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Direct approach to perturbation theory for bright solitons*

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A direct approach to perturbation theory for the nonlinear Schrödinger equation is developed based on the separation of variables. The first-order effects of perturbation on a bright soliton, i.e. the slow time dependence of soliton parameters and the first-order correction are derived.

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1. Introduction

It is well known that the nonlinear Schrödinger equation (NLSE)

$$\mathrm{i}u_t + su_{xx} + 2|u|^2u = 0\,, \qquad s = \pm 1 \qquad (1)$$

is one of the most important nonlinear evolution equations, since it has a lot of applications in many fields of physics. For example, it governs the evolution of optical solitons in waveguides, with $s = \pm 1$ corresponding to the case of bright solitons propagating in self-focusing media or dark solitons in self-defocusing media, respectively.[1−3] In recent years, a great deal of study on the solitons (both bright and dark solitons) in Bose–Einstein condensates (BEC) has been undertaken.[4−7] In a BEC, the evolution equation governing the soliton is also the NLSE with $s = \pm 1$ corresponding to attraction or repulsion between the particles respectively. However, in physics, the standard soliton equation is highly idealized. Therefore, in this paper, we study a NLSE slightly different from the standard one due to small additional terms that are called perturbations. One of the powerful techniques in dealing with the perturbed soliton equations is based on the inverse scattering transformation (IST).[8,9] Of course, this technique is inconvenient to those who are not familiar with IST. Another alternative way to study soliton perturbations is the direct method.[10−15] For example, Herman has studied the perturbed NLSE for $s = 1$ (perturbation for bright soliton,) in a direct approach,[13] where the squared Jost solutions are employed as the basis for perturbation expansion, and so this method is still dependent on the IST. Eight years later, Chen Xiang-Jun *et al* dealt with the dark soliton in a similar way.[14] At the same time, we developed a direct approach to perturbations on solitons,[15−19] which relies on no tools of IST. Among them Ref.[15] studied perturbations on a bright soliton. Although in Ref.[15] the mathematics is somewhat simplified in comparison with that in Ref.[13], we think it is still complicated artificially. Therefore, from the viewpoint of practice, making further simplification of the mathematics in Ref.[15] is of significance, owing to the widespread applications of NLSE. The main improvement to Ref.[15], in this paper, is that the perturbed NLSE is studied together with its complex conjugate, and as a result, we shall deal with equations of two components throughout. Although in this paper we merely study the perturbation for bright solitons, we believe that our method can be extended directly to deal with the dark soliton, which will be performed in the near future.

2. Linearization

Now we consider the perturbed NLSE

$$\mathrm{i}u_t + u_{xx} + 2|u|^2u = \mathrm{i}\epsilon f[u], \qquad (2)$$

where $\mathrm{i}=\sqrt{-1}$, the subscripts stand for partial differentiation with respect to time t and space x, ϵ is a

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small positive constant measuring the weakness of the perturbation ($0 < \epsilon \ll 1$). The perturbation term $f[u]$ is a known functional of u. It is commonly known that the standard NLSE has a single-soliton solution:

$$u_0(x,t) = 2\beta\ \mathrm{sech}[2\beta(x - x_0 + 4\alpha t)] \times \exp[-2\mathrm{i}\alpha x - 4\mathrm{i}(\alpha^2 - \beta^2)t - \mathrm{i}\theta_0], \quad (3)$$

where α, β, x_0 and θ_0 are four real parameters. In this paper, we study the effects of perturbation on a single soliton up to the first order, i.e. we seek for an approximate solution of Eq.(2):

$$u(x,t) = u_0(x,t) + \epsilon v(x,t), \quad (4)$$

with initial conditions $u(x,0) = u_0(x,0)$, $v(x,0) = 0$. Besides, in order to eliminate the so-called secular terms, we introduce a slow time variable $\tau = \epsilon t$, and then the derivative with respect to t becomes $\partial_t \to \partial_t + \epsilon\partial_\tau$. As a result, Eq.(2) is linearized into the following two equations (with appropriate initial conditions)

