Vol 13 No 9, September 2004
1009-1963/2004/13(09)/1588-03

Chinese Physics

© 2004 Chin. Phys. Soc.
and IOP Publishing Ltd

Mass neutrino oscillations in Reissner–Nordstrom space–time*

Huang Xiu-Ju(黄秀菊) and Wang Yong-Jiu(王永久)†

Institute of Physics, Hunan Normal University, Changsha 410081, *China*

(Received 17 January 2004; revised manuscript received 16 February 2004)

Along the geodesic we calculate the interference phase of the mass neutrinos propagating in the radial direction in the Reissner–Nordstrom field. From the calculation we find that, though there exists electric charge in the gravitational source, the phase increases very little compared with that in the Schwarzschild field.

Keywords: interference phase, mass neutrino, Reissner–Nordstrom space–time
PACC: 9760L, 0460K, 0470D

1. Introduction

Neutrino oscillations have been a hot topic recently. Neutrino oscillations in a flat space–time have been extensively studied in the past by using both plane waves[1] and wave packets.[2] Many authors have calculated gravitational effects on neutrino oscillations in curved space–time. Also, mass neutrino oscillations have aroused great interest in high energy experimental and theoretical physics recently, as a consequence of, in particular, the high confidence atmospheric neutrino experiment of Super-Kamiokande to assure the neutrino mass.[3] In 1996, Fornengo and co-workers[4] studied the interference phase of the high-energy mass neutrinos when they propagate in the radial and non-radial directions respectively in Schwarzschild field. In 2001, Zhang and Beesham[5] also studied the propagation of neutrinos in the Schwarzschild field with a different method, and they obtained that the phase calculated along the null is equivalent to the half phase along the geodesic in the high energy limit, which means that the correct relative phase of the mass neutrinos is either the null phase or the half geodesic phase, and they also pointed out that the convenient use of the energy condition was very important in calculating the interference phase of the mass neutrinos. With the same method, in 2003, we calculated the interference phase in Kerr space–time.[6]

In this paper, we will discuss the interference phase of the mass neutrinos in the Reissner–Nordstrom (RN) field, and discuss the influence on the phase because of the existence of the electric charge in the gravitational source. We set $G = \eta = c = 1$ throughout this paper.

2. Interference phase in the RN field

In curved space–time, the phase factor is conventionally written as[7]

$$\Phi = \int m \mathrm{d}s, \tag{1}$$

where the phase factor Φ is also the classical Lagrange function for the particle motion, and the route is along the geodesic determined by the variational principle or Jacob–Hamilton equation,[8] and $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$.

Now we follow the Zhang and Beesham method to study the phase of the neutrino oscillation in the radial direction in RN space–time, and the route is along the geodesic. In the RN field, the spherically symmetric static space–time can be globally represented as

$$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 + g_{11}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2, \tag{2}$$

and

$$g_{00} = (-g_{11})^{-1} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}.$$

*Project supported by the National Basic Research Programme of China (Grant No 2003CB716300) and Hunan Education Department (NSF 03A028, 02C203).

†Corresponding author, E-mail: wyj@hunnu.edu.cn

Here M and Q are, respectively, the gravitational mass and charge of the star.

We consider the phase of the mass neutrinos in RN space–time by means of the particle geodesic equation

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}s^2}+\Gamma^\mu_{\alpha\beta}\times\frac{\mathrm{d}x^\alpha}{\mathrm{d}s}\times\frac{\mathrm{d}x^\beta}{\mathrm{d}s}=0. \tag{3}$$

Along the radial direction with $\mathrm{d}\theta=\mathrm{d}\varphi=0$, the nontrivial timelike equation is

$$\frac{\mathrm{d}^2t}{\mathrm{d}s^2}+2\Gamma^0_{01}\times\frac{\mathrm{d}t}{\mathrm{d}s}\times\frac{\mathrm{d}r}{\mathrm{d}s}=0. \tag{4}$$

Solving Eq.(4), we obtain

$$g_{00}\frac{\mathrm{d}t}{\mathrm{d}s}=\text{const}, \tag{5}$$

or we rewrite Eq.(5) as

$$E=P_0=mg_{00}\frac{\mathrm{d}t}{\mathrm{d}s}=\text{const}. \tag{6}$$

Equation (6) represents that the covariant energy is the motion constant along the geodesic, by which the calculation is proceeded. It is stressed that it is the covariant energy P_0 (not P^0) that is the constant of motion, which is important in the later calculation of the neutrino phase.

Considering $\dfrac{\mathrm{d}\theta}{\mathrm{d}s}=\dfrac{\mathrm{d}\varphi}{\mathrm{d}s}=0$, Eq.(2) can be rewritten as

$$g_{00}\left(\frac{\mathrm{d}t}{\mathrm{d}s}\right)^2+g_{11}\left(\frac{\mathrm{d}r}{\mathrm{d}s}\right)^2=1. \tag{7}$$

