z}_3 = -x_2 z_2 - \left(1 - \dfrac{c+1}{b}\right)z_1 z_2 - z_4 - z_3, \\ \dot{z}_4 = z_3 - z_4. \end{cases} \tag{18}$$

Since $\dot{V}_4$ is negative and definite, it follows from the LaSalle–Yoshizawa theorem[18] that in the (z_1, z_2, z_3, z_4) coordinates the equilibrium $(0, 0, 0, 0)$ is global asymptotically stable. There exist the following properties: $z_1 = e_1$, $z_2 = e_2 - \alpha_1(z_1) = e_2 + \dfrac{c+1}{b}z_1$, $z_3 = e_3 - \alpha_2(z_1, z_2) = e_3$, and $z_4 = e_4 - \alpha_3(z_1, z_2, z_3) = e_4$, and they imply that e_1, e_2, e_3 and e_4 go to zero asymptotically. In other words, the trajectory of the slave system (2) asymptotically approaches the one of the master system (1).

4. Synchronization of chaotic system with hyperchaotic system

In this section we will discuss how to choose an active control law so that we can synchronize a chaotic system with a hyperchaotic system. Take the Chua chaotic system and Rössler hyperchaotic system into consideration.

Here the Rössler hyperchaotic system is regarded as the master system expressed as Eq.(1), and as the slave system, the Chua chaotic system[30] is given by

$$\begin{cases} \dot{y}_1 = -\beta y_2 + u_1, \\ \dot{y}_2 = y_1 - y_2 + y_3 + u_2, \\ \dot{y}_3 = p(y_2 - h(y_3)) + u_3, \end{cases} \tag{19}$$

where $\beta = \dfrac{100}{7}$, $p = 9$, $m_0 = -\dfrac{1}{7}$, $m_1 = \dfrac{2}{7}$ and $h(y_3) = m_1 y_3 + \dfrac{1}{2}(m_0 - m_1)(|y_3 + 1| - |y_3 - 1|)$.

Similarly, subtracting Eq.(1) from Eq.(19) yields the error dynamics

$$\begin{cases} \dot{e}_1 = -\beta e_2 + f_1 + u_1, \\ \dot{e}_2 = e_1 - e_2 + e_3 + f_2 + u_2, \\ \dot{e}_3 = pe_2 - ph(e_3 + x_3) + f_3 + u_3, \end{cases} \tag{20}$$

where $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$, $f_1 = -(\beta + b)x_2 - cx_1$, $f_2 = x_1 - x_2 + x_3 - x_2 x_3 - 3$ and $f_3 = (p+1)x_2 + x_4$.

The active backstepping controller will be designed as follows.

Step 1 Assume that $z_1 = e_1$, then we can obtain its derivative

$$\dot{z}_1 = -\beta e_2 + f_1 + u_1, \tag{21}$$

where $e_2 = \alpha_1(z_1)$ is regarded as a virtual controller.

For the design of $\alpha_1(z_1)$ to stabilize the z_1-subsystem (21), we can choose the following Lyapunov function:

$$V_1 = \frac{1}{2}z_1^2. \tag{22}$$

The derivative of V_1 is

$$\begin{aligned} \dot{V}_1 = z_1\dot{z}_1 =& z_1[-\beta\alpha_1(z_1) + f_1 + u_1] \\ =& -\beta z_1\alpha_1(z_1) + z_1(f_1 + u_1). \end{aligned} \tag{23}$$

We can choose $\alpha_1(z_1) = 0$ and $u_1 = -f_1 - z_1$ so that $\dot{V}_1$ is negative and definite. This implies that the z_1-subsystem (21) is asymptotically stable.

Since the virtual control function $\alpha_1(z_1)$ is estimative, the error between e_2 and $\alpha_1(z_1)$ is

$$z_2 = e_2 - \alpha_1(z_1). \tag{24}$$

Thus, we can obtain the following (z_1, z_2)-subsystem:

$$\begin{cases} \dot{z}_1 = -\beta z_2 - z_1, \\ \dot{z}_2 = z_1 - z_2 + e_3 + f_2 + u_2, \end{cases} \tag{25}$$

where $e_3 = \alpha_2(z_1, z_2)$ is regarded as a virtual controller.

