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Numerical simulation for characterizing femtosecond optical pulses with the SPIDER algorithm*

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In this article based on the spectral phase interferometry for direct electric-field reconstruction (SPIDER), the femtosecond pulses with various phase characters are numerically simulated. The spectral phases and amplitudes of the transform-limited pulse, the linear chirped pulse, the cubic dispersion pulse, the quartic dispersion pulse, the self-phase modulation pulse and the pulses with the combination of different chirped characters are retrieved. These characterized pulses are applicable to the real-time measurement as samples for diagnosing the chirped characters of pulses quickly.

Keywords: SPIDER, spectral shear, phase measurement
PACC: 4280W, 4285F, 0760L

1. Introduction

In last decade, the techniques of femtosecond optical pulse measurement have been noticeably developed with the significant advance in femtosecond optical pulse generation and its applications. The accurate solutions to characterize the shape and phase of a femtosecond optical pulse have been demonstrated by experiments, such as the frequency-domain phase measurement (FDPM),[1] the frequency-resolved optical gating (FROG),[2] the spectral and temporal resolved upconversion technique (STRUT),[3] the temporal analysis of spectral components (TASC),[4] the spectral phase interferometry for direct electric-field reconstruction (SPIDER),[5] and the time-resolved optical gating based on dispersive propagation (DP-TROG).[6] Two more popular techniques among them are FROG and SPIDER, which have shown the capability of characterizing few-cycle pulses.[7] Comparing SPIDER with FROG, we know that SPIDER has an advantage of fast measurement time due to the non-iterative reconstruction algorithms.[8] Rigorous methods to obtain quantitatively the complete intensity and phase information of an arbitrary ultrashort optical pulse are assuredly important. However, it may be more essential to diagnose qualitatively the chirp information of pulses with the capability of real-time and single-shot, especially in adjusting and optimizing an ultra-strong laser system with a low repetition rate. From this point of view, the two-dimensional spectrograms of FROG[9] are more convenient and intuitive for characterizing the pulses qualitatively than SPIDER.

In this paper, based on SPIDER in order to apply them to the real-time measurement as the samples for diagnosing the chirped characters of pulses quickly, the femtosecond pulses with various phase characters are numerically simulated.

2. Inversion algorithm of SPIDER

SPIDER uses spectral shearing interferometry to retrieve the spectral phase of the test pulse.[10] As described in Ref.[10] (see Fig.1 of Ref.[10]), a test pulse is split into two replicas with a time delay τ which is generated by using a Michelson interferometer. The two test replicas are then upconverted in a BBO crystal with a third strongly chirped pulse from a portion of the same test pulse. As a result, the two test replicas

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experience a frequency shift denoted by Ω with respect to one another. The two frequency-upconverted replicas are spectrally resolved in a spectrometer and the spectrum of the interference is recorded with a CCD camera. The recorded interferogram is then analyzed by a computer.

This interferogram can be expressed as follows:[11]

$$D(\omega,\tau) = \left|\tilde{E}(\omega)\right|^2 + \left|\tilde{E}(\omega+\Omega)\right|^2 + 2\left|\tilde{E}(\omega)\,\tilde{E}(\omega+\Omega)\right| \cos\{\phi(\omega+\Omega) - \phi(\omega) + \omega\tau\}, \quad (1)$$

$$\theta(\omega) = \phi(\omega) - \phi(\omega+\Omega), \quad (2)$$

$$\Omega = -\frac{\tau}{2\ddot{\varphi}}, \quad (3)$$

where $\tilde{E}(\omega)$ and $\tilde{E}(\omega+\Omega)$ are the frequency representation of the electric fields of the two test pulse replicas with the spectral shear Ω, τ is the time delay between the two replicas and $\ddot{\varphi}$ is the second-order dispersion provided to the third chirped pulse by a stretcher. $\phi(\omega)$ is the spectral phase of the test pulse which needs to recover from the interferogram, $\theta(\omega)$ is the relative phase between successive pairs of frequency components separated by Ω.

