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Local discrete cosine transformation domain Volterra prediction of chaotic time series*

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In this paper a local discrete cosine transformation (DCT) domain Volterra prediction method is proposed to predict chaotic time series, where the DCT is used to lessen the complexity of solving the coefficient matrix. Numerical simulation results show that the proposed prediction method can effectively predict chaotic time series and improve the prediction accuracy compared with the traditional local linear prediction methods.

Keywords: chaotic time series, local prediction, DCT, phase-space reconstruction
PACC: 0545

1. Introduction

With the development of chaos theory and applied technique research, the modelling and prediction of chaotic time series has become a major research domain of chaotic signal processing.[1] Based on the Takens embedding theorem and phase-space reconstruction,[2] many prediction methods have been proposed in the past few years, and these methods are classified into three categories: global prediction method,[3−7] local prediction method[8−11] and adaptive nonlinear filter prediction method.[12−16] Previous studies have shown that local method is the most accurate of these three categories.

Local methods such as local zero-order method and local first-order prediction method are widely used in chaotic time series because of their simplicity. However, local methods are often criticized because of their poor prediction accuracy and great computation demand compared with other methods, especially when the order of the local polynomial model exceeds two.

In recent years, adaptive Volterra filters and variant types have been successfully used to predict chaotic time series[12−16] and control chaos,[17,18] and local polynomial models have been successfully applied to predict chaotic time series.[10] One main point of this paper is to propose a local discrete cosine transformation (DCT) domain Volterra model for chaotic time series prediction, where the DCT is used to lessen the complexity of solving coefficient matrix. This method can effectively predict chaotic time series and reduce the complexity of solving the coefficient matrix based on DCT, and improve the prediction accuracy compared with the traditional local linear prediction methods.

This paper is organized as follows. The local DCT domain Volterra predictive model is described in Section 2, and the effectiveness of the proposed prediction model is illustrated by comparing with the local first-order prediction method in Section 3. Section 4 is devoted to a brief summary and discussion.

2. Local DCT domain Volterra prediction method

Suppose a scalar time series $\{x(t), t = 1, 2, ..., n\}$ is a measurement on the chaotic dynamical system in the state space. If the embedding dimension m and the time delay τ have been chosen for the phase-space reconstruction, a reconstructed vector can be written as follows:

$$X_i = \{x(i), x(i-\tau), ..., x(i-(m-1)\tau)\}. \quad (1)$$

According to Takens' embedding theorem or its extensions, there exists a function F such that

$$x(t+T) = F(X_t), \quad (2)$$

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where the integer T is the look-ahead time. Let the reconstructed attractor be denoted by S, and it will be necessary to fit F only in the domain S by the prediction model. A model of chaotic time series can be obtained by finding a functional form $\hat{F}$ for the training set pairs, $(X(t), d(t))$, $t = 1, \cdots, n$, where $d(t) = x(t+T)$ is the output of the model, and $X(t)$ is the input of the model. $\hat{F}$ maps $X(t)$ to $d(t)$:

$$x(t+T) = \hat{F}(X_t) + \varepsilon(t), \tag{3}$$

where $\varepsilon(t)$ represents the noise with finite length and fitting error.

There are several possible approaches to construction of $\hat{F}$. For the local methods, an attempt is made to approximate the map (Eq.(3)) by subdivision of the $\hat{F}$ domain into many subsets. In this way, the dynamics of the system is described step by step locally in the phase-space. A local method can alleviate considerably the complexity of the representation $\hat{F}$ without degrading the quality of the prediction for the very short term; it generally provides better results than those obtainable with a global method.

For the local prediction method, to predict $x(t+T)$ we first set up a metric in the state space, which is denoted by Euclidian distance $\|.\|$, and find the k nearest neighbours $\{X_t^1, X_t^2, ..., X_t^k\}$ of X_t by calculating the Euclidean distance between the query vector X_t and the other reconstruction vector X_i:

$$\begin{aligned} R_i(X_t, X_i) =& \|X_t - X_i\| \\ =& \sqrt{\sum_{j=1}^{m} [x(t-(j-1)\tau) - x(i-(j-1)\tau)]^2}, \end{aligned} \tag{4}$$

