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Phase synchronization of Chua circuit induced by the periodic signals*

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Phase synchronization of chaotic systems induced by periodic signals is studied experimentally and numerically. Three kinds of periodic signals are chosen to drive Chua circuits, including the periodic signal in the Chua circuit, sinusoidal and pulsed signals. Under conditions of small mismatch between the natural frequencies of the periodic and the chaotic signals, appropriate amplitudes of the driving signals, and appropriate duty cycles of the pulse, phase synchronization can be achieved. Even when the duty circle reduces to 3%, it can still drive the chaotic signals into phase synchronization, which is interesting for applications.

Keywords: phase synchronization, chaotic oscillator
PACC: 0545

1. Introduction

Synchronization in chaotic systems has been an active research field during the past decade because of its importance in nonlinear dynamics and potential applications, such as laser system,[1,2] secure communication,[3−6] and so on. Different chaos synchronizations have been distinguished as complete synchronization,[7] generalized synchronization,[8] and phase synchronization (PS).[9] PS is defined as the phase locking between nonidentical systems whose amplitudes remain uncorrelated, but whose phase difference $|\theta_1 - \theta_2| \leq$ const remains. Recently PS in coupled chaotic oscillators[10−12] and chaotic oscillators driven by periodic forces[13−15] have been studied theoretically, and some experiments have been carried out with the circuits and the coupled Chua-plasma systems.[16,17] In this paper we investigate the PS in the Chua circuit driven by periodic signals experimentally and numerically. In Section 2, we introduce the experimental setup. Section 3 presents the experimental results that relate the PS to controlling parameters, such as frequency and amplitude of the driving signals, and the duty cycles of pulsed signals. Section 4 shows the simulation by taking pulse as the driving signal, and the conclusion is given in the last section.

2. Experiential setup

Our experimental setup is shown in Fig.1. It contains the Chua circuit with a driving supplementary circuit, through which the driving signal V can be input into the Chua circuit. The driving strength ε is controlled by two resistors R_1 and R_2 with $\varepsilon = R_2/(R_1 + R_2)$. The nonlinear resistor $R_{\rm NL}$ is composed of two operational amplifiers (Texas Instruments type TL082C) and six resistors. Equations for the Chua circuit are

$$\begin{aligned} C_1\frac{{\rm d}V_{\rm C_1}}{{\rm d}t} &= \frac{1}{R}(V_{\rm C_2} - V_{\rm C_1}) - f(V_{\rm C_1}),\\ C_2\frac{{\rm d}V_{\rm C_2}}{{\rm d}t} &= \frac{1}{R}(V_{\rm C_1} - V_{\rm C_2}) + i_{\rm L},\\ L\frac{{\rm d}i_{\rm L}}{{\rm d}t} &= -V_{\rm C_2} + \varepsilon V,\\ f(x) &= m_0 x + 0.5m_1[|x+a| - |x-a|]\\ &\quad + 0.5m_2[|x+b| - |x-b|], \end{aligned} \tag{1}$$

where the variables $V_{\rm C_1}$ and $V_{\rm C_2}$ are the voltages across the capacitors $\rm C_1$ and $\rm C_2$, respectively, and $i_{\rm L}$ is the current through the inductor L. We take the following values for parameters: m_0=2.05mA/V, m_1=–0.35mA/V, m_2=–2.45mA/V, a=2.0V, b=10.2V. A function generator 33220A (Agilent) is used to pro-

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duce the sinusoidal and pulsed signals. Data are acquired by using 6110/6111E DAQ card of National Instruments connected to a computer and a software LABVIEW is used to analyse the experimental data. To decrease the error of the measured frequency, 45 data sequences are statistically analysed and we obtain the average frequency f and the standard deviation $\Delta f = \sqrt{\sum_{i=1}^{n}(f_i - f)^2/(n(n-1))}$, n=45. In this paper, we fix the parameters as shown in Fig.1 and take resistor R as a variable.