The frequency domain interferogram is first Fourier transformed (FT) to the temporal domain. Three peaks, which are separated by the time delay τ, are obtained. The central peak in $t = 0$ is the 'DC' portion of the interferogram, and the two sideband peaks in $t = \pm\tau$ are the 'AC' portions corresponding to the cosine term in Eq.(1). Both of the two 'AC' sidebands contain the same phase information. Therefore, one 'AC' sideband peak in $t = \tau$ is picked by a super Gaussian filter and is then inverse Fourier transformed (IFT) back to the frequency domain. The argument of the frequency data is exactly the spectral phase of the cosine term in Eq.(1). The phase difference between two frequencies of the test pulse separated by Ω is shown in Eq.(2), after the linear $\omega\tau$ term is subtracted. Set a reference point ω_c, which is the central frequency of the test pulse spectrum, then the spectral phase of the test pulse can be retrieved at discrete frequencies $\{\ldots\phi(\omega_c - 2\Omega), \phi(\omega_c - \Omega), \phi(\omega_c), \phi(\omega_c + \Omega), \phi(\omega_c + 2\Omega)\ldots\}$ by a concatenation procedure. It is convenient to let the phase $\phi(\omega_c)$ be equal to zero, because the relative phase information of the test pulse can be obtained by this method only. If information of the test pulse in the temporal domain is desired, the temporal electric field is given by the Fourier transform of the spectral electric field as follows:[12]

$$E(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\omega \left\{\sqrt{I(\omega)}\exp\left[\mathrm{i}\phi(\omega)\right]\right\}\exp(\mathrm{i}\omega t), \quad (4)$$

where $I(\omega)$ is the spectrum of the test pulse. The temporal intensity and phase can be recovered from Eq.(4).

3. Results for reconstruction simulations

The test pulses are considered as the Gaussian pulses with variable chirps.[13] The electric field is written as

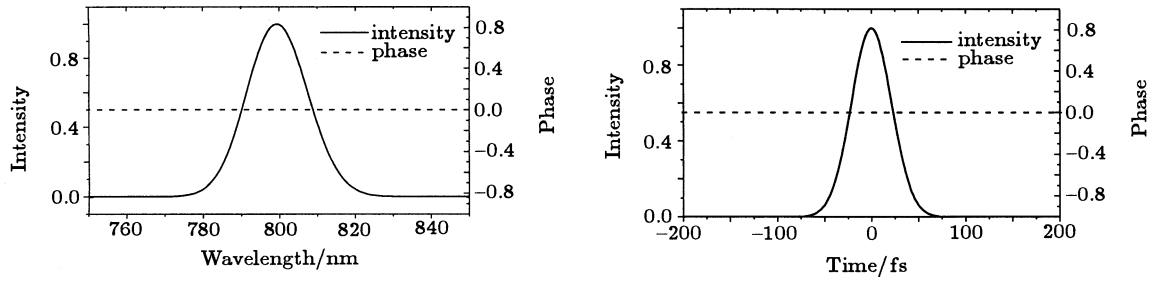
$$E(t) = \exp[-at^2 + \mathrm{i}(bt^2 + ct^3 + \ldots)]\exp(-\mathrm{i}\omega_c t). \quad (5)$$

