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Chaotifying a stable Takagi–Sugeno fuzzy system by a sawtooth nonlinearity*

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The problem of making a stable Takagi–Sugeno (TS) fuzzy system chaotic by using the state feedback control of arbitrarily small magnitude is studied. The feedback controller used is a simple sawtooth function of the system state. The improved Marotto theorem has been obtained and applied to mathematically prove that the controlled system is chaotic in the sense of Li and Yorke. In particular, an explicit formula for the computation of chaotification parameters is obtained. A numerical example is used to visualize and illustrate the theoretical results.

Keywords: chaos, chaotification, TS fuzzy system, chaos control
PACC: 0545

1. Introduction

The study of chaos suppression and chaos synchronization has progressed rapidly in the last decade.[1,2] The chaotification (also called "anticontrol of chaos") by means of making an originally nonchaotic dynamical system chaotic, or enhancing and retaining existing chaos, has also attracted some special attention from the engineering and physics communities lately due to the great potential of chaos in nontraditional applications.[2,3] Chaos, when appropriately monitored, can provide a system designer with a variety of special properties, flexibilities, and opportunities.[2,3] Dynamical systems (maps) with chaotic and/or bifurcate behaviours have been reported to be very useful in some real applications, such as high-performance circuits and devices, liquid mixing, chemical reactions, biological systems, secure information processing, and critical decision-making.[2] These provide a strong motivation for the current research on chaotification of the discrete-time systems[3−11] and the continuous-time systems.[12,13]

Interactions between fuzzy logic and chaos theory have been explored since the 1990s. The exploration has been carried out mainly on fuzzy modelling of chaotic systems using the Takagi–Sugeno (TS) fuzzy model,[14] suppression control of chaos via a linear matrix inequality (LMI) based fuzzy control system design,[15,16] and linguistic description.[17,18] Recently, Li *et al*[10] proposed a method for chaotifying discrete-time TS fuzzy systems and applied the Marotto theorem[19] to prove that their approach could generate chaos in the sense of Li and Yorke[20] with the control $u_k = [\sigma \sin(\pi\beta x_k(1)/\sigma), \cdots, \sigma \sin(\pi\beta x_k(n)/\sigma)]^{\mathrm{T}}$ if $\beta > \bar{\beta}$ for a sufficiently large $\bar{\beta} > 0$ and $\sigma > 0$, but they did not indicate how to obtain $\bar{\beta}$. Moreover, Refs.[21–24] have pointed out that the Marotto theorem[19] has a problem, for Marotto used the following property $A' \Rightarrow B'$:

A': all eigenvalues of the real matrix A are larger than 1 in absolute value:

B': A satisfies $||Ax||_2 \geq \mu||x||_2$ for some $\mu > 1$, for $x \in R^n$.

In this paper, the problem of making a stable TS fuzzy system chaotic by using the state feedback controller that is a simple sawtooth function of the system states is studied. We apply the improved Marotto theorem to mathematically prove that the controlled system is chaotic in the sense of Li and Yorke.[20] In particular, an explicit formula for the computation of chaotification parameters is obtained. Finally, a numerical example is used to visualize and illustrate the theoretic analysis and the controller design.

All vector inequalities such as $x \geq 0$ are component-wise inequalities. $\|\cdot\|$ denotes any vec-

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tor norm in R^n and the operator norm in $R^{n\times n}$. For a vector $x \in R^n$, its 2-norm and infinity-norm are defined respectively by $\|x\|_2 = \sqrt{\left(\sum_{i=1}^{n}|x_i|^2\right)}$ and $\|x\|_\infty = \max_{1\le i\le n}|x_i|$, and their corresponding induced norms are defined respectively by

$$\|A\|_2 = \sqrt{\max_i \lambda_i(A^{\mathrm{T}}A)},$$

$$\|A\|_\infty = \max_{1\le i\le n}\left\{\sum_{j=1}^{n}|a_{ij}|\right\}.$$

2. Chaos generator design for a discrete TS fuzzy system

2.1. The TS fuzzy model

A general TS fuzzy system[24] is described as follows.