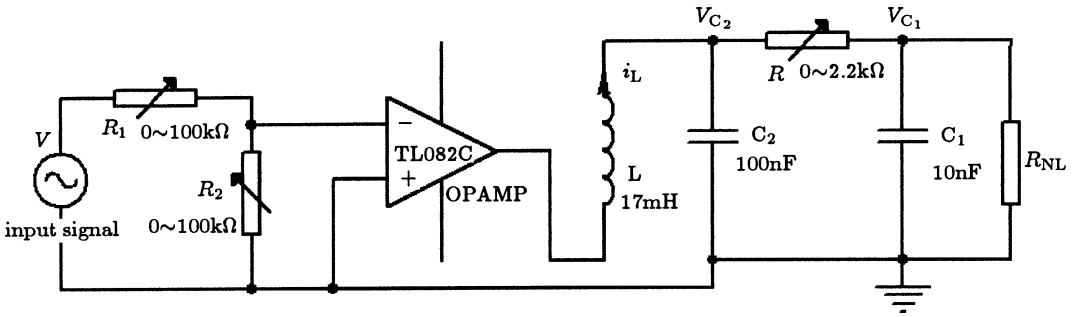


Fig.1. The experimental setup including Chua circuit and a driving supplementary circuit.

3. Experimental results

In this section we investigate the PS behaviours experimentally. In the circuit, the Chua circuit is driven by three kinds of periodic signals, i.e. the sinusoidal, pulsed signals and the periodic signal from another Chua circuit. We study the PS through plotting the Poincaré section map, i.e. V_{C_1} versus V_{C_2} trajectory sampled during the driving signal changing from negative to positive, and confirm the parameter range within which the PS is achieved. It is evident that for a free chaotic oscillator, the curves of the Poincaré section map do not change with time because of time translation invariability and show ergodicity. But the curves of the Poincaré section map are contracted and become localized when periodic signals drive the chaotic oscillator and the PS happens.

Figure 2 shows the case with a fixed driving strength ε=0.8 (R_1=10kΩ, R_2=40kΩ) and sinusoidal driver V=10sin($2\pi f_d t$) in units of mV. With R=1.8759kΩ, the Chua oscillator is a single attractor. The frequency of the free chaotic oscillator is measured to be f_c=2803.2±1.4Hz. Figures 2(a) and 2(b) show the trajectories of V_{C_1} versus V_{C_2} (grey lines) and corresponding Poincaré section map (circles) with the driver frequency f_d being 2800.0Hz and 2820.0Hz. One can observe that the curves of Poincaré section map in Fig.2(a) are localized and contracted, thus we can say that the respond system reaches the PS with the driver, otherwise the plots of the Poincaré section map in Fig.2(b) are global, indicating that the two systems are in phase nonsynchronization. From the curves of the Poincaré section map, we can see that the frequency range of the driving signal is approximately from 2784.0Hz to 2814.0Hz for obtaining the PS. In order to manifest further the PS phenomena, the frequency of the respond system is measured and the curves of the average frequency f_r (circles) of the respond system and the standard deviation (vertical lines) are plotted against the driver frequency f_d in Fig.2(c). When the PS happens, the driver locks the frequency of the respond system and the deviations of the frequency of the respond system are small (corresponding to the middle region in Fig.2(c)), otherwise the deviations become larger and the locking frequency fails.

We set up also another Chua circuit, of which the attractor is period-1 limit cycle, to output the voltage V_{C_2} as the driver. The measured frequencies of this driver and the free chaotic oscillator are 2813.9±0.03Hz and 2816.0±3.5Hz, respectively. We also study the PS against the driving strength. At an appropriate driving strength, the oscillator can be controlled to reach the PS with the driver. Fix R_1=90kΩ and choose resistor R_2 as a variable. Figure 3(a) shows that the responder realizes the PS with the driver at R_2=129Ω. We observe the curves of Poincaré section map through changing resistor R_2 and confirm the range of R_2 being approximately from 59 to 149Ω, in this region the PS can be realized. To detect PS in the respond system, we compute the ratio of the average frequency of the respond system to the frequency

of the driver. In Fig.3(b) we plot f_r/f_d versus R_2, and the variation range of parameter R_2 in the PS region is approximately the same as the above. We can see that all data fluctuations including PS and non-PS regions are larger in Fig.3(b) than in Fig.2(c), and the main reason for this is that the driver circuit is least disturbed by the chaotic system through the R_2 resistor.