where $i = 1, 2, ..., (t-1)$. Regarding each neighbour $X_t^i, i = 1, 2, ..., k$ of the query vector X_t as a point in the domain and $x(t+T)$ as the corresponding point in the range, we can construct a local predictor. The simplest method is the zero-order approximation, which uses the corresponding value of the nearest neighbour as the prediction value of the query vector X_t. Farmer and Sidorowich[8] present a superior approach which is the first-order or linear approximation. This approach takes $k > m$ nearest neighbours, and constructs a linear polynomial with the pairs $(X_t^i, x(t_i+T))$; and the fit can be made in the least square sense by using singular-value decomposition. For the approximation using higher-order polynomials, the performance is not significantly better than the one obtained with the first-order approximation; moreover, the complexity of computation increases quadratically.

With the k nearest neighbours of the query vector X_t, the local second-order Volterra prediction model can be defined as follows:

$$\begin{aligned} x(t_i+T) =& a + \sum_{k=1}^{m} b_k x(t_i - (k-1)\tau) \\ &+ \sum_{j=1}^{m}\sum_{k=1}^{m} c_{j,k} x(t_i - (j-1)\tau) \\ &\times x(t_i - (k-1)\tau), \end{aligned} \tag{5}$$

where a, b_k and $c_{j,k}$ are constants, and there are $1 + m + m^2/2$ coefficients to be determined, and this would be a very large computational burden.

Now we re-write Eq.(5) as the following matrix norm:

$$x(t_i+T) = A + BX + X^{\mathrm{T}} C X, \tag{6}$$

where $X = [x(t_i)\ x(t_i-\tau)\ \cdots\ x(t_i-(m-1)\tau)]^{\mathrm{T}}$, $A = [a_i]$, $B = [b_1\ b_2\ \cdots\ b_m]$ and C each are a $m \times m$ symmetric matrix,[19] and may be diagonalized by unitary matrix Q as follows:

$$\begin{aligned} C &= \begin{bmatrix} c_{10} & c_{11} & \cdots & c_{1m} \\ c_{20} & c_{21} & \cdots & c_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ c_{m0} & c_{m1} & \cdots & c_{mm} \end{bmatrix} \\ &= Q^{\mathrm{T}} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \lambda_m \end{bmatrix} Q = Q^{\mathrm{T}} \Lambda Q, \end{aligned} \tag{7}$$

where Λ is a diagonal matrix and its elements are the eigenvalues of matrix C. Column vectors of unitary matrix Q are orthogonal to each other, and $Q^{\mathrm{T}}Q = QQ^{\mathrm{T}} = I$. Combining Eq.(6) with Eq.(7) yields the local DCT domain Volterra model as follows:

$$\begin{aligned} x(t_i+T) =& A + BX + X^{\mathrm{T}} Q^{\mathrm{T}} \Lambda Q X \\ =& A + BX + \sum_{j=1}^{m} \lambda_j x^2(t_i - (j-1)\tau), \end{aligned} \tag{8}$$

where DCT is selected to obtain unitary matrix Q and implement the orthogonal transformation.[16] The coefficient number in Eq.(8) is $2m+1$ which is significantly less than that of Eq.(5). The structure of this local DCT domain Volterra predictor is depicted in Fig.1; the coefficients can be estimated by the least squares method combined with the nearest neighbours searched.