Rule i: IF $x_1(k)$ is $M_{i1},\cdots$, and $x_n(k)$ is M_{in}, THEN

$$x_{k+1} = A_i x_k + u_k, \tag{1}$$

where $x_k = [x_k(1),\cdots, x_k(n)]^{\mathrm{T}}$, $u_k = [u_k(1),\cdots, u_k(n)]^{\mathrm{T}}$, $i = 1,2,\cdots,r$, r is the number of IF-THEN rules, M_{ij} ($i = 1,2,\cdots,r$ and $j = 1,2,\cdots,n$) are fuzzy sets, and the equation $x_{k+1} = A_i x_k + u_k$ is the output from the ith IF-THEN rule. Assume that $\|A_i\|_\infty < 1$, $i = 1,2,\cdots,r$, then all A_i are $n\times n$ Schur stable.

Now, given a pair of (x_k, u_k), the final output of the fuzzy system is inferred from

$$x_{k+1} = \frac{\sum_{i=1}^{r} w_i(k)\{A_i x_k + u_k\}}{\sum_{i=1}^{r} w_i(k)}, \tag{2}$$

where $w_i(k) = \prod_{j=1}^{n} M_{ij}(x_k(j))$, $M_{ij}(x_k(j))$ is the degree of membership of $x_k(j)$ in M_{ij}, with

$$\begin{cases} \sum_{i=1}^{r} w_i(k) > 0 \\ w_i(k) \ge 0, \quad i = 1,2,\cdots,r. \end{cases}$$

By using $h_i(k) = w_i(k)/\sum_{i=1}^{r} w_i(k)$ instead of $w_i(k)$, Eq.(2) is rewritten as

$$x_{k+1} = \left[\sum_{i=1}^{r} h_i(k)A_i\right]x_k + u_k. \tag{3}$$

Note that

$$\begin{cases} \sum_{i=1}^{r} h_i(k) = 1 \\ h_i(k) \ge 0, \quad i = 1,2,\cdots,r, \end{cases} \tag{4}$$

in which $h_i(k)$ can be regarded as the normalized weight of the IF-THEN rules.

2.2. Problem description and controller design

The chaotification problem is to design a control input sequence, $\{u_k\}_{k=0}^{\infty}$ with an arbitrarily small magnitude, $\bar{u} > 0$, namely

$$\|u_k\|_\infty \le \bar{u}, \quad \text{for} \quad k = 1,2,\cdots, \tag{5}$$

so that the controlled system (3) becomes chaotic. For this purpose, we design the controller

$$u_k = [\beta\varphi(\sigma x_k^1),\cdots,\beta\varphi(\sigma x_k^n)]^{\mathrm{T}} \equiv \beta\Phi(\sigma x_k), \tag{6}$$

where $x_k = [x_k^1, x_k^2,\cdots, x_k^n]^{\mathrm{T}}$, σ and β are two positive constants and $\varphi: R \to R$ is a continuous sawtooth function defined by (see Fig.1)

$$\varphi(y) = (-1)^m(y - 2m\varepsilon)$$

for

$$(2m-1)\varepsilon \le y \le (2m+1)\varepsilon, \quad m = 0, \pm1, \pm2,\cdots. \tag{7}$$

Obviously, $|\varphi(y)| \le \varepsilon$ for any $y \in R$, so that $\|u_k\| \le \beta\varepsilon$, where ε can be chosen to be small enough so that $\|u_k\| \le \beta\varepsilon \le \bar{u}$, as required by the condition (5). Similar to Lemma 1 in Ref[10], we obtain the following lemma.

Lemma 1 (Boundedness Theorem). The state vector of the controlled system (3), under the control of the controller (6), is uniformly bounded by a constant, $\beta\varepsilon(1-\alpha)^{-1}$, where the constant α is defined by $\alpha = \max_{1\le i\le r}\{\|A_i\|_\infty\} < 1$.

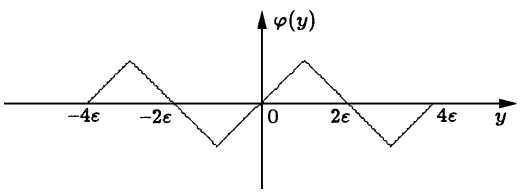


Fig.1. The sawtooth function.

3. Verification of chaos in the controlled TS fuzzy system with a sawtooth function

In this section, the improved Marotto theorem is first reviewed, and then applied to prove that the controlled system (3) with the controller (6) is chaotic in the sense of Li and Yorke.