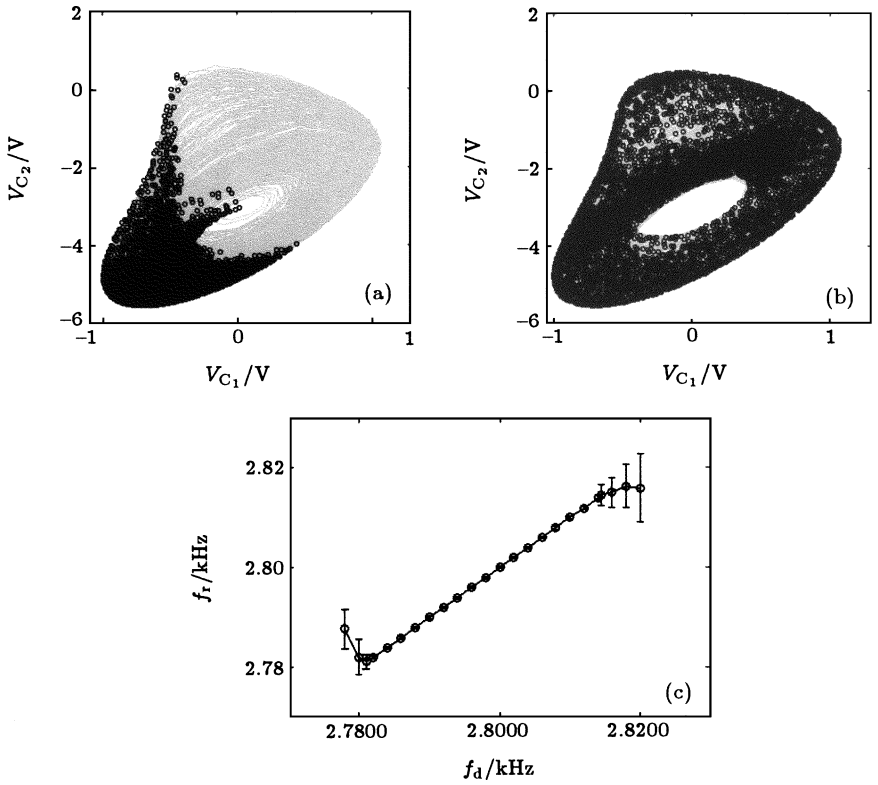


Fig.2. (a)–(b) Trajectories of V_{C_1} versus V_{C_2} (grey lines) and corresponding curves of Poincaré section map (circles) sampled during the sinusoidal signal changing from negative to positive. (a) PS with f_d=2800.0Hz. (b) Phase nonsynchronization with f_d=2820.0Hz. (c) The curve of the average frequency f_r (circles) of the respond system and the standard deviation (vertical lines) against the driver frequency f_d.

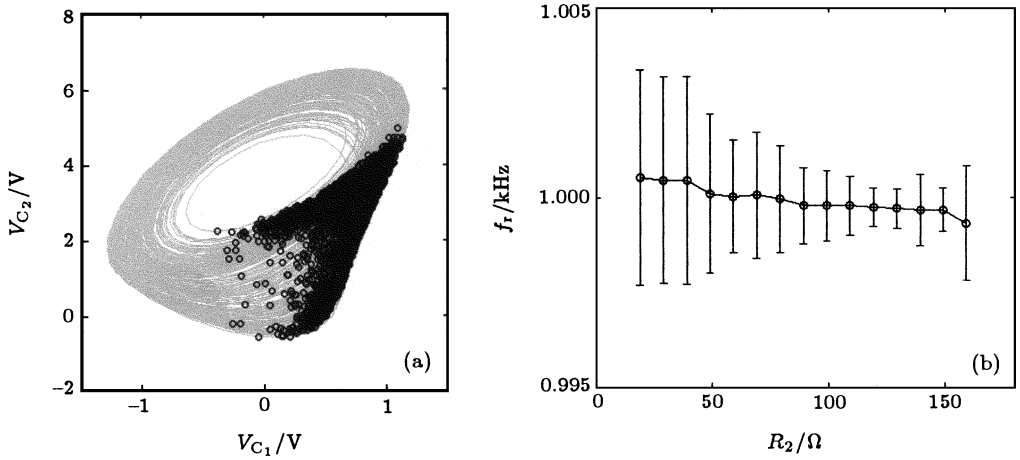


Fig.3. The chaotic system driven by the period-1 signal from the Chua circuit with the frequency being 2813.9±0.03Hz. (a) The curves are the same as Fig.2 (a) at R_2=129Ω. (b) Curves of f_r/f_d (circles) and the standard deviation (vertical lines) versus resistor R_2.

Pulsed waves are usually used to control the respond system in application. Next we investigate the PS versus duty cycles of pulsed signals. The frequency of the free chaotic oscillator is 2803.2 ± 1.4Hz. We fix the frequency and the amplitude of the pulsed signal, and the driving strength as 2811.0Hz, 10mV, and 0.8, respectively. Figures 4(a) and 4(b) plot the trajectories of V_{C_1} versus V_{C_2} (grey lines) and corresponding curves of Poincaré section map (circles). One can see the PS phenomenon with the duty cycle being 40% (Fig.4(a)) and 20% (Fig.4(b)). In Fig.4(c) we plot f_r/f_d (circles) and the standard deviation (vertical lines) versus the duty cycle d, where one can observe that the chaotic oscillator can reach the PS with the pulsed signal lying within the middle horizontal plateau (corresponding to the duty cycle ranging from 20% to 80%). To show the details of the plateau, we magnify this plot in the inset and find the differences between the frequency ratio and unity, even when the PS happens. The differences result from the limited number of turns for chaotic orbits acquired, and are in inverse proportion to the number of turns.

