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Entanglement and photon statistics of output fields from beam splitter for binomial state inputs*

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The entanglement properties are investigated based on linear entropy, and nonclassicalities are examined of output fields from a beam splitter for pure binomial state inputs. It is shown that the properties of the entanglement and the photon statistics of output fields are not only strongly dependent on the parameters of input binomial states but also quite involved with the nature of the beam splitter. The best entanglement can be obtained when the parameters of both input states and the beam splitter are chosen appropriately. Finally, we analyse briefly the distinguishability between the joint input state and the joint output state.

Keywords: entanglement, photon statistics, distinguishability, beam splitter, binomial states, linear entropy, Q-parameter

PACC: 4280H, 4250

1. Introduction

One of the most outstanding representations of the properties of quantum systems is entanglement which was first elucidated by EPR[1] and Schrödinger.[2] Entangled quantum states as a useful information 'resource' have played important roles in much of the current development of quantum computation and quantum information processing.[3−8] While a beam splitter is one of the few key elements for the preparation of the entangled state. The entanglement properties of a beam splitter for various input states including pure states and mixed states have been studied extensively.[9−14] From these works, it follows that the nonclassicality of the input fields is a necessary condition for entanglement of the output field modes with the help of a beam splitter. For instance, an input Fock state can generate entangled state at the output ports due to its nonclassicality,[12] and a coherent state passing through one arm of a beam splitter is a product state with zero entanglement[15] owing to its classicality. However, as we know, when the inputs are 'intermediate states' between the number state and the coherent state, the nonclassicality of the output fields from a beam splitter has not been examined. It is well known that a binomial state can be defined as[16]

$$|p,N\rangle=\sum_{n=0}^{N}F_p^N(n)|n\rangle,\quad (N>0,0<p<1),\quad (1)$$

where

$$F_p^N(n)=\left[\frac{N!}{n!(N-n)!}p^n(1-p)^{N-n}\right]^{1/2},$$

N is the maximum number of photons of the field, and p is the characteristic probability distribution parameter of each photon occurring, both the coherent state and the Fock state are its limit cases. By continuously varying p, we are able to interpolate between the coherent state $|\alpha\rangle$, which is formed in the limit $p\to 0$, $N\to\infty$ and keeping $pN=\alpha^2$ constant, and the Fock state $|N\rangle$, formed when $p=1$. Hence the binomial states share the properties of both in this sense. Some nonclassical properties of the binomial state, such as squeezing, sub-Poissonian, etc., have been widely discussed.[16−21] Particularly, the binomial states may be generated by Jaynes–Cummings

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Hamiltonian[17,22] or a classical current interacting with two quantized radiation fields.[16,21] In this paper we investigate the properties of entanglement and photon statistics of the binomial state after interacting with a lossless beam splitter to reveal the nonclassicalities of output fields from a beam splitter. In Section 2, we briefly review some properties of a lossless beam splitter and give the expression of the density operator of the output fields from the beam splitter for the binomial state inputs. In Section 3, we investigate the entanglement properties of the output fields by introducing linear entropy as a measure of the entanglement for pure binomial state inputs. We also study the photon statistics properties of the output fields from the beam splitter for the binomial state inputs, including sub-Poissonian and the quadrature squeezing of photons in the output fields in Section 4. In Section 5, we give an analysis of the distinguishability between the joint input binomial state and the joint output state. We summarize our results in Section 6.

2. General properties of beam splitter

Figure 1 shows the schematic diagram of a beam splitter operation. Both the input and the output are two-mode fields. The different modes are distinguished by the propagation direction of the field.