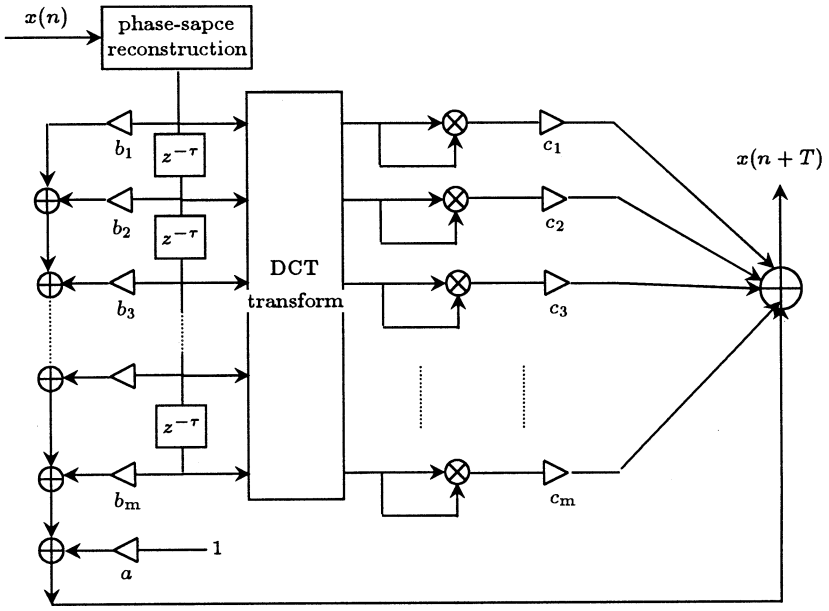


Fig.1. The structure of local DCT domain Volterra predictor.

3. Simulation and results

To illustrate the effectiveness of the local DCT domain Volterra predictor, the Henon map, Lorenz system and Mackey–Glass differential equation are chosen as examples of chaotic time series predictions. Each example uses the corresponding 2000 values after omitting initial transient points. The experimental data are divided into two parts: the first 1800 values are the training set used to search nearest neighbours, and the others are treated as the test set. The mean-square-error (MSE) and the relative mean-square-error (RMSE) are used to evaluate the prediction accuracy, where

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}[x(i) - \hat{x}(i)]^2, \tag{9}$$

$$\mathrm{RMSE} = \sum_{i=1}^{n}[x(i) - \hat{x}(i)]^2 \Big/ \sum_{i=1}^{n} x^2(i). \tag{10}$$

In the simulation 2, 3 and 3 are selected as the embedding dimensions of Henon map, Lorenz system, and Mackey–Glass differential equation chaotic time series respectively, and the time delay τ is 1 in order to reconstruct the phase space of chaotic time series given above. Numerical experiments are given in Figs.2–7.

Example 1. Henon map with initial conditions $a = 1.4$, $b = 0.3$ and $x_0 = 0.36$, $x_1 = 0.64$.

$$\begin{cases} x_{t+1} = 1 - ax_t^2 + y_t, \\ y_{t+1} = bx_t. \end{cases} \tag{11}$$

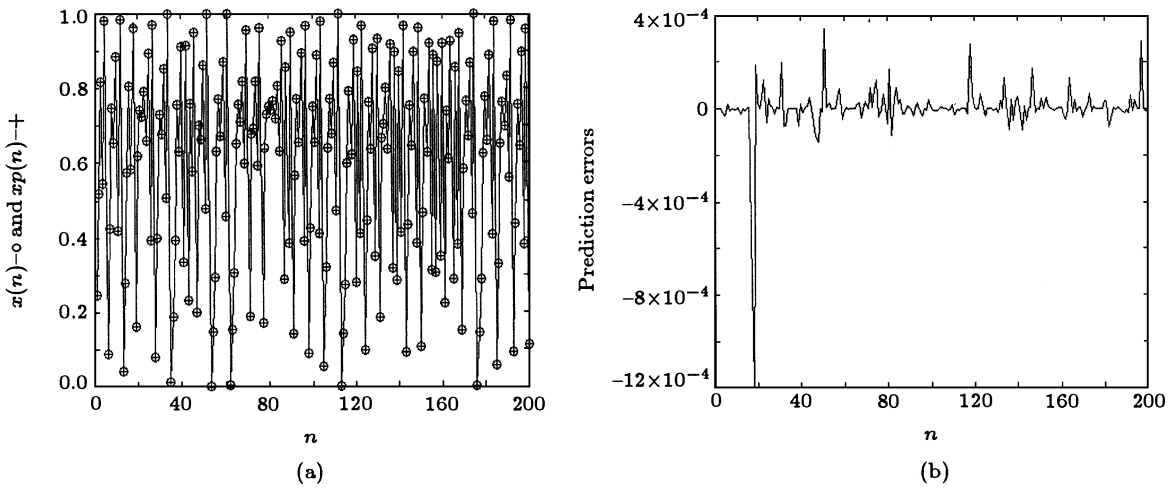


Fig.2. Prediction step=1:(a) Results of the original and predicted Henon maps, (b) Prediction errors. (MSE=1.2029×10^{-8}, RMSE=2.7461×10^{-8}).