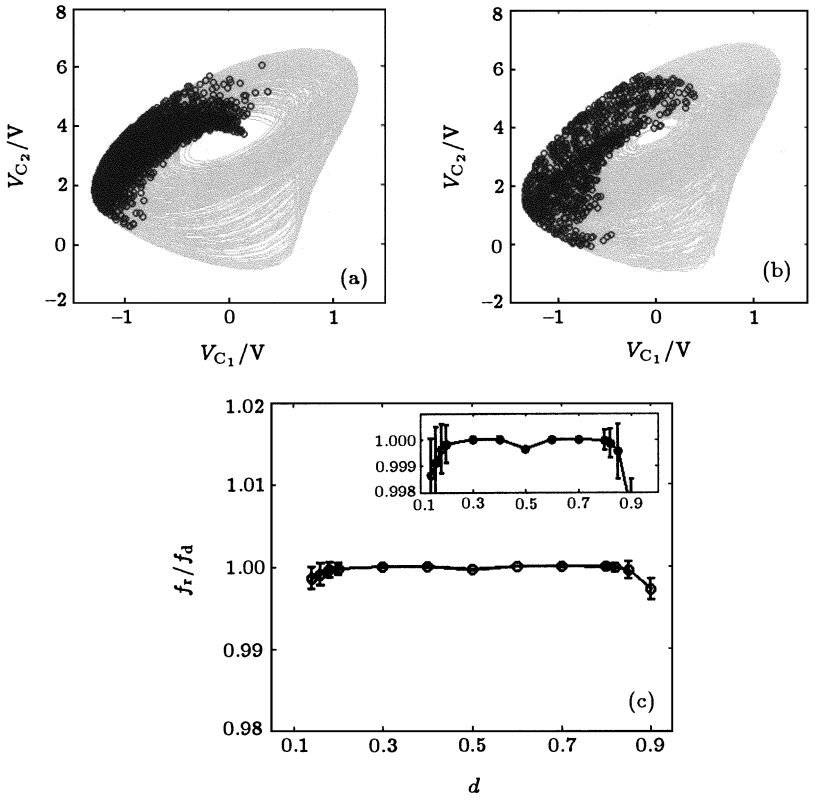


Fig.4. The chaotic system driven by the pulsed signal with the frequency and amplitude being 2811.0Hz, 10mA, and driving strength being 0.8. In Figs.(a) and (b), the curves in Fig.2(a) are plotted with the duty cycle being 40% and 20%, respectively. (c) Curve of f_r/f_d (circles) and the standard deviation (vertical lines) versus duty cycle d.

4. Simulation

The equations used for numerical investigation are as follows:

$$\begin{aligned} \frac{dx}{d\tau} &= \alpha[r_1(y-x) - f(x)], \\ \frac{dy}{d\tau} &= r_1(x-y) + z, \\ \frac{dz}{d\tau} &= -\beta x + \varepsilon u(f_d), \end{aligned} \tag{2}$$

where $u(f_d)$ is the driving signal, f_d is the frequency of the driver, and ε is the driving strength. In the simulation the parameters are fixed as r_1=0.53, α=10, β=5.55. The average frequency of a coherence oscillator can be defined as the average of the phase variable, expressed as

$$\begin{aligned} f_r &= \lim_{T\to\infty} \frac{1}{2\pi T} \int_0^T \dot{\theta}(\tau) d\tau, \\ \theta &= \tan^{-1}\left[\frac{y(\tau)}{x(\tau)}\right]. \end{aligned} \tag{3}$$

When the frequency of the respond system is equal to that of the driver, i.e. $f_r = f_d$, the response system can achieve the PS. The frequency of the system

described by Eq.(2) with $u = 0$ is simulated to be f_c=0.3105.

Without losing generality, the driving signal adopted is the pulsed wave

$$u = \begin{cases} 1, & nT_0 < \tau \leq nT_0 + dT_0; \\ 0, & nT_0 + dT_0 < \tau \leq (n+1)T_0, \end{cases} \tag{4}$$

where T_0 is the period with $T_0 = \dfrac{1}{f_d}$, and d is the duty cycle.