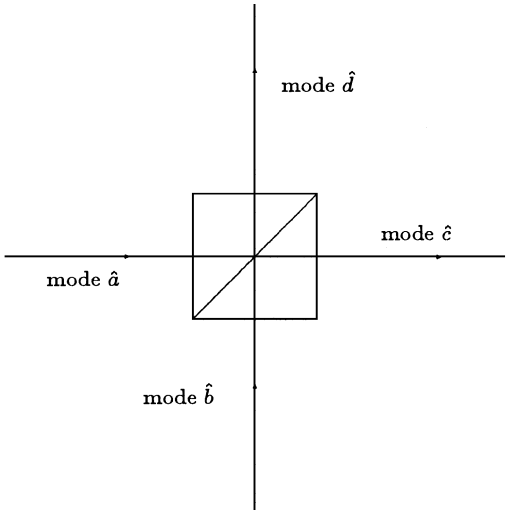


Fig.1. Geometry diagram of beam splitter.

The quantized input field modes are represented by the annihilation operators $\hat{a}$ and $\hat{b}$, and two outputs are described by the corresponding operators $\hat{c}$ and $\hat{d}$, respectively. The action of the beam splitter may be corresponding to the unitary transformation for input fields,[12,14] then the unitary evolution beam splitter operator $\hat{U}$ in the representation of the angular momentum can be written as[23−27]

$$\hat{U} = \exp\left[\theta(\hat{a}^{+}\hat{b}\mathrm{e}^{\mathrm{i}\phi} - \hat{a}\hat{b}^{+}\mathrm{e}^{-\mathrm{i}\phi})\right]. \tag{2}$$

This is the well-known two-mode mixing operator of Schumaker[28] for describing a process where the annihilation of one photon in one of the modes leads to the creation of a photon in the other mode, and vice versa. Here θ is the single angular parameter of the beam splitter, which satisfies the equations $T \equiv t^2 = \cos^2\theta$ and $R \equiv r^2 = \sin^2\theta$, where R and T are the reflection and transition coefficients respectively. The phase difference ϕ between the reflected and transmitted fields is given by the beam splitter. Most generally, the property of the beam splitter operator $\hat{U}$ can be summarized by the following equations:[14,23]

$$\begin{pmatrix} \hat{c} \\ \hat{d} \end{pmatrix} = \hat{U}^{+} \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} \hat{U}, \tag{3}$$

$$\hat{\rho}_{\mathrm{out}} = \hat{U}\hat{\rho}_{\mathrm{in}}\hat{U}^{+}, \tag{4}$$

$$\hat{U}|00\rangle = |00\rangle, \tag{5}$$

where $|00\rangle$ is the vacuum state for both modes, ρ_{in} and ρ_{out} are density operators for input and output states, respectively. From Eqs.(3) and (4) we can see that the effect of the beam splitter operator on the joint input state is opposite to its effect on the explicit field operators. Equation (5) shows the simple case of no input and no output.

Suppose that input states are two independent Fock states (i.e. $|\psi_{\mathrm{in}}\rangle = |m,n\rangle$), with the help of Eqs.(2)–(5), the joint output state is given by

$$\hat{U}|m,n\rangle = \sum_{j=0}^{m}\sum_{k=0}^{n} G(m,n,j,k)|j+k, m+n-j-k\rangle \tag{6}$$

for the beam splitter. Here the output photon number probability amplitude G is given by

$$G(m,n,j,k) = (-1)^{m-j}\frac{\sqrt{m!n!(j+k)!(m+n-j-k)!}}{j!(m-j)!k!(n-k)!} \times t^{n+j-k}r^{m-j+k}\mathrm{e}^{-\mathrm{i}(m-j-k)\phi}.$$

When two independent binomial states are simultaneously incident on a beam splitter, namely, the joint input state is in $|q,M;p,N\rangle$, after straightforward algebra, the joint output state can be written

as

$$|\psi_{\rm out}\rangle = \hat{U}|q, M; p, N\rangle = \sum_{m=0}^{M}\sum_{n=0}^{N}\sum_{j=0}^{m}\sum_{k=0}^{n} F_q^M(m)F_p^N(n) \times G(m,n,j,k)|j+k, m+n-j-k\rangle. \quad (7)$$