Step 2 In order to stabilize the (z_1, z_2)-subsystem (25), we can choose a Lyapunov function V_2 defined by

$$V_2 = V_1 + \frac{1}{2}z_2^2. \tag{26}$$

Its derivative is given by

$$\begin{aligned}\dot{V}_2 =& \dot{V}_1 + z_2\dot{z}_2 \\ =& -z_1^2 + z_2\alpha_2(z_1, z_2) \\ &+ z_2[(1-\beta)z_1 - z_2 + f_2 + u_2].\end{aligned} \tag{27}$$

If $\alpha_2(z_1, z_2) = 0$ and $u_2 = (\beta - 1)z_1 - f_2$, then $\dot{V}_2 = -z_1^2 - z_2^2 < 0$, which makes (z_1, z_2)-subsystem (25) asymptotically stable.

Similarly, assume that $z_3 = e_3 - \alpha_2(z_1, z_2)$, then we can obtain the following (z_1, z_2, z_3)-system:

$$\begin{cases} \dot{z}_1 = -\beta z_2 - z_1, \\ \dot{z}_2 = \beta z_1 - z_2 + z_3, \\ \dot{z}_3 = pz_2 - ph(z_3 + x_3) + f_3 + u_3. \end{cases} \tag{28}$$

Step 3 In order to stabilize the (z_1, z_2, z_3)-system (28), we can choose a Lyapunov function V_3 defined by

$$V_3 = V_2 + \frac{1}{2}z_3^2. \tag{29}$$

Its derivative is given by

$$\begin{aligned}\dot{V}_3 =& \dot{V}_2 + z_3\dot{z}_3 \\ =& -z_1^2 - z_2^2 + z_3[(p+1)z_2 + f_3 \\ &- ph(z_3 + x_3) + u_3].\end{aligned} \tag{30}$$

If $u_3 = -(p+1)z_2 - f_3 + ph(z_3 + x_3) - z_3$, then $\dot{V}_3 = -z_1^2 - z_2^2 - z_3^2 < 0$, which makes the (z_1, z_2, z_3)-system (28) asymptotically stable.

Finally, we can obtain the fill dimension (z_1, z_2, z_3)-system

$$\begin{cases} \dot{z}_1 = -\beta z_2 - z_1, \\ \dot{z}_2 = \beta z_1 - z_2 + z_3, \\ \dot{z}_3 = -z_2 - z_3. \end{cases} \tag{31}$$

Likewise, since $\dot{V}_4$ is negative and definite, it follows from the LaSalle–Yoshizawa theorem[18] that in the (z_1, z_2, z_3) coordinates the equilibrium (0,0,0) is globally stable asymptotically. The properties $z_1 = e_1$, $z_2 = e_2$ and $z_3 = e_3$, imply that the master system (1) and the slave system (19) are eventually synchronized with each other.

5. Numerical simulations

In what follows, numerical experiments are given to verify the effectiveness of the novel active backstepping control approach.

5.1. Case 1 Synchronization of two identical Rössler hyperchaotic systems with each other

The four-order Runge–Kutta algorithm is used in all of our simulations with time step 0.001. The initial values are $x = (15, 9, -19, -4)^{\mathrm{T}}$ and $y = (14, 2, -17, 7)^{\mathrm{T}}$. After 40 seconds, the motion trajectories enter into the hyperchaotic attractors. From then on we activate the above active backstepping controller to control the slave hyperchaotic system. The simulation results are shown in Fig.2. From Figs.2 (a)–2(e), one can see that the error vector e converges to zero as time t goes to infinity. This shows that the trajectories of the slave system asymptotically approach the ones of the master system, i.e. the master system (1) and the slave system (2) are eventually synchronized with each other.

5.2. Case 2 Synchronization of Rössler hyperchaotic system with Chua chaotic system

Similarly, let the initial conditions be $x = (15, 9, -19, -4)^{\mathrm{T}}$ and $y = (1, 0.5, -2)^{\mathrm{T}}$. After 40s, the motion trajectories enter into the chaotic and hyperchaotic attractors, and the active backstepping controller starts to be applied to the slave Chua chaotic system. As expected, one can observe that the trajectories of the slave system asymptotically approach the ones of the master system as illustrated in Figs.3(a)–3(c). Furthermore, Fig.3(d) shows that the error vector e eventually converges to zero after the controller is activated. This implies that the Chua chaotic system and Rössler hyperchaotic system are synchronized with each other.