Moreover, for the self-phase modulated pulse, the electric field is described as

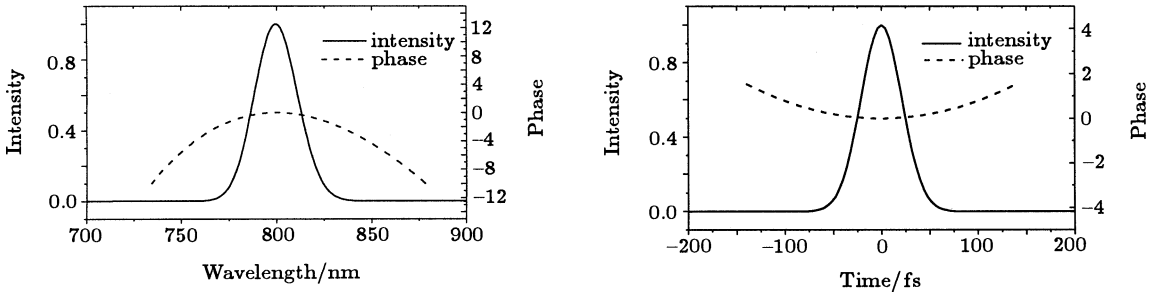
$$E(t) = \exp[-at^2 + \mathrm{i}Q\,|E(t)|^2], \quad (6)$$

where $a = 2\ln 2/t_{\mathrm{pin}}^2$, t_{pin} is the width of the incident pulse, and the coefficients b, c and d, denote respectively the quadratic, cubic and quartic components of the chirped pulse, Q is the coefficient of self-phase modulation. All the coefficients are dimensionless.

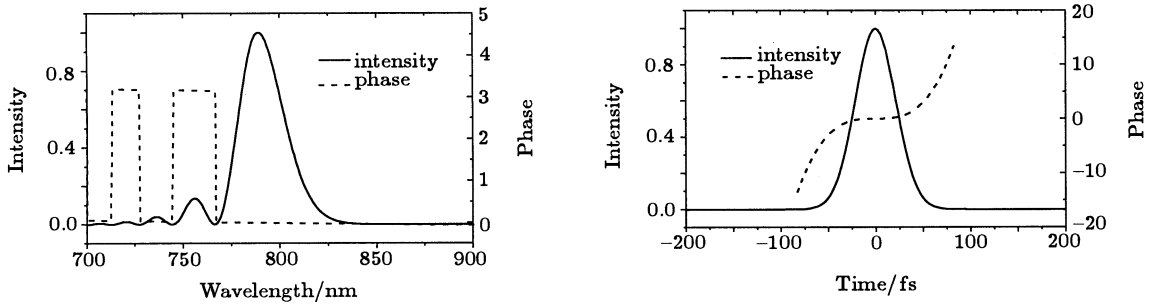
For the test pulse with a pulsewidth t_{pin}=50fs, the central wavelength λ_c=800nm, we choose the time delay between the two replicas, τ=2.1ps, and the second-order dispersion is provided to the third chirped pulse by a stretcher, $\ddot{\varphi} = 3.36 \times 10^5\mathrm{fs}^2$ corresponds to a stretched chirped pulse of 20ps. Based on the SPIDER inversion algorithm, the spectral phases and amplitudes of the transform-limited pulse, linear chirped pulse, the cubic dispersion pulse, the quartic dispersion pulse, the self-phase modulation pulse and the pulses with the combination of different chirped characters are retrieved as shown in Fig.1. These characterized pulses are applicable to the real-time measurement as samples for quick diagnosis of the chirped characters of pulses.



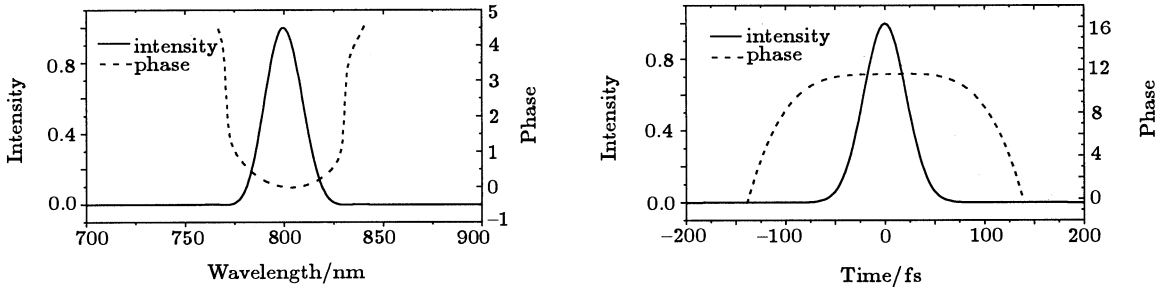
(a) Transform limited pulse: $b = c = d = 0$.