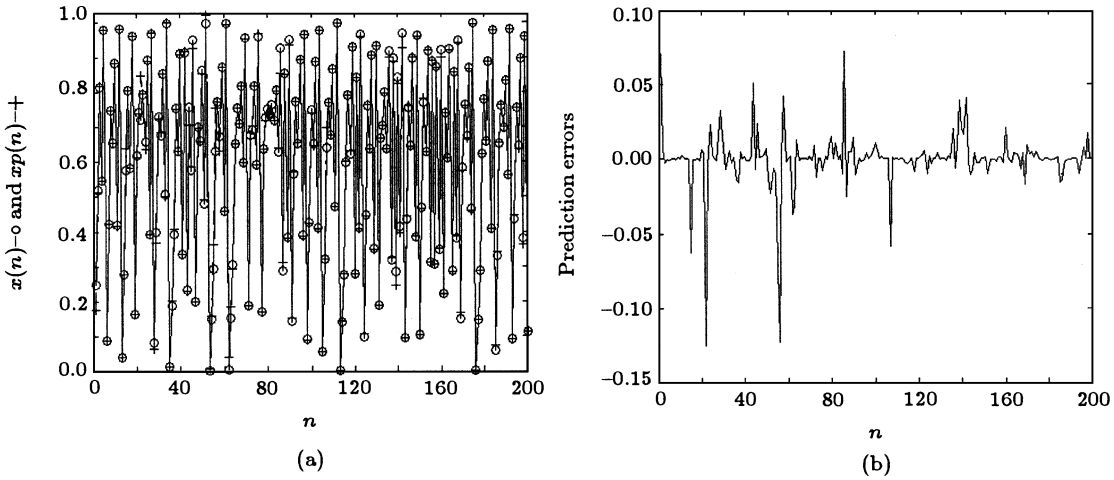


Fig.3. Prediction step=6:(a) Results of the original and predicted Henon maps, (b) Prediction errors (MSE=0.00036356, RMSE=0.00082997).

Example 2. Lorenz chaotic system with initial conditions $\sigma = 10$, $r = 28$, $b = 8/3$.

$$\begin{cases} \dot{x} = \sigma(x + y), \\ \dot{y} = -xz + rx - y, \\ \dot{z} = xy - bz. \end{cases} \tag{12}$$

Example 3. Mackey–Glass differential equation with $\Delta = 17$.

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -0.1x(t) + \frac{0.2x(t-\Delta)}{1 + x(t-\Delta)^{10}} \tag{13}$$

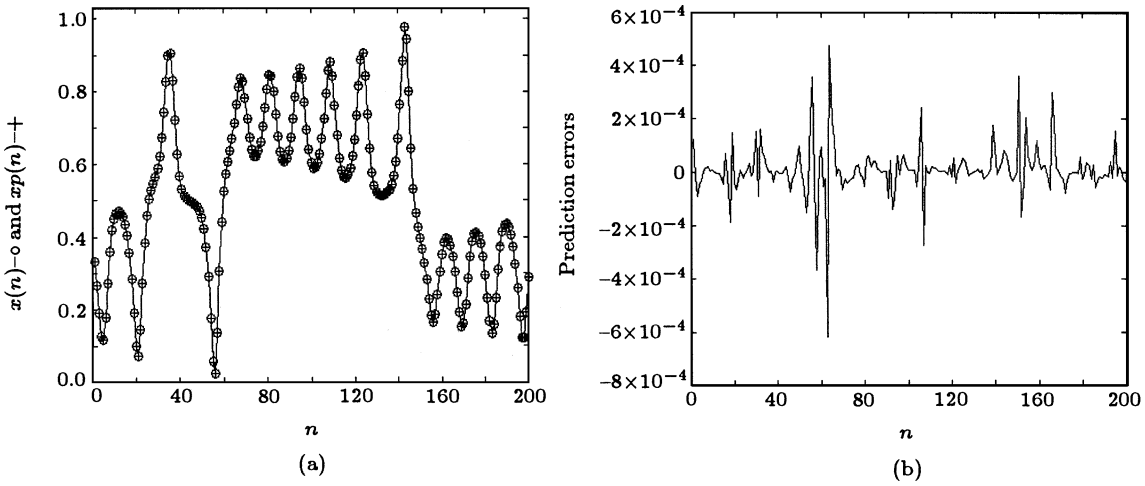


Fig.4. Prediction step=1: (a) results of the original and predicted Lorenz systems, (b) prediction errors (MSE=9.4095×10^{-9}, RMSE=3.065×10^{-8}).