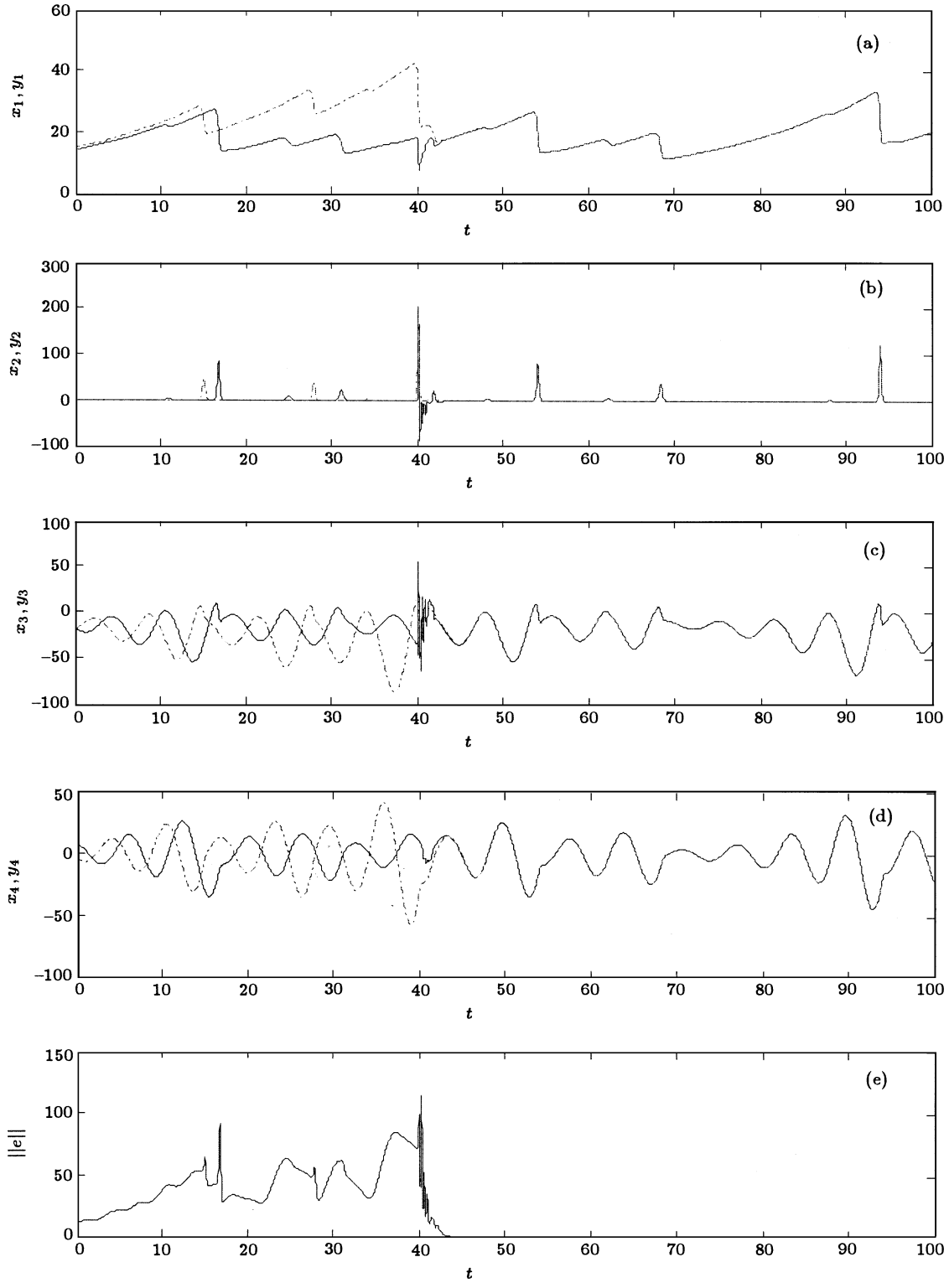


Fig.2. Synchronization of two identical Rössler hyperchaotic systems with each other. (a) Time waveforms of the states x_1 (dashed) and y_1 (solid), (b) time waveforms of the states x_2 (dashed) and y_2 (solid), (c) time waveforms of the states x_3 (dashed) and y_3 (solid), (d) time waveforms of the states x_4 (dashed) and y_4 (solid), (e) time waveforms of the norm $||e||$ of error vector.