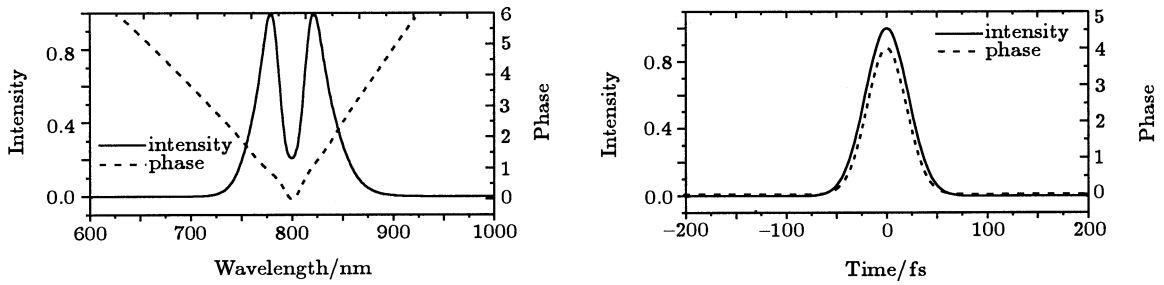
(b) Linear chirped pulse: $b = \sqrt{2}a$, $c = d = 0$.



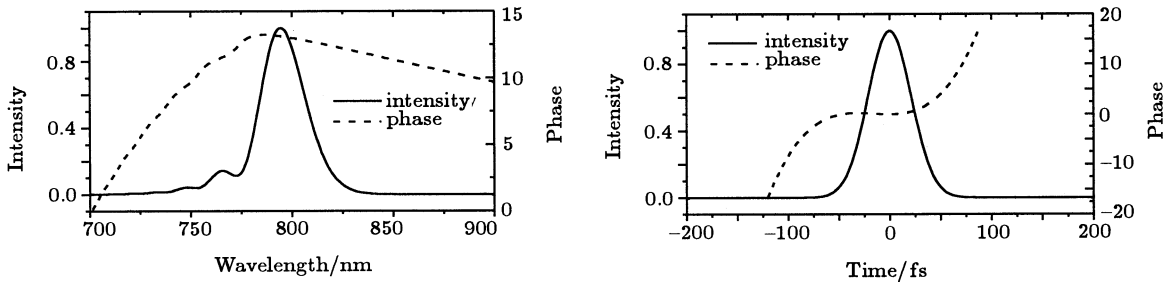
(c) Cubic dispersion pulse: $c = 3$, $b = d = 0$.



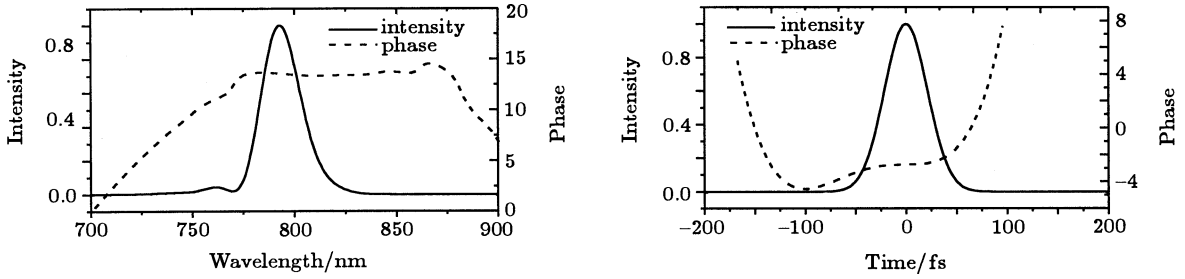
(d) Quartic dispersion pulse: $d = 1$, $b = c = 0$.