$$\mathrm{i}\frac{\partial u_0}{\partial t} + \frac{\partial u_0}{\partial x^2} + 2|u_0|^2 u_0 = 0,$$

$$u_0(x,0) = 2\beta\ \mathrm{sech}[2\beta(x - x_0)]\exp(-2\mathrm{i}\alpha x - \mathrm{i}\theta_0), \quad (5)$$

$$\mathrm{i}v_t + v_{xx} + 4|u_0|^2 v + 2u_0^2\overline{v} = \mathrm{i}f[u_0] - \mathrm{i}\frac{\partial u_0}{\partial \tau} \equiv \mathrm{i}r[u_0],$$

$$v(x,0) = 0, \quad (6)$$

where an overbar signifies the complex conjugate. The zeroth-order approximation equation (5) is just the standard NLSE. It has a single-soliton solution that is formally the same as Eq.(3):

$$u_0(x,t) = 2\beta\ e^{-\mathrm{i}\theta}\ \mathrm{sech}z \quad (7)$$

with

$$\begin{aligned} z &= 2\beta(x - \xi), \\ \theta &= 2\alpha(x - \xi) + \delta = \alpha z/\beta + \delta, \\ \xi_t &= -4\alpha, \\ \delta_t &= -4(\alpha^2 + \beta^2)\ . \end{aligned} \quad (8)$$

Due to perturbation, the soliton parameters α, β, ξ and δ are now supposed to be functions of the slow time variable τ, but α and β are independent of t, and the t dependence of ξ and δ are given by the last two equations in Eq.(8), respectively. It follows from Eq.(7) that

$$\frac{\partial u_0}{\partial \tau} = \mathrm{e}^{-\mathrm{i}\theta}[2\mathrm{i}\beta(2\alpha\xi_\tau - \delta_\tau)\phi^{(1)}(z) - 2\mathrm{i}\alpha_\tau\phi^{(2)}(z) + 2\beta_\tau\phi^{(3)}(z) + 4\beta^2\xi_\tau\phi^{(4)}(z)], \quad (9)$$

where

$$\begin{aligned} \phi^{(1)}(z) &= \mathrm{sech}z, \\ \phi^{(2)}(z) &= z\ \mathrm{sech}z, \\ \phi^{(3)}(z) &= (1 - z\ \tanh z)\mathrm{sech}z, \\ \phi^{(4)}(z) &= \tanh z\ \mathrm{sech}z\ . \end{aligned} \quad (10)$$

It is more convenient to use z (a space variable in a coordinate system moving with the soliton) as a new independent variable in place of x. Then Eq.(6) and its complex conjugate together with their initial conditions are reduced to the following form:

$$\begin{aligned} &\mathrm{i}v_t + 8\mathrm{i}\alpha\beta v_z + 4\beta^2 v_{zz} + 16\beta^2\mathrm{sech}^2 z\ v \\ &+ 8\beta^2\mathrm{sech}z\mathrm{e}^{-2\mathrm{i}\theta}\ \overline{v} = \mathrm{i}r, \quad v(z,0) = 0, \\ &\mathrm{i}\overline{v}_t + 8\mathrm{i}\alpha\beta\overline{v}_z - 4\beta^2\overline{v}_{zz} - 16\beta^2\mathrm{sech}^2 z\ \overline{v} \\ &- 8\beta^2\mathrm{sech}^2 z\mathrm{e}^{2\mathrm{i}\theta}\ v = \mathrm{i}\overline{r}, \quad \overline{v}(z,0) = 0\ . \end{aligned} \quad (11)$$

Letting

$$v = \mathrm{e}^{-\mathrm{i}\theta}w, \qquad \overline{v} = \mathrm{e}^{\mathrm{i}\theta}\overline{w}, \quad (12)$$

one can rewrite Eq.(11) in a form of matrix differential equation

$$\mathrm{i}W_t + 4\beta^2\hat{L}W = \mathrm{i}\mathrm{e}^{\mathrm{i}\sigma_3\theta}R, \qquad W(z,0) = 0\ . \quad (13)$$