3.1. Chaos in the sense of Li and Yorke: the improved Marotto theorem

Consider a discrete-time autonomous system of the form

$$x_{k+1} = g(x_k), \quad k = 0, 1, 2, \cdots, \tag{8}$$

where $x_k \in R^n$ and $g : R^n \to R^n$ is continuous mapping. Let g^t denote the t-folded compositions of g itself. A point, x^*, is said to be a p-periodic point of g if $g^p(x^*) = x^*$ but $g^t(x^*) \neq x^*$ for $1 \leq t < p$. If $p = 1$, that is, $g(x^*) = x^*$, x^* is called a fixed point. Let $Dg(x)$ and $\det(Dg(x))$ be the Jacobian of g evaluated at the point x and its determinant, respectively, and let $B_r(x)$ denote a closed ball in R^n with a radius of r centered at the point x. The improved Marotto theorem is given as follows.

Theorem 1[19] If the system (8) has a fixed point x^*, which satisfies that

(i) there exist a real numbers $r > 0$ so that g is differentiable in $B_r(x^*)$ with

$$||(\mathrm{Dg}(\xi))^{-1}|| < 1$$

where $\mathrm{Dg}(\xi) \in R^{n\times n}$ is an $n \times n$ matrix whose ith row vector is $\nabla g_i(\xi^i)$, $i = 1, 2, \cdots, n$, and $\forall \xi^1, \xi^2, \cdots, \xi^n \in B_r(x^*)$;

(ii) there exists a point $x^0 \in B_r(x^*)$ with $x^0 \neq x^*$ so that $g^M(x^0) = x^*$ and $\det(Dg^M(x^0)) \neq 0$ for some positive integer M, then the system (8) is chaotic in the sense of Li–Yorke.

3.2. Verification of chaos in the designed controlled systems

Theorem 2 Suppose that $h_i(k), i = 1, \cdots, r$, are continuously differentiable in the neighbourhood of the fixed point, $x^* = 0$ of the controlled system (3) with the controller (6). If

$$\sigma\beta > (3 + \alpha), \tag{9}$$

where $\alpha = \max\limits_{1\leq i\leq r}\{\|A_i\|_\infty\} < 1$, then the controlled TS fuzzy system (3) with the controller (6) is chaotic in the sense of Li and Yorke.

Proof The controlled system (3) with the controller (6) is

$$x_{k+1} = \left[\sum_{i=1}^{r} h_i(k)A_i\right]x_k + \beta\,\Phi(\sigma x_k) \equiv g(x_k). \tag{10}$$

Obviously, $x^* = 0$ is a fixed point of the system (10). Let $b = [1\ 1\ \cdots\ 1]^{\mathrm{T}} \in R^n$. In what follows, we prove the following statements.

(a) There exists a point x_0 in the interval $0 < x_0 < \sigma^{-1}\varepsilon b$ so that $g^2(x_0) = 0 = x^*$.

(b) There exist constants r in the interval $0 < r < \sigma^{-1}\varepsilon$ so that $x_0 \in B_r(0) \equiv \{x \in R^n \mid ||x||_\infty \leq r\}$, $g(x)$ is differentiable for all $x \in B_r(0)$ and $||(\mathrm{Dg}(\xi))^{-1}||_\infty < 1$ for $\xi = [\xi^1, \cdots, \xi^n] \in R^{n\times n}$ with $\xi^i \in B_r(0)$.

(c) $\det(D(g^2)(x_0)) \neq 0$.

Proof of (a) The proof is completed in two steps.

Step (1) It is to prove that there exists a point x_1 in the interval $\sigma^{-1}\varepsilon b < x_1 < 3\sigma^{-1}\varepsilon b$ so that $g(x_1) = 0$. Let $g(x) = [g_1(x), \cdots, g_n(x)]^{\mathrm{T}}$ and $\left[\sum\limits_{i=1}^{r} h_i(k)A_i\right]x = [f_1(x), \cdots, f_n(x)]^{\mathrm{T}}$. It follows from the system (10) that $g_i(x) = f_i(x) + \beta\varphi(\sigma x^i)$ $(i = 1, 2, \cdots, n)$ are continuous. To prove that there exists a point x_1 in the interval $\sigma^{-1}\varepsilon b < x_1 < 3\sigma^{-1}\varepsilon b$ so that $g(x_1) = 0$, we consider the equation

$$H(x) \equiv \left[\sum_{i=1}^{r} h_i(k)A_i\right]x + \beta(2\varepsilon b - \sigma x) = 0.$$