Obviously, the joint output state from the beam splitter is a superposition state consisting of a set of possible two-mode Fock states. The beam splitter operator in Eq.(2) may displace the input binomial states via Eq.(4) to generate an $(M+N+1)$-dimensional output entangled state. We can consider the simple case of *Bernoulli states*,[16] which is a class of special binomial states consisting of combinations of the vacuum and the first excited state, as input states. When the joint input state is the product state of two independent *Bernoulli states* (i.e. $|\psi_{in}\rangle = |q,\ M=1;\ p, N=1\rangle$), in terms of Eq.(7) the joint output state can be written as

$$\begin{aligned}\hat{U}|q,1;p,1\rangle =& \sqrt{(1-q)(1-p)}\ |0,0\rangle \\ &+ (t^2-r^2)\sqrt{qp}\ |1,1\rangle \\ &+ rt\sqrt{2qp}\ (\mathrm{e}^{\mathrm{i}\phi}|2,0\rangle - \mathrm{e}^{-\mathrm{i}\phi}|0,2\rangle) \\ &+ (t\sqrt{(1-q)p} - r\mathrm{e}^{-\mathrm{i}\phi}\sqrt{q(1-p)}\ )|0,1\rangle \\ &+ (t\sqrt{q(1-p)} + r\mathrm{e}^{\mathrm{i}\phi}\sqrt{(1-q)p}\ )|1,0\rangle. \end{aligned} \quad (8)$$

From Eq.(8) it is seen that the joint output state is a photon-number superposition state which is obviously nonmaximally entangled state for two independent *Bernoulli state* inputs. When both q and p are equal to 1, Eq.(8) is simplified as $|\psi_{\rm out}\rangle = (t^2-r^2)\sqrt{qp}\ |1,1\rangle + rt\sqrt{2qp}\ (\mathrm{e}^{\mathrm{i}\phi}|2,0\rangle - \mathrm{e}^{-\mathrm{i}\phi}|0,2\rangle)$, which means two photons can always be detected at the output ports of the beam splitter for two independent single-photon inputs. Especially for the 50:50 beam splitter, this resulting state becomes $\frac{1}{\sqrt{2}}\ (\mathrm{e}^{\mathrm{i}\phi}|2,0\rangle - \mathrm{e}^{-\mathrm{i}\phi}|0,2\rangle)$ which is actually a nonmaximally entangled state in the Hilbert space composed of $|0\rangle$ and $|2\rangle$,[29] while the odd number states are removed from the output modes because of destructive interference arising from transmission or reflection of both beams with the same number of photons for the 50:50 beam splitter. In addition, when the joint input state is the product of the vacuum state and the *Bernoulli state*, the joint output state is given by

$$\hat{U}|0;p,1\rangle = \sqrt{1-p}\ |0,0\rangle + \sqrt{p}\ (t|0,1\rangle + r\mathrm{e}^{\mathrm{i}\phi}|1,0\rangle). \quad (9)$$

Clearly, it is a superposition of the vacuum and the entangled states. When $p=1$ (only one photon input), the output state in Eq.(9) is purified to a maximally entangled *Bell state*[30] $\frac{1}{\sqrt{2}}(|0,1\rangle + \mathrm{e}^{\mathrm{i}\phi}|1,0\rangle)$ for the 50:50 beam splitter.

With the help of Eqs.(4) and (7), the density operator $\hat{\rho}_{\rm out}$ of the joint output state can be written as

$$\begin{aligned}\hat{\rho}_{out} = & \sum_{m_1=0}^{M}\sum_{m_2=0}^{M}\sum_{n_1=0}^{N}\sum_{n_2=0}^{N}\sum_{j_1=0}^{m_1}\sum_{j_2=0}^{m_2}\sum_{k_1=0}^{n_1}\sum_{k_2=0}^{n_2} \\ &\times F_q^M(m_1)F_q^M(m_2)F_p^N(n_1)F_p^N(n_2) \\ &\times G(m_1,n_1,j_1,k_1) \\ &\times G^*(m_2,n_2,j_2,k_2) \\ &\times |j_1+k_1, m_1+n_1-j_1-k_1\rangle \\ &\times \langle m_2+n_2-j_2-k_2, j_2+k_2|. \end{aligned} \quad (10)$$

In the next section we investigate the entanglement properties of the binomial states after passing through the beam splitter by using the reduced density operator ρ_c given by Eq.(10).