Here σ_3 is the third Pauli matrix, W and R are column matrices with two components, and $\hat{L}$ is a 2×2 matrix. They are defined as follows:

$$W = \begin{pmatrix} w \\ \overline{w} \end{pmatrix}, \quad R = \begin{pmatrix} r \\ \overline{r} \end{pmatrix},$$

$$\hat{L} = \begin{pmatrix} \partial_z^2 + 4\ \mathrm{sech}^2 z - 1 & 2\ \mathrm{sech}^2 z \\ -2\ \mathrm{sech}^2 z & -\partial_z^2 - 4\ \mathrm{sech}^2 z + 1 \end{pmatrix}. \quad (14)$$

3. Eigenvalue problem

To find the solutions of Eq.(13) based on the separation of variables, the key is to solve the following two eigenvalue problems:

$$\hat{L}\,\Phi = \lambda\Phi, \qquad \hat{L}^\dagger\Psi = \lambda'\Psi, \quad (15)$$

where $\hat{L}^\dagger$ is the adjoint operator of $\hat{L}$:

$$\hat{L}^\dagger = \begin{pmatrix} \partial_z^2 + 4\ \mathrm{sech}^2 z - 1 & -2\ \mathrm{sech}^2 z \\ 2\ \mathrm{sech}^2 z & -\partial_z^2 - 4\ \mathrm{sech}^2 z + 1 \end{pmatrix}. \quad (16)$$

We have derived all the eigenfunctions of $\hat{L}$ and $\hat{L}^\dagger$ in a way somewhat different from that used in Ref.[15].

However, the process of derivation is omitted since space is limited. The eigenfunctions for continuous eigenvalues $\lambda = \lambda' = \lambda_{\pm}(k) = \pm(k^2+1),\ -\infty < k < \infty$ are derived as

$$\Phi^{(+)}(z,k) = \frac{\mathrm{e}^{\mathrm{i}kz}}{\sqrt{2\pi}(k^2+1)}\begin{pmatrix} -\mathrm{sech}^2 z \\ (\tanh z - \mathrm{i}k)^2 \end{pmatrix},$$

$$\Phi^{(-)}(z,k) = \frac{\mathrm{e}^{\mathrm{i}kz}}{\sqrt{2\pi}(k^2+1)}\begin{pmatrix} (\tanh z - \mathrm{i}k)^2 \\ -\mathrm{sech}^2 z \end{pmatrix}, \quad (17)$$

$$\Psi^{(+)}(z,k) = \frac{\mathrm{e}^{\mathrm{i}kz}}{\sqrt{2\pi}(k^2+1)}\begin{pmatrix} \mathrm{sech}^2 z \\ (\tanh z - \mathrm{i}k)^2 \end{pmatrix},$$

$$\Psi^{(-)}(z,k) = \frac{\mathrm{e}^{\mathrm{i}kz}}{\sqrt{2\pi}(k^2+1)}\begin{pmatrix} (\tanh z - \mathrm{i}k)^2 \\ \mathrm{sech}^2 z \end{pmatrix}. \quad (18)$$

It is not difficult to prove that $\Phi^{(\pm)}(z,k)$ and $\Psi^{(\pm)}(z,k)$ are really solutions of Eq.(15). Moreover, $\hat{L}$ and $\hat{L}^{\dagger}$ also have eigenfunctions corresponding to discrete eigenvalue $\lambda_0 = 0$, which can be constructed by $\phi^{(j)}(z),\ j = 1,2,3,4$ given in Eq.(10):

$$\Phi^{(1)}(z) = \frac{1}{\sqrt{2}}\phi^{(1)}(z)\begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

$$\Phi^{(4)}(z) = \frac{1}{\sqrt{2}}\phi^{(4)}(z)\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (19)$$

$$\Psi^{(2)}(z) = \frac{1}{\sqrt{2}}\phi^{(4)}(z)\begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

$$\Psi^{(3)}(z) = \frac{1}{\sqrt{2}}\phi^{(1)}(z)\begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (20)$$