Since $\left\|\sum\limits_{i=1}^{r} h_i(k)A_i\right\|_\infty \leq \alpha < 1$, $|f_i(x)| \leq \alpha\|x\|_\infty$. Since $H(x)$ is continuous and

$$\begin{aligned}
&H_1(2\beta\varepsilon(\beta\sigma - \alpha)^{-1}, \omega_2, \cdots, \omega_n)\\
=&f_1(2\beta\varepsilon(\beta\sigma - \alpha)^{-1}, \omega_2, \cdots, \omega_n)\\
&+ 2\beta\varepsilon - 2\beta\varepsilon(\beta\sigma - \alpha)^{-1}\beta\sigma\\
\leq&2\alpha\beta\varepsilon(\beta\sigma - \alpha)^{-1} + 2\beta\varepsilon\\
&- \beta\sigma \times 2\beta\varepsilon(\beta\sigma - \alpha)^{-1} = 0,
\end{aligned}$$

$$\begin{aligned}
&H_1(2\beta\varepsilon(\beta\sigma + \alpha)^{-1}, \omega_2, \cdots, \omega_n)\\
=&f_1(2\beta\varepsilon(\beta\sigma + \alpha)^{-1}, \omega_2, \cdots, \omega_n)\\
&+ 2\beta\varepsilon - 2\beta\varepsilon(\beta\sigma + \alpha)^{-1}\beta\sigma\\
\geq&-2\alpha\beta\varepsilon(\beta\sigma + \alpha)^{-1} + 2\beta\varepsilon\\
&- 2\beta^2\sigma\varepsilon(\beta\sigma + \alpha)^{-1} = 0,
\end{aligned}$$

$\forall\omega_i \in [2\beta\varepsilon(\beta\sigma+\alpha)^{-1}, 2\beta\varepsilon(\beta\sigma-\alpha)^{-1}]$, $i = 2, 3, \cdots, n$, there must exist a number $x_1^1(\omega_2, \cdots, \omega_n) \in [2\beta\varepsilon(\beta\sigma+\alpha)^{-1}, 2\beta\varepsilon(\beta\sigma-\alpha)^{-1}]$ so that $(x_1^1, \omega_2, \cdots, \omega_n)$ satisfies $H_1(x_1^1, \omega_2, \cdots, \omega_n) = 0$.

Furthermore, since

$$\begin{aligned}
&H_2(x_1^1, 2\beta\varepsilon(\beta\sigma - \alpha)^{-1}, \omega_3, \cdots, \omega_n)\\
\leq&2\alpha\beta\varepsilon(\beta\sigma - \alpha)^{-1} + 2\beta\varepsilon - 2\beta\varepsilon(\beta\sigma - \alpha)^{-1}\beta\sigma = 0,\\
&H_2(x_1^1, 2\beta\varepsilon(\beta\sigma + \alpha)^{-1}, \omega_3, \cdots, \omega_n)\\
\geq&-2\alpha\beta\varepsilon(\beta\sigma + \alpha)^{-1} + 2\beta\varepsilon - 2\beta\varepsilon(\beta\sigma + \alpha)^{-1}\beta\sigma = 0,
\end{aligned}$$

$\forall\omega_i \in [2\beta\varepsilon(\beta\sigma+\alpha)^{-1}, 2\beta\varepsilon(\beta\sigma-\alpha)^{-1}], i=3,4,\cdots,n$, there must exist a number $x_1^2(\omega_3,\cdots,\omega_n) \in [2\beta\varepsilon(\beta\sigma+\alpha)^{-1}, 2\beta\varepsilon(\beta\sigma-\alpha)^{-1}]$ so that $(x_1^1, x_1^2, \omega_3, \cdots, \omega_n)$ satisfies $H_2(x_1^1, x_1^2, \omega_3, \cdots, \omega_n) = 0$.

Now, by repeating the same process $(n-3)$ times, eventually, since

$$\begin{aligned} &H_n(x_1^1, x_1^2, \cdots, 2\beta\varepsilon(\beta\sigma-\alpha)^{-1}) \\ \le& 2\alpha\beta\varepsilon(\beta\sigma-\alpha)^{-1} + 2\beta\varepsilon \\ &- 2\beta\varepsilon(\beta\sigma-\alpha)^{-1}\beta\sigma = 0, \\ &H_n(x_1^1, x_1^2, \cdots, 2\beta\varepsilon(\beta\sigma+\alpha)^{-1}) \\ \ge& -2\alpha\beta\varepsilon(\beta\sigma+\alpha)^{-1} + 2\beta\varepsilon \\ &- 2\beta\varepsilon(\beta\sigma+\alpha)^{-1}\beta\sigma = 0, \end{aligned}$$

there must exist a number $x_1^n \in [2\beta\varepsilon(\beta\sigma+\alpha)^{-1}, 2\beta\varepsilon(\beta\sigma-\alpha)^{-1}]$ so that $x_1 = (x_1^1, x_1^2, \cdots, x_1^n)$ satisfies

$$H(x_1) = \left[\sum_{i=1}^{r} h_i(k)A_i\right]x_1 + \beta(2\varepsilon b - \sigma x_1) = 0.$$