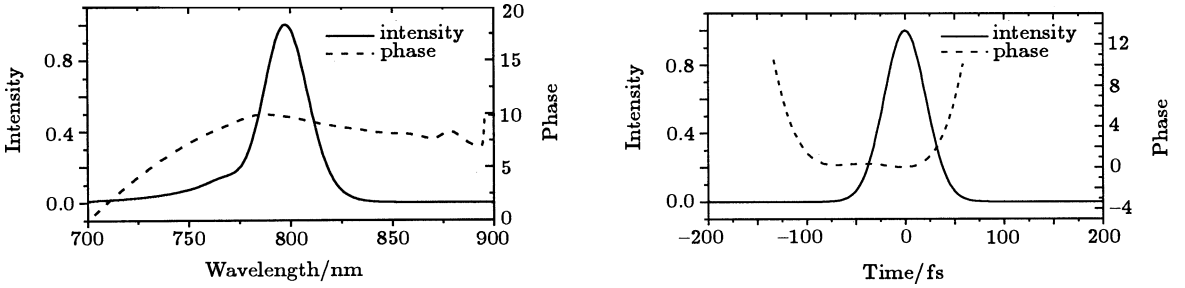
(e) Self-phase modulation pulse: $Q = 1$.



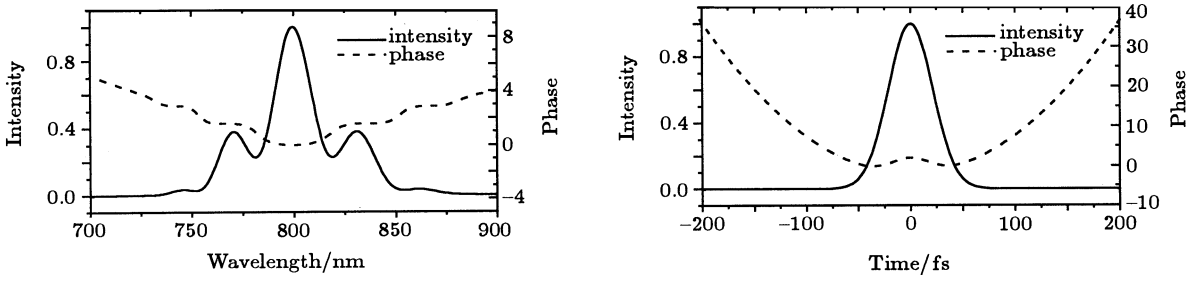
(f) Linear chirp + cubic dispersion pulse: $b = 2$, $c = 2$, $d = 0$.



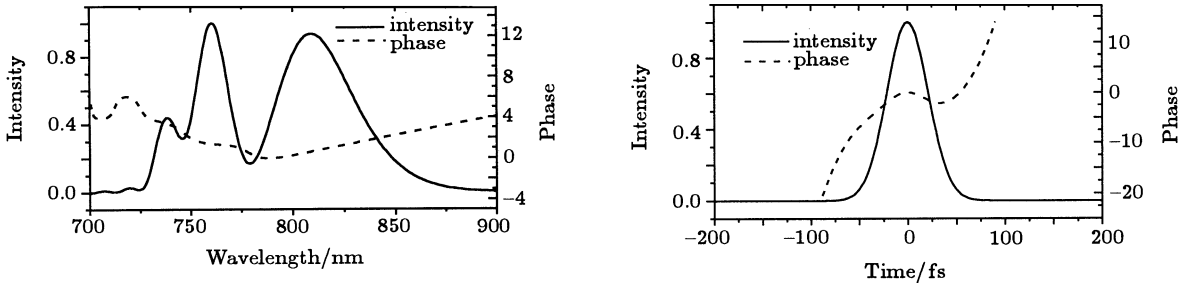
(g) Cubic dispersion + quartic dispersion pulse: $b = 0$, $c = 2$, $d = 1$.