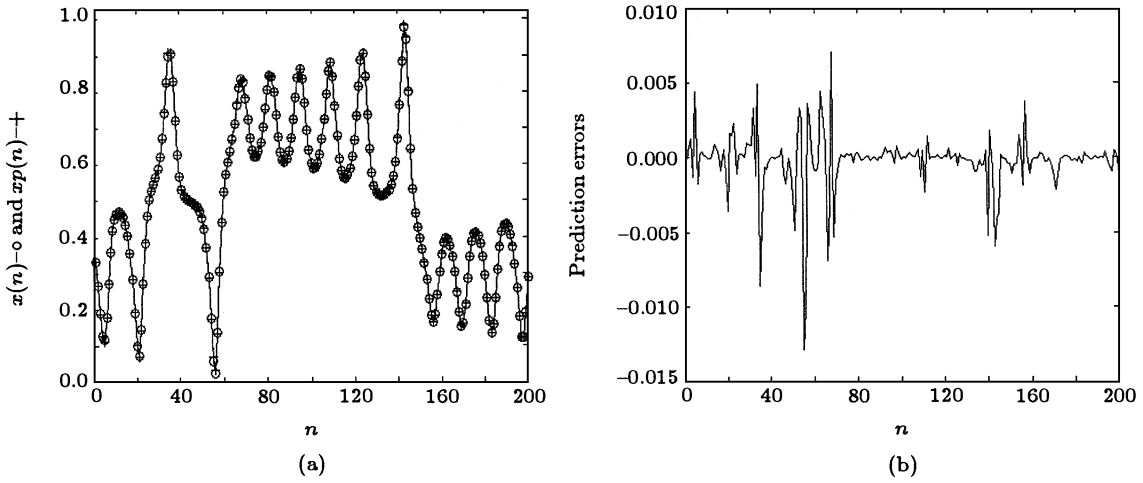


Fig.5. Prediction step=6: (a) results of the original and predicted Lorenz systems, (b) prediction errors (MSE= 4.1642×10^{-6}, RMSE=1.3565×10^{-5}).

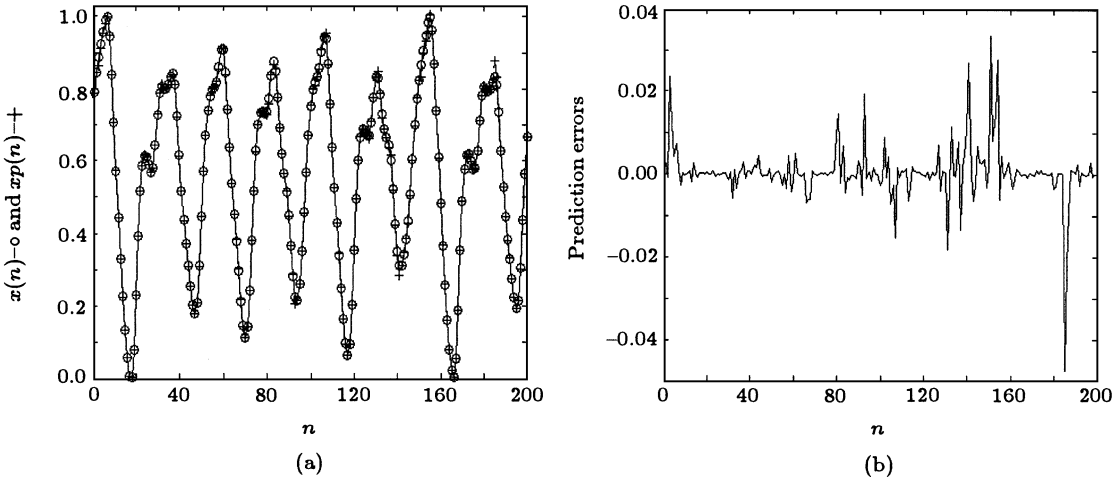


Fig.6. Prediction step=1: (a) results of the original and predicted Mackey–Glass differential equations, (b) prediction errors (MSE=4.641×10^{-5}, RMSE=1.1957×10^{-4}).