3. Entanglement properties of output fields

In the papers concerning entanglement-measure, two commonly used entanglement measures for pure states are the von Neumann entropy E_N[31,32] and the linear entropy E_L.[33] The former (usually called the entropy of entanglement) is a uniquely good measure of entanglement for a pure state (Eq.(7)). The latter can be regarded as a kind of 'linearized' version of the former. In other words, the linear entropy is a simple lower bound of the von Neumann entropy. As a measure of entanglement of output density operator Eq.(10), with respect to the von Neumann entropy E_N, the linear entropy E_L has a definite advantage of being a polynomial in the pure state $|\psi_{\rm out}\rangle$.[34] For a bipartite state $\hat{\rho}_{\rm out}$ in a D-dimensional Hilbert space, the linear entropy can be defined as[35–37]

$$E_L(|\psi_{\rm out}\rangle) = \frac{D}{D-1}\left[1 - \mathrm{Tr_c}(\hat{\rho}_{\rm c}^2)\right], \quad (11)$$

with the reduced density operator $\hat{\rho}_{\rm c} = \mathrm{Tr_d}\,(\hat{\rho}_{\rm out})$. Then one has $0 \le E_L \le 1$, where the lower (upper) bound is reached if and only if $|\psi_{\rm out}\rangle$ is a disentangled state (maximally entangled state).

In the following we will concentrate on two cases of input states by using numerical calculation. In the

first case, we suppose that one binomial state is incident on one input port of the beam splitter while no photon is injected into the other. That is, let $M = 0$ in Eq.(10), together with Eq.(11), then one has the linear entropy E_L for the reduced density operator ρ_c of the beam splitter output mode c as a function of parameters N, R and p, as shown in Fig.2. We find that the entanglement of the output fields depends highly on the reflection coefficient R, the binomial parameters p and N. In Fig.2(a) one can see first a very rapid and then an exceedingly slow fall following the maximum entanglement with increasing N for $p = 0.1$, and it is almost zero for very large N. It is interesting to note from Fig.2(b) that an initial higher peak can be achieved around $N = 6$ for $p = 0.9$. Thereafter it tails off very slowly from the maximum entanglement with increasing N but drops to zero hardly for finite N. This shows that the output state will always be entangled for any binomial input state with large p, even if N is very large (but not infinite). From Fig.2(c) we find that the entanglement of output fields increases rapidly with increasing p. This means that we may increase p to bring about high entanglement of the output fields. For given binomial parameters p and N from Figs.2(a)–(c) the entanglement of the output fields is always maximum for the 50:50 beam splitter. In other words, when the parameters p and N are given, in order to generate the maximum entanglement the 50:50 beam splitter should be chosen.

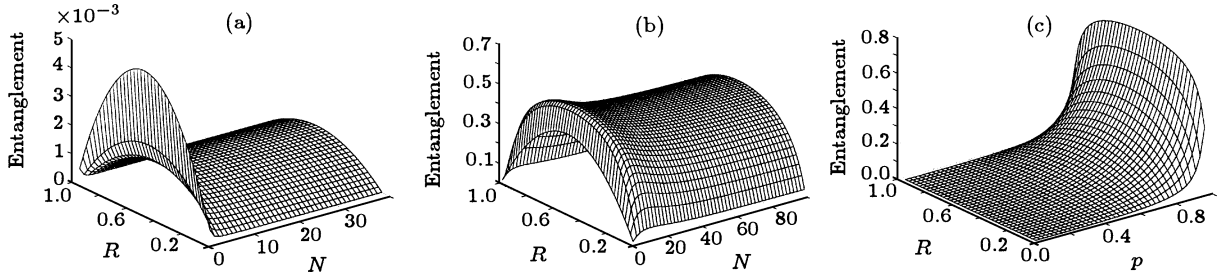


Fig.2. The measure of entanglement $E_L(\hat{\rho}_c)$ based on the linear entropy for the reduced density operator of the beam splitter output field for the joint input state $|0; p, N\rangle$ against N and R for (a) $p = 0.1$ and (b) $p = 0.9$, and (c) vs p and R for $N = 40$.