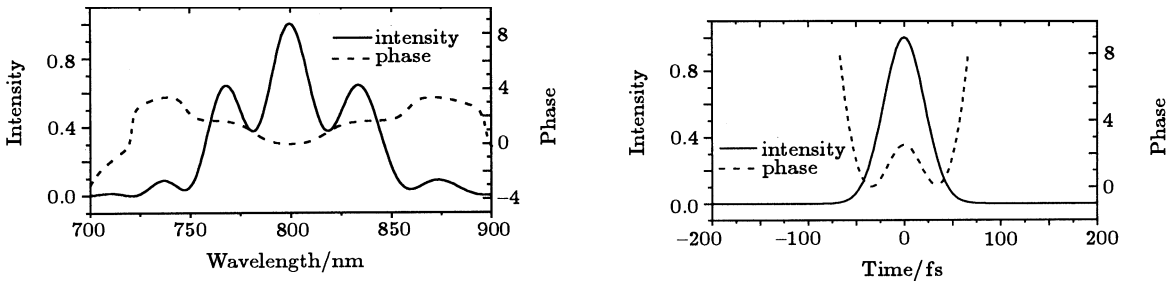
(h) Linear chirp + cubic dispersion + quartic dispersion pulse: $b = 3$, $c = 3$, $d = 1$.



(i) Linear chirp + self-phase modulation pulse: $b = 3$, $Q = 1$, $c = d = 0$.



(j) Cubic dispersion + self-phase modulation pulse: $c = 3$, $Q = 1$, $b = d = 0$.



(k) Quartic dispersion + self-phase modulation pulse: $d = 3$, $Q = 1$, $b = c = 0$.

Fig.1. The characteristics for the pulses with various types of phase distortion based on the SPIDER inversion algorithm. Intensity is in arb. units, wavelength in unit of nm, phase in unit of rad and time in units of fs.

4. Experimental demonstration

A home-made SPIDER apparatus is illustrated in Fig.2. The two pulse replicas are generated in a Michelson interferometer, while a portion of the incident pulse is chirped and expanded by a stretcher composed of a 1200groove/mm grating and roof prisms. The femtosecond optical pulses from a Kerr-lens mode-locked Ti:sapphire laser are characterized by the SPIDER. The measured spectrum and the reconstructed spectral phase, temporal intensity profile and