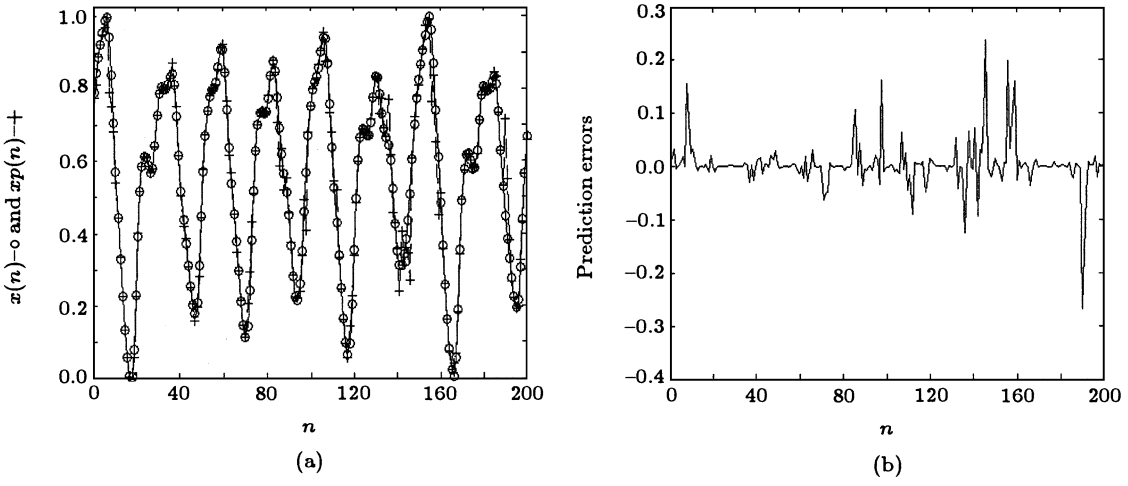


Fig.7. Prediction step=6: (a) results of the original and predicted Mackey–Glass differential equations, (b) prediction errors, (MSE=2.0223×10^{-3}, RMSE=5.2101×10^{-3}).

From these experimental results listed above, we can see clearly that this local DCT domain Volterra prediction method can effectively predict chaotic time series and have better prediction accuracy. The MSEs of three kinds of chaotic time series are less than 5.000×10^{-5} for single-step prediction, and MSEs of their six-step prediction are below 0.0025. In the meantime, the prediction performance degrades while moving from one-step prediction to multi-step prediction, which affirms that the chaotic time series are locally predictable and have short-term prediction characteristics.

The following issue is to explore the prediction accuracy of the proposed method in this paper and local first-order prediction method[8,9] using the same chaotic time series given above. Table 1 presents the comparative results of two kinds of local prediction methods.

Table 1. The comparative results of two kinds of local prediction methods.

Chaotic time series	Local first-order prediction method		Local DCT domain Volterra prediction method	
	Single step	Six step	Single step	Six step
Henon map	3.1317×10^{-8}	0.0011576	1.2029×10^{-8}	0.00036356
Lorenz system	8.6308×10^{-8}	1.0706×10^{-5}	9.4095×10^{-9}	4.1642×10^{-6}
Mackey–Glass equation	5.1165×10^{-5}	0.0022894	4.641×10^{-5}	0.0020223

The results presented in Table 1 illustrate that the proposed local DCT domain Volterra prediction method provides superior prediction accuracy compared to local first-order prediction method not only for one-step prediction but also for multi-step prediction, which is in accordance with the theoretical analytical results that quadratic polynomials have better nonlinear approximation capability than the first-order linear prediction method. At the same time, the proposed method overcomes the scenario that traditional local quadratic polynomial is difficult to implement because of too many coupling coefficients. Consequently, there exists a tradeoff between prediction accuracy and calculation complexity.

4. Conclusion

In this paper, a local DCT domain Volterra prediction method is proposed to predict chaotic time series, where the DCT is used to reduce the complexity of solving the coefficient matrix. Computer simulations show that the proposed local DCT domain Volterra model can not only provide good prediction performance and high fitting accuracy compared with the local first-order models, but can also reduce computation complexity through orthogonal DCT compared with previous local polynomial models.

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