Next we study the PS versus three parameters, i.e. the frequency of driver f_d, driving strength ε, and the duty cycle d. In Fig.5(a), we first show the frequency difference between the driver and responder $\Delta f = |f_d - f_r|$ against f_d, and the two parameters are fixed as $\varepsilon = 0.20$, $d = 5\%$ in this case. It can be found that the frequency differences become near zero under the condition that f_d is changed from 0.3100 to 0.3111. From the above simulation, we can observe the PS under the condition of the small mismatch of the frequency between the driver and the responder. In Figs.5(b) and 5(c) we show the frequency difference Δf against the driving strength with $f_d = 0.3111$. The duty cycles are chosen to be 5% and 30% in Fig.5(b) and 5(c), respectively. One can see that there exists the PS within appropriate driving strengths in Figs.5(b) and 5(c). We can understand this point from the view point of power, i.e. the driver must output an appropriate power in order to control the response system, otherwise this controlling fails at larger and smaller powers. Comparing Fig.5(b) with Fig.5(c), one can find that the minimal and the maximal driving strengths for attaining PS decrease when the duty cycle increases. To show the range of input powers from the driver, we define the minimal power $P_{\min}$ and the maximum power $P_{\max}$ in one cycle as

$$P_{\min} = d\varepsilon_{\min}, \quad P_{\max} = d\varepsilon_{\max}$$

with $\varepsilon_{\min}$ and $\varepsilon_{\max}$ being the minimal and the maximal driving strengths when PS happens. In Fig.5(d) we plot the minimal power $P_{\min}$ (squares) and maximum power $P_{\max}$ (circles) against the duty cycle to show the control efficiency. We are interested in the curve of the minimal power, and find that the minimal power needed to control the PS decreases with the decrease of the duty cycle and tends to be constant. This control of pulsed signal, especially when the duty cycle is small, is interesting because it is not only simple, but also is evidently effective.

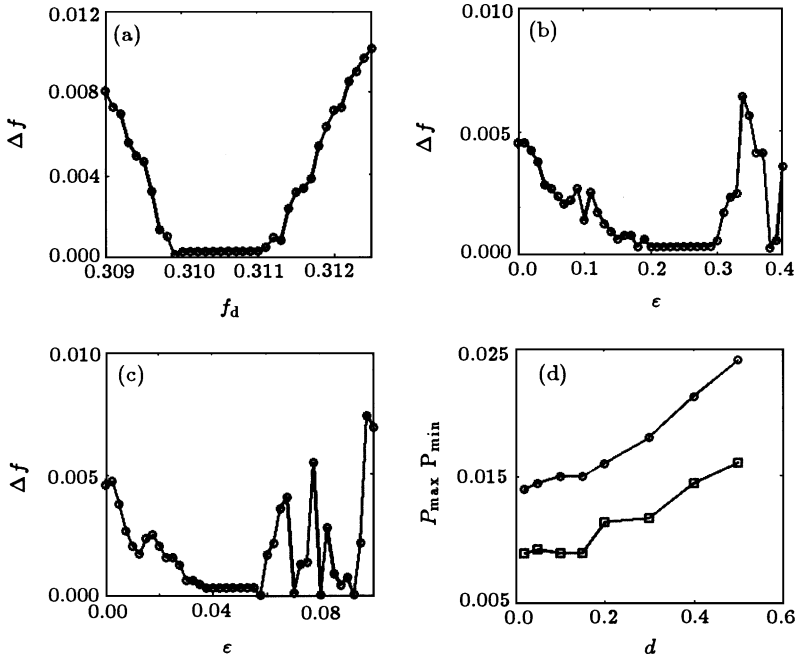


Fig.5. (a) Curve of the frequency difference between the driver and responder against the frequency of the pulsed signal. (b) Frequency difference between the driver and responder versus the driving strength with the duty cycle being 5%. (c) The curve same as (b) with the duty cycle being 30%. (d) The minimal and the maximum powers versus the duty cycles when the PS happens.

5. Conclusion

In this paper we have investigated experimentally and numerically the PS of the chaotic Chua circuit driven by periodic signals. We obtain the relation between the PS and the parameters, i.e. the driver's frequency, amplitude and the duty cycle of the pulsed signal. For small frequency differences between the chaotic oscillator and the driver, the PS can be observed. For larger and smaller values of amplitudes and duty cycles of the driver, the PS fails. From our experimental investigation we can conclude that the PS can be accomplished in chaotic systems by selecting different periodic drivers.

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