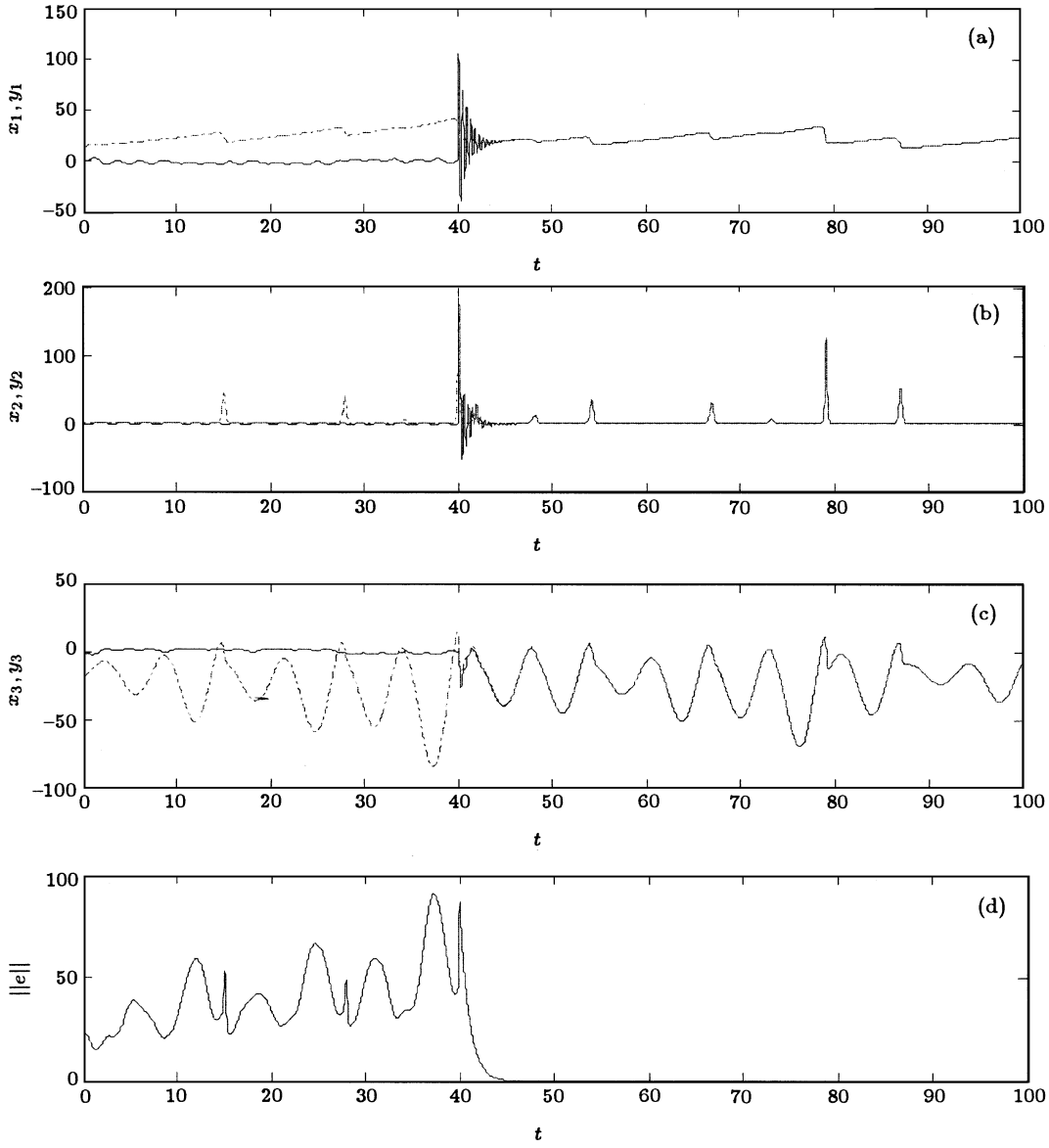


Fig.3. Synchronization of Rössler hyperchaotic system with Chua chaotic system. (a) Time waveforms of the states x_1 (dashed) and y_1 (solid), (b) time waveforms of the states x_2 (dashed) and y_2 (solid), (c) time waveforms of the states x_3 (dashed) and y_3 (solid), (d) time waveforms of the norm $||e||$ of error vector.

6. Conclusions

An effective control method for synchronizing two identical Rössler hyperchaotic systems with each other has been proposed based on an active backstepping design. In particular, the proposed control approach can achieve the generalized synchronization of Chua chaotic system with Rössler hyperchaotic system. A comparison of our proposed method with the aforementioned methods clearly highlights the advantages of the proposed approach in the following respects. (i) The design method can solve the controller singularity problem resulting from the nonlinear term of quadratic type.[23–25] (ii) The proposed approach provides a systematic design approach for chaos control and synchronization, and guarantees the global stability of the closed-loop system. (iii) Especially, flexibility features in this control approach in constructing a control law so that the control strategy can be easily extended to other higher-dimensional hyperchaotic systems.

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