It can be deduced from the condition (9) that

$$\begin{aligned} &\sigma x_1 > 2\beta\sigma\varepsilon(\beta\sigma+\alpha)^{-1}b > b\varepsilon, \\ &\sigma x_1 < 2\beta\sigma\varepsilon b(\beta\sigma-\alpha)^{-1} < 3b\varepsilon. \end{aligned}$$

Therefore, it follows from the definition of the sawtooth function that

$$g(x_1) = \left[\sum_{i=1}^{r} h_i(k)A_i\right]x_1 + \beta(2\varepsilon b - \sigma x_1) = 0 \equiv x^*.$$

Step (2) It is to prove that there exists a point x_0 in the interval $0 < x_0 < \sigma^{-1}\varepsilon b$, so that $F(x_0) = x_1$. To do so, we consider the equation

$$\bar{H}(x) \equiv \left[\sum_{i=1}^{r} h_i(k)A_i\right]x + \beta\sigma x - x_1.$$

Similar to the above proof in step (1), one can prove that there exists a point x_0 in the interval $0 < x_0 < \sigma^{-1}\varepsilon b$ so that

$$\bar{H}(x_0) = \left[\sum_{i=1}^{r} h_i(k)A_i\right]x_0 + \beta\sigma x_0 - x_1 = 0.$$

Consequently, it follows from the definition of the sawtooth function that

$$g(x_0) = \left[\sum_{i=1}^{r} h_i(k)A_i\right]x_0 + \beta\sigma x_0 = x_1.$$

Steps (1) and (2) together imply that there exists a point x_0 in the interval $0 < x_0 < \sigma^{-1}\varepsilon b$ so that after two-step iterations, $g^2(x_0) = g(x_1) = 0$. Q.E.D.

Proof of (b) Now, let r be chosen so that $\|x_0\|_\infty \le r < \sigma^{-1}\varepsilon$, then $x_0 \in B_r(0)$ and it follows from the definition of g that, for any $x \in B_r(0)$, $g(x)$ is differentiable and $\mathrm{Dg}(x) = \left[\sum_{i=1}^{r} h_i(k)A_i\right] + \beta\sigma I$. Furthermore, $\mathrm{Dg}(\xi) = \left[\sum_{i=1}^{r} h_i(k)A_i\right] + \beta\sigma I$, where $\xi \in B_r^n(0) = \{\xi = [\xi^1, \cdots, \xi^n] \in R^{n\times n} | \xi^i \in B_r(0)\}$. The condition (9) implies that $||(\mathrm{Dg}(\xi))^{-1}||_\infty < 1$ for $\xi = [\xi^1, \cdots, \xi^n] \in R^{n\times n}$ with $\xi^i \in B_r(0)$. Q.E.D.

Proof of (c) g^2 is differentiable at x_0 with

$$\begin{aligned} \det(Dg^2(x_0)) &= \det(Dg(x_0))\det(Dg(x_1)) \\ &= \det\left(\left[\sum_{i=1}^{r} h_i(k)A_i\right] + \beta\sigma I\right) \\ &\quad \times \det([\sum_{i=1}^{r} h_i(k)A_i] - \beta\sigma I) \ne 0. \text{Q.E.D.} \end{aligned}$$

Thus, based on the improved Marotto theorem, the statements (a), (b) and (c) together imply that the controlled system (3) with the controller (6) is chaotic in the sense of Li and Yorke.