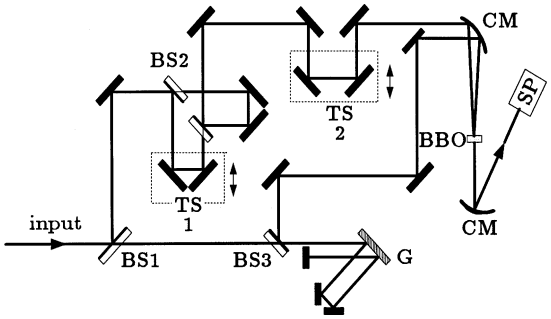
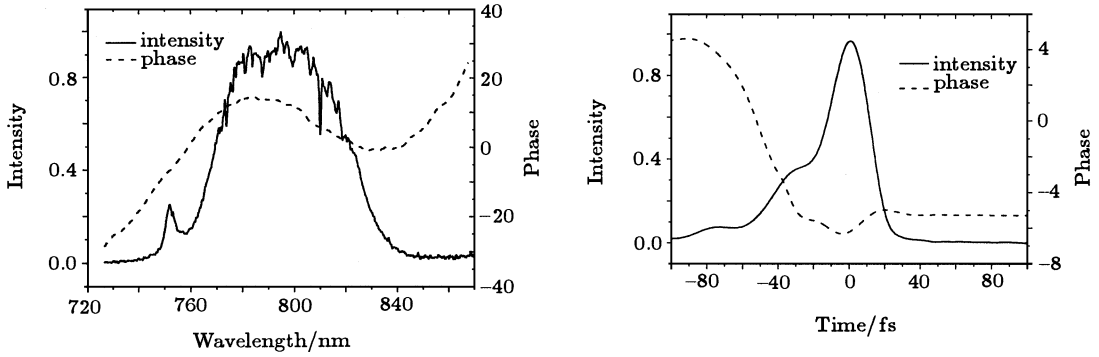
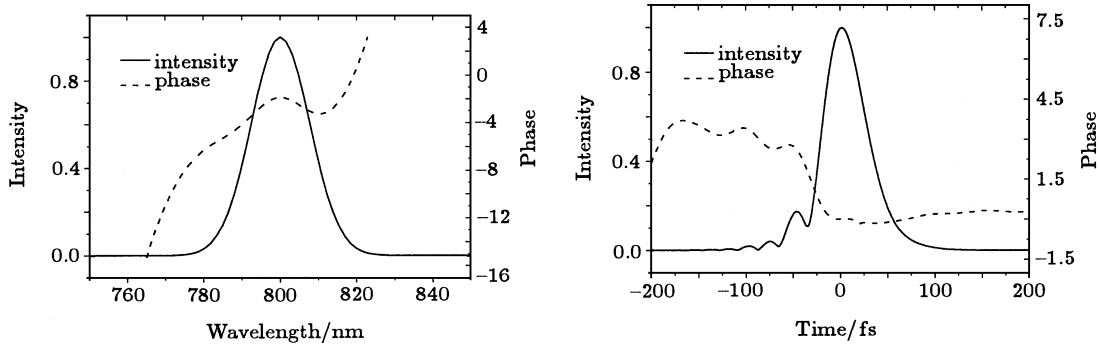


Fig.2. Schematic layout of the SPIDER experimental setup.



(a) Measured results from a Ti:sapphire oscillator.



(b) Simulated results of a pulse with linear chirp +cubic dispersion+SPM: $b = 1$, $c = 3$, $Q = 0.5$.

Fig.3. Comparison between the experimental results and the simulated results. (a) The independently measured spectrum and retrieved spectral phase (left), the reconstructed temporal intensity profile and phase (right) from a Ti:sapphire femtosecond laser. (b) The numerical simulation for a pulse containing the linear chirp (GDD), the cubic dispersion (TOD) and the SPM. The spectral phase and intensity (left), the temporal intensity profile and phase (right). There are similar characteristics between (a) and (b). Intensity is in arb. units, phase in unit of rad, wavelength in unit of nm and time in unit of fs.

temporal phase of the experimental pulses are shown in Fig.3(a). The measurements show that the pulses have a duration of 26fs, and spectra width of 52nm with a central wavelength at 795nm. It is obvious that the pulses are non-transform limited and contain the complex chirp. To analyse the chirp of the pulses, numerical simulations for the pulse with the linear chirp phase, the cubic phase and the phase with SPM, which are mostly the influence factors in a Kerr-lens modelocked laser, are performed and shown in Fig.3(b).

Comparing Fig.3(a) with Fig.3(b), we know that both figures exhibit good similar character under consideration of the complicated pulse structure, so we can confirm qualitatively that the pulses contain the group delay dispersion, the third-order dispersion and the self-phase modulation.

5. Conclusion

In this paper, based on the inversion algorithm of SPIDER, we have numerically simulated the femtosecond pulses with various phase characters. The characteristics of the spectral phase for the transform-limited pulse, linear chirp pulse, the cubic dispersion pulse, the quartic dispersion pulse, the self-phase modulation pulse and the pulses with the combination of different phase distortion are retrieved. A practical measurement has demonstrated that these characteristics of the representative pulses as samples are helpful to obtain qualitatively and quickly the chirped information of a femtosecond optical pulse in real-time diagnosis applications.

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