It is easy to check directly that

$$\begin{aligned} &\hat{L}\Phi^{(1)}(z) = \hat{L}\Phi^{(4)}(z) = 0, \\ &\hat{L}^{\dagger}\Psi^{(2)}(z) = \hat{L}^{\dagger}\Psi^{(3)}(z) = 0\ . \end{aligned} \quad (21)$$

Since operators $\hat{L}$ and $\hat{L}^{\dagger}$ are not self adjoint, their eigenfunctions are not complete. Therefore, more functions must be added in order to construct a complete basis for perturbation expansion. For this purpose, we introduce

$$\Phi^{(2)}(z) = \frac{1}{\sqrt{2}}\phi^{(2)}(z)\begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

$$\Phi^{(3)}(z) = \frac{1}{\sqrt{2}}\phi^{(3)}(z)\begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$\Psi^{(1)}(z) = \frac{1}{\sqrt{2}}\phi^{(3)}(z)\begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

$$\Psi^{(4)}(z) = \frac{1}{\sqrt{2}}\phi^{(2)}(z)\begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (22)$$

It must be pointed out emphatically that the functions given in Eq.(22) are all not eigenfunctions. In fact, they satisfy the following equations:

$$\begin{aligned} &\hat{L}\Phi^{(2)}(z) = -2\Phi^{(4)}(z), \\ &\hat{L}\Phi^{(3)}(z) = 2\Phi^{(1)}(z), \\ &\hat{L}^{\dagger}\Psi^{(1)}(z) = 2\Psi^{(3)}(z), \\ &\hat{L}^{\dagger}\Psi^{(4)}(z) = -2\Psi^{(2)}(z)\ . \end{aligned} \quad (23)$$

Now we have obtained two complete sets: $\Phi = \{\Phi^{(+)}(z,k), \Phi^{(-)}(z,k),\ \Phi^{(j)}(z); j = 1,2,3,4\}$ and $\Psi = \{\Psi^{(+)}(z,k), \Psi^{(-)}(z,k),\ \Psi^{(j)}(z); j = 1,2,3,4\}$. They satisfy the following completeness relation:

$$\begin{aligned} &\int_{-\infty}^{\infty} \mathrm{d}k[\Phi^{(+)}(z,k)\overline{\Psi}^{(+)}(z',k) \\ &+ \Phi^{(-)}(z,k)\overline{\Psi}^{(-)}(z',k)] \\ &+ \sum_{j=1}^{4} \Phi^{(j)}(z)\overline{\Psi}^{(j)}(z') = I\ \delta(z-z'), \end{aligned} \quad (24)$$

where an overbar stands for the transposed conjugate and I is a 2×2 unit matrix. The completeness relation (24) can be derived in a similar way to that in Ref.[15], but the derivation is omitted for the same reason. Besides, sets $\{\Phi\}$ and $\{\Psi\}$ are orthonormal in the following meaning:

$$\int_{-\infty}^{\infty} \mathrm{d}z\ \overline{\Psi}^{(\pm)}(z,k')\Phi^{(\pm)}(z,k) = \delta(k-k'), \quad (25)$$

$$\int_{-\infty}^{\infty} \mathrm{d}z\ \overline{\Psi}^{(i)}(z)\Phi^{(j)}(z) = \delta_{ij}, \quad (26)$$

$$\begin{aligned} &\int_{-\infty}^{\infty} \mathrm{d}z\ \overline{\Psi}^{(\mp)}(z,k')\Phi^{(\pm)}(z,k) \\ &= \int_{-\infty}^{\infty} \mathrm{d}z\ \overline{\Psi}^{(j)}(z)\Phi^{(\pm)}(z,k) = 0\ . \end{aligned} \quad (27)$$

4. First-order effects of perturbation on a soliton

Now we return to Eq.(13). To solve it by the separation of variables, we expand W on the basis $\{\Phi\}$

as follows:

$$W(z,t)=\int_{-\infty}^{\infty}\mathrm{d}k\,[a^{(+)}(t,k)\,\Phi^{(+)}(z,k) + a^{(-)}(t,k)\,\Phi^{(-)}(z,k)] + \sum_{j=1}^{4}a^{(j)}(t)\,\Phi^{(j)}(z)\,. \tag{28}$$