4. A simulation example

Consider a nonchaotic discrete-time TS fuzzy model, given by the rule IF x_k is M_i, THEN $X_{k+1} = \begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = A_i \begin{bmatrix} x_k \\ y_k \end{bmatrix} + u_k$, $i = 1, 2$, where $A_1 = \begin{bmatrix} d & 0.1 \\ 0.1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} -d & 0.1 \\ 0.1 & 0 \end{bmatrix}$, $x_k \in [-d, d]$, $d > 0$, and membership functions

$$M_1 = 0.5(1 - x_k/d), \quad M_2 = 0.5(1 + x_k/d).$$

The controlled TS fuzzy system is described as follows:

$$X_{k+1} = \left[\sum_{i=1}^{2} h_i(k)A_i\right]X_k + \beta\varphi(\sigma X_k). \qquad (11)$$

The magnitude of the control input is arbitrarily chosen to be $\bar{u} = 0.1$. In the simulation, we have chosen $\beta = 1$, $\varepsilon = 0.1$, $X_0 = [0.1, 0.1]^{\mathrm{T}}$, and $d = 0.7$. Since $\alpha = \max\limits_{1\le i\le r}\{\|A_i\|_\infty\} = 0.8$, the uncontrolled TS fuzzy system (11) with $u_k = 0$ is stable. From Theorem 1, we know that the controlled TS fuzzy system (11) is chaotic if $\sigma\beta > 3.8$. When $\sigma = 4.0$, i.e. $\sigma\beta = 4.0$, the phase portrait and time wave diagrams are shown in Fig.2, respectively. From Fig.2, we can see that $\|u_k\|_\infty \le 0.1$ and the controlled TS fuzzy system (11) displays complex chaotic behaviours. These numerical results verify the theoretical analysis in this paper.

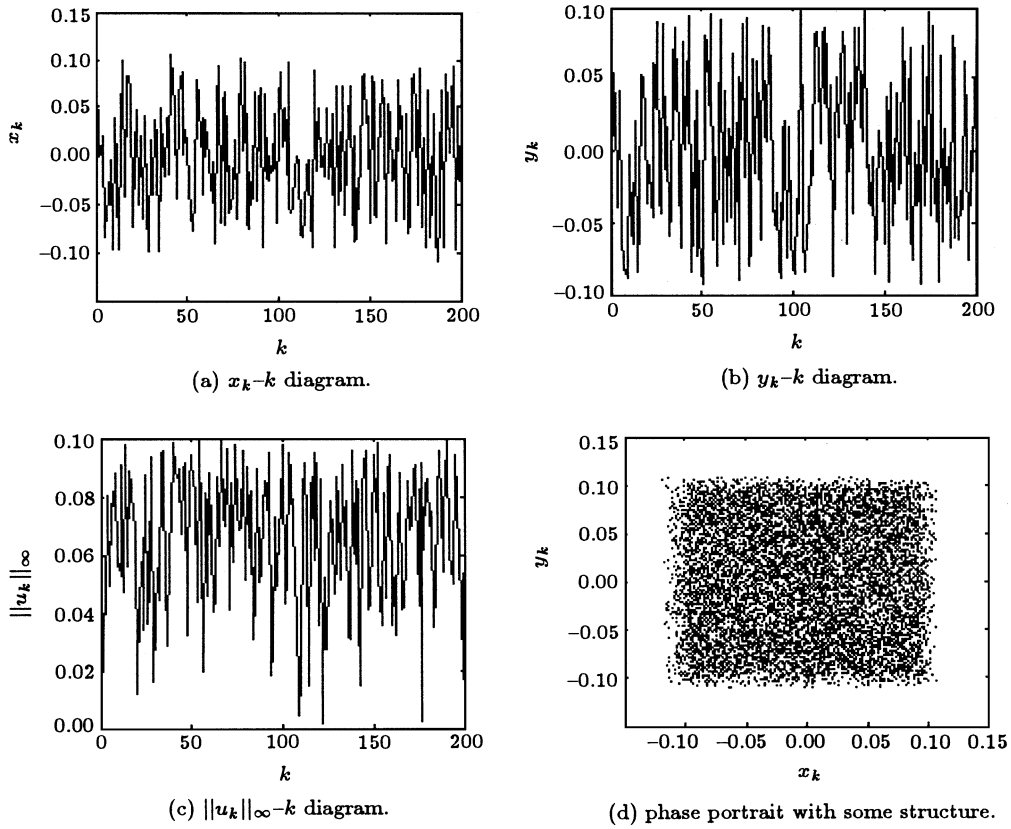


Fig.2. Simulation results with control.

5. Conclusions

The problem of making a stable TS fuzzy system chaotic by using the state feedback control of an arbitrarily small magnitude has been studied. The feedback controller used is a simple sawtooth function of the system states. We have mathematically and rigorously proved that the controlled TS fuzzy system designed is indeed chaotic in the sense of Li and Yorke by using the improved Marotto theorem. In particular, an explicit formula for the computation of control parameters has been obtained. Simulation results have been used to demonstrate the effectiveness of the theoretical results.

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