Substituting Eq.(6) into Eq.(7), then we obtain

$$\begin{aligned}\frac{\mathrm{d}s}{\mathrm{d}r}=&\left[\left(\frac{p_0}{m}\right)^2-1+\frac{2M}{r}-\frac{Q^2}{r^2}\right]^{-1/2}\\=&\left[k+\frac{2M}{r}-\frac{Q^2}{r^2}\right]^{-1/2},\end{aligned} \tag{8}$$

where $k=\left(\dfrac{p_0}{m}\right)^2-1$. Thus the accurate phase factor in RN space–time in the radial direction will be acquired by the next calculation:

$$\begin{aligned}\Phi=&\int m\mathrm{d}s=\int m\left(\frac{\mathrm{d}s}{\mathrm{d}r}\right)\mathrm{d}r\\=&\int m\left(k+\frac{2M}{r}-\frac{Q^2}{r^2}\right)^{-1/2}\mathrm{d}r\\=&\frac{m}{k}(\sqrt{kr^2+2Mr+Q^2}-\sqrt{kr_0^2+2Mr_0+Q^2})\\&-\frac{Mm}{k^{3/2}}\ln\frac{kr+M+\sqrt{k(kr^2+2Mr+Q^2)}}{kr_0+M+\sqrt{k(kr_0^2+2Mr_0+Q^2)}}.\end{aligned} \tag{9}$$

Expanding Eq.(8) in terms of electric charge Q, we have

$$\begin{aligned}\Phi=&\frac{m}{k}(\sqrt{kr^2+2Mr}-\sqrt{kr_0^2+2Mr_0})\\&-\frac{Mm}{k^{3/2}}\ln\frac{kr+M+\sqrt{k(kr^2+2Mr)}}{kr_0+M+\sqrt{k(kr_0^2+2Mr_0)}}\\&+\frac{m}{2\sqrt{k}}\left[\frac{\sqrt{kr^2+2Mr}+\sqrt{k}r}{\sqrt{kr^2+2Mr}(M+kr+\sqrt{k(kr^2+2Mr)})}\right.\\&\left.-\frac{\sqrt{kr_0^2+2Mr_0}+\sqrt{k}r_0}{\sqrt{kr_0^2+2Mr_0}(M+kr_0+\sqrt{k(kr_0^2+2Mr_0)})}\right]\\&\times Q^2+o(Q^4).\end{aligned} \tag{10}$$

The first two terms in the right-hand side of Eq.(10) represent the contribution to the phase by the gravitational field relative to the mass of the star. They also exactly equal the neutrino phase along the geodesic in the Schwarzschild field. This is our inevitable result. The third term represents the contribution by the gravitational field relative to the electric charge of the star. If we consider the case of a weak field, the phase will be approximately rewritten as

$$\begin{aligned}\Phi=&\frac{m}{\sqrt{k}}(r-r_0)-\frac{Mm}{k^{3/2}}\ln\frac{kr+M}{kr_0+M}\\&+\frac{mQ^2}{k^2}\left(\frac{1}{r}-\frac{1}{r_0}\right)+o(Q^4).\end{aligned} \tag{11}$$

Because $\dfrac{Q^2}{r^2}\Big/\dfrac{2M}{r}\propto\dfrac{1}{r}$, the third term is very small compared with the other two terms. That is, the contribution to the phase by the existence of the electric charge is very little.

3. Conclusion

In this paper, we have calculated the interference phase of mass neutrinos propagating in the radial direction along the geodesic in the RN field. Solving the nontrivial timelike equation, we obtain Eq.(7), from which we know that the covariant energy is the motion constant along the geodesic. This point has been stressed in Refs.[5,6]. Then we give the phase in the field. We find that among the terms of the phase, the term relative to the electric charge is very small compared with the other two terms. This means that the electric charges in the gravitational source have little influence on the phase. Furthermore, if $Q=0$, the phase in the RN field will naturally reduce to that in the Schwarzschild field. This is an inevitable result. This gravitational effect[9−14] on neutrino oscillations is intimately related to the redshift.

References

[1] Pontecorvo B 1957 *Zh. Eksp. Teor. Fiz.* **33** 549
[2] Giunti C, Kim C W and Lee U W 1991 *Phys. Rev.* D **44** 3635
[3] Fukuda Y *et al* 1998 *Phys. Rev. Lett.* **81** 1158
[4] Fornengo N *et al* 1997 *Phys. Rev.* D **56** 1895
[5] Zhang C M and Beesham A 2001 *Gen. Rel. Grav.* **33** 1011
[6] Huang X J and Wang Y J 2003 *Commun. Theor. Phys.* (Beijing, China) **40** 742
[7] Cardall C Y and Fuller G M 1996 *Phys. Rev.* D **55** 7960
[8] Piriz D, Roy M and Wudka J 1996 *Phys. Rev.* D **54** 1587
[9] Wang Y J and Tang Z M 2002 *Astrophys. Space Sci.* **281** 689
[10] Wang Y J and Tang Z M 2002 *Sci. Chin.* Ser. A **45** 528
[11] Wang Y J and Tang Z M 2002 *Sci. Chin.* Ser. A **44** 801
[12] Huang X J and Wang Y J 2002 *Chin. Phys. Lett.* **19** 755
[13] Wang B B and Liu L 2002 *Acta Phys. Sin.* **51** 1654 (in Chinese)
[14] He H and Zhao Z 2002 *Acta Phys. Sin.* **51** 2661 (in Chinese)

Erratum

In the paper entitled "Morphological study of the localized growth of materials in dielectric barrier discharge" published on page 1076 in Vol.13 No.7 of Chinese Physics, the affiliation of Li Xue-Chen should be changed from "a" to "b", namely, "Institute of Physics, Chinese Academy of Sciences, Beijing 100080, China". And we are sorry to have made such a mistake.