Substituting Eq.(28) into Eq.(13), and making use of the orthonormal relations (25)–(27), we obtain the following ordinary differential equations with the appropriate initial conditions

$$\mathrm{i}\dot{a}^{(\pm)}(t,k)\pm 4\beta^2(k^2+1)a^{(\pm)}(t,k)=\mathrm{i}b^{(\pm)}(k)$$
$$=\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}z\;\overline{\Psi}^{(\pm)}(z,k)\mathrm{e}^{\mathrm{i}\sigma_3\theta}R[u_0(z)],$$
$$a^{(\pm)}(0,k)=0, \tag{29}$$
$$\mathrm{i}\dot{a}^{(1)}(t)+8\beta^2a^{(3)}(t)=\mathrm{i}b^{(1)}$$
$$=\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}z\;\overline{\Psi}^{(1)}(z)\mathrm{e}^{\mathrm{i}\sigma_3\theta}R[u_0(z)],\quad a^{(1)}(0)=0, \tag{30}$$
$$\mathrm{i}\dot{a}^{(2)}(t)=\mathrm{i}b^{(2)}$$
$$=\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}z\;\overline{\Psi}^{(2)}(z)\mathrm{e}^{\mathrm{i}\sigma_3\theta}R[u_0(z)],\quad a^{(2)}(0)=0, \tag{31}$$
$$\mathrm{i}\dot{a}^{(3)}(t)=\mathrm{i}b^{(3)}$$
$$=\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}z\;\overline{\Psi}^{(3)}(z)\mathrm{e}^{\mathrm{i}\sigma_3\theta}R[u_0(z)],\quad a^{(3)}(0)=0, \tag{32}$$
$$\mathrm{i}\dot{a}^{(4)}(t)-8\beta^2a^{(2)}(t)=\mathrm{i}b^{(4)}$$
$$=\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}z\;\overline{\Psi}^{(4)}(z)\mathrm{e}^{\mathrm{i}\sigma_3\theta}R[u_0(z)],\quad a^{(4)}(0)=0, \tag{33}$$

where the overdot denotes the derivative with respect to t. Since all $b^{(j)}$, $j=1,2,3,4$ are constants, Eqs.(30)–(33) will lead to secularity. In fact, Eqs.(31) and (32) yield $a^{(2)}=b^{(2)}t$ and $a^{(3)}=b^{(3)}t$ respectively, and then $a^{(2)}$ and $a^{(3)}$ grow infinitely in time. To eliminate these secularities, we must demand that

$$b^{(2)}=0\longrightarrow a^{(2)}(t)=0,\quad b^{(3)}=0\longrightarrow a^{(3)}(t)=0\,. \tag{34}$$

Then Eqs.(30) and (33) are reduced to $\dot{a}^{(1)}=b^{(1)}$ and $\dot{a}^{(4)}=b^{(4)}$, respectively, and similarly

$$b^{(1)}=0\longrightarrow a^{(1)}(t)=0,\quad b^{(4)}=0\longrightarrow a^{(4)}(t)=0\,. \tag{35}$$

Now, inserting Eqs.(34) and (35) into Eqs.(30)–(33), we obtain the following four important formulae immediately:

$$\alpha_\tau=\frac{1}{2\sqrt{2}\mathrm{i}}\int_{-\infty}^{\infty}\mathrm{d}z\;\overline{\Psi}^{(2)}(z)\mathrm{e}^{\mathrm{i}\sigma_3\theta}F[u_0(z)],$$

$$F[u_0]=\begin{pmatrix} f[u_0]\\ \overline{f}[u_0]\end{pmatrix}, \tag{36}$$

$$\beta_\tau=-\frac{1}{2\sqrt{2}}\int_{-\infty}^{\infty}\mathrm{d}z\;\overline{\Psi}^{(3)}(z)\mathrm{e}^{\mathrm{i}\sigma_3\theta}F[u_0(z)], \tag{37}$$

$$\xi_\tau=-\frac{1}{4\sqrt{2}\beta^2}\int_{-\infty}^{\infty}\mathrm{d}z\;\overline{\Psi}^{(4)}(z)\mathrm{e}^{\mathrm{i}\sigma_3\theta}F[u_0(z)], \tag{38}$$

$$\delta_\tau=\frac{1}{2\sqrt{2}\beta}\int_{-\infty}^{\infty}\mathrm{d}z\;[-\alpha\overline{\Psi}^{(4)}(z) + \mathrm{i}\beta\overline{\Psi}^{(1)}(z)]\mathrm{e}^{\mathrm{i}\sigma_3\theta}F[u_0(z)]\,. \tag{39}$$