In the second case, two binomial states are supposed to be simultaneously incident on the beam splitter. i.e. the joint input state is in $|q, M; p, N\rangle$. According to Eqs.(10) and (11), we may produce the plots of the linear entropy E_L as a function of parameters of the two binomial states and the beam splitter, as shown in Fig.3. From Fig.3(a) it can be seen that a large M corresponds to a high entanglement of output fields for 50:50 beam splitter. When we keep $M = N = 1$ (i.e. the *Bernoulli state* inputs), the dependence of the entanglement of output fields on q is higher for small p than that for large p, and the entanglement of output fields increases with increasing q. A matter worthy of note is that the linear entropy at dot A is equal to the value at dot B when the joint input state is in $|q = 0.9,\ M = 3;\ p = 0.8,\ N = 1\rangle$ and $|q = 0.8,\ M = 1;\ p = 0.9,\ N = 3\rangle$. This shows that when the binomial states are exchanged between two input ports of the 50:50 beam splitter, the entanglement of the output fields can be invariant, due to the symmetries of the number state output probabilities of the beam splitter. Figure 3(b) gives the linear entropy against the binomial parameters q and p for 50:50 beam splitter when M and N are given (see the figure caption). We find that for given M, N and R, the entanglement of the output fields increases with increasing q and p, and when the value of the binomial parameter q (or p) is large, a high entanglement of the output fields is obtained for the 50:50 beam splitter. A similar case occurs for M and N when both q and p are given. This indicates that the degree of entanglement of the output fields is governed by the parameters of the two input binomial states. From Fig.3(c) it can be seen that when $M = N = 1$ and the values of both q and p are large, the entanglement of the output fields shows a dip for the 50:50 beam splitter (the dotted curve), while it displays multi-peak when $M = N > 1$ (compare the solid curve with the dotted curve in Fig.3(c)). We also note that the larger the values of q and p are, the more obvious the multi-peak displays (compare the '+' curve with the '−' curve), and the dip or the multi-peak vanishes when $M \neq N$, regardless of the values of q, p and R (as indicated by the '△' curve in Fig.3(c)). Figure 3(d) displays that

the dip weakens quickly with decreasing q and p. The results mentioned above are caused by the two-photon interference effect which arises from destructive interference between the amplitudes of both reflected photons and both transmitted photons. It is a consequence of the bosonic nature of the photons.[38,39] The destructive interference has been experimentally confirmed by the coincidence measurement.[29,40] Figures 3(a)–(d) show clearly that the most important impact factor on the entanglement of the output fields is the binomial parameters q and p of the input states. The second one is the reflection coefficient R of the beam splitter, and then is the maximum number of photons of the input fields (i.e. M and N).