Equations (36)–(39) give the τ-dependence of all the soliton parameters. They determine how the shape and position of soliton are affected by the perturbation. As to Eq.(29), it can be solved easily by the standard method:

$$a^{(\pm)}(t,k)=\pm\frac{\mathrm{i}}{4\beta^2(k^2+1)}\{1-\exp[\pm 4\mathrm{i}\beta^2(k^2+1)t]\} \times\int_{-\infty}^{\infty}\mathrm{d}z'\;\overline{\Psi}^{(\pm)}(z',k)\mathrm{e}^{\mathrm{i}\sigma_3\theta}R[u_0(z')]\,. \tag{40}$$

Substituting Eq.(40) into Eq.(28), and noting that $\alpha^{(j)}(t)=0, j=1,2,3,4$, we obtain

$$W(z,t)=\frac{\mathrm{i}}{4\beta^2}\int_{-\infty}^{\infty}\frac{\mathrm{d}k}{k^2+1}[1-\mathrm{e}^{4\mathrm{i}\beta^2(k^2+1)t}]\,\Phi^{(+)}(z,k) \times\int_{-\infty}^{\infty}\mathrm{d}z'\overline{\Psi}^{(+)}(z',k)\mathrm{e}^{\mathrm{i}\sigma_3\theta'}R[u_0(z')] - \frac{\mathrm{i}}{4\beta^2}\int_{-\infty}^{\infty}\frac{\mathrm{d}k}{k^2+1}[1-\mathrm{e}^{-4\mathrm{i}\beta^2(k^2+1)t}]\,\Phi^{(-)}(z,k) \times\int_{-\infty}^{\infty}\mathrm{d}z'\overline{\Psi}^{(-)}(z',k)\mathrm{e}^{\mathrm{i}\sigma\theta'}R[u_0(z')], \tag{41}$$

where $\theta'=\theta(z')=\alpha z'/\beta+\delta$. Making use of Eq.(12), we can rewrite Eq.(41) as

$$V(z,t)=\begin{pmatrix} v(z,t)\\ \overline{v}(z,t)\end{pmatrix}=\int_{-\infty}^{\infty}\mathrm{d}z'G(z,z';t)R[u_0(z')], \tag{42}$$

where $G(z,z';t)$ is the so-called Green's function:

$$G(z,z';t)=\mathrm{e}^{-\mathrm{i}\sigma_3\theta}\int_{-\infty}^{\infty}\frac{\mathrm{i}\mathrm{d}k}{4\beta^2(k^2+1)} \times\{[1-\mathrm{e}^{4\mathrm{i}\beta^2(k^2+1)t}]\,\Phi^{(+)}(z,k)\overline{\Psi}^{(+)}(z',k) - [1-\mathrm{e}^{-4\mathrm{i}\beta^2(k^2+1)t}] \times\Phi^{(-)}(z,k)\overline{\Psi}^{(-)}(z',k)\}\mathrm{e}^{\mathrm{i}\sigma_3\theta'}\,. \tag{43}$$

Obviously, Eq.(42) gives an additional correction for a soliton due to perturbation.

Finally, as an example, we study the dissipation NLSE in which $f[u_0(z)]=-u_0(z)=-2\beta\,\mathrm{sech}z\;\mathrm{e}^{-\mathrm{i}\theta}$. From Eqs.(36)–(39), one can easily calculate that $\beta_\tau=2\beta$, and $\alpha_\tau=\xi_\tau=\delta_\tau=0$. These results mean that the height of soliton decreases while the width increases with time until it finally disappears.

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