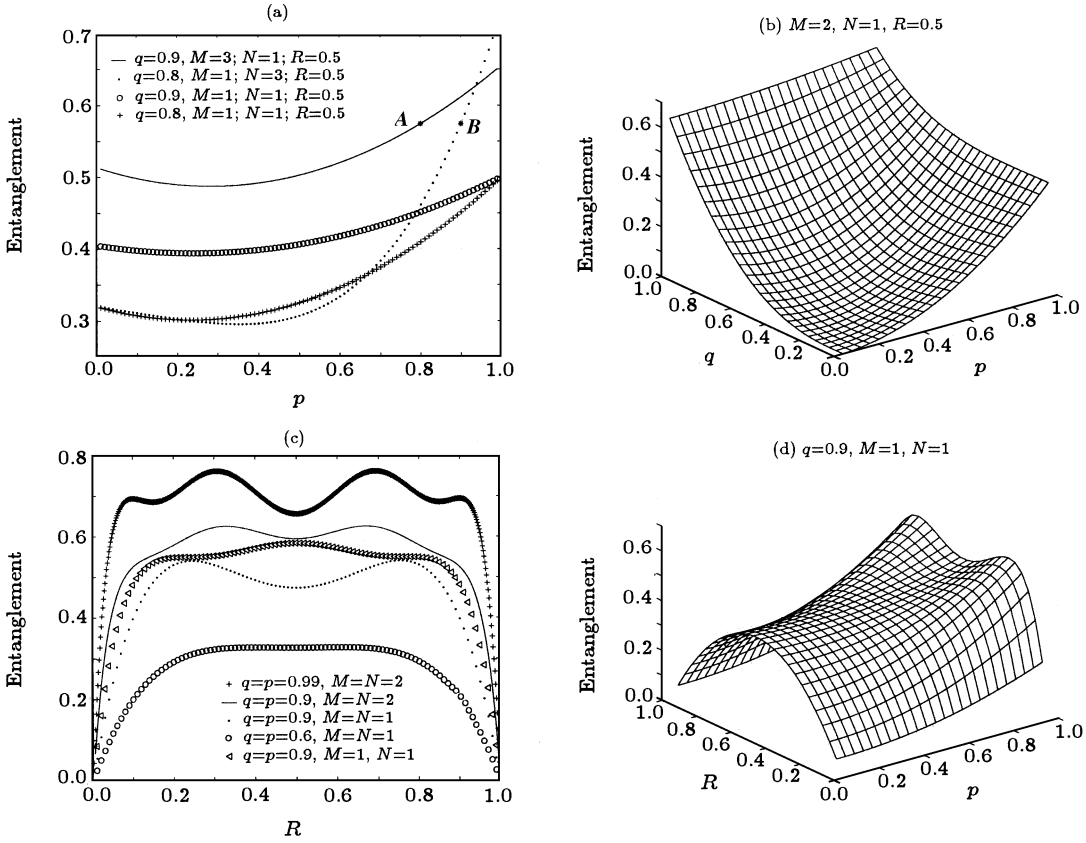


Fig.3. The measure of entanglement $E_L(\hat{\rho}_c)$ for the beam splitter output field (a) as a function of the binomial parameter p for the given q, M, N, and $R = 0.5$, (b) against the input binomial parameters q and p for $M = 2$, $N = 1$, and $R = 0.5$, (c) versus the reflected coefficient R for the given input binomial parameters q, M, p, and N, (d) with respect to the binomial parameter p and the reflected coefficient R for $q = 0.9$ and $M = N = 1$.

4. Photon statistics of output fields

The entanglement of the output fields is clearly an indication of nonclassicality and the lack of it can be construed as a signature of classicality.[41] The photon statistical behaviour of the output fields from the beam splitter, such as sub-Poissonian and squeezing effect, etc., is an essential tool in quantum information and quantum communication.[4]

4.1. Q-parameter

With the help of Eqs.(2) and (3), the mean photon number of the output field c can be written as

$$\langle \hat{N}_c \rangle = t^2 \langle \hat{N}_a \rangle + r^2 \langle \hat{N}_b \rangle + rt \langle \hat{I} \rangle \qquad (12)$$

for the joint input state consisting of two independent binomial states, where $\hat{N}_i = \hat{i}^+ \hat{i}$ $(i = a, b, c, d)$ is the photon number operator of the ith mode. $\hat{I}$, named as the interference operator, is given by $\hat{I} = \mathrm{e}^{\mathrm{i}\phi} \hat{a}^+ \hat{b} + \mathrm{e}^{-\mathrm{i}\phi} \hat{a} \hat{b}^+$. According to the law of conservation of energy, the mean photon number of the output mode d is $\langle \hat{N}_d \rangle = \langle \hat{N}_a \rangle + \langle \hat{N}_b \rangle - \langle \hat{N}_c \rangle$ for the lossless beam splitter. From Eq.(12) it can be seen that the mean number of photons in the output mode c contains the transmitted part of mode a, reflected mode b,

and the most important, an interference term between the two input fields.

The quantum variance of the photon number is an important parameter in photon statistics. When the binomial states are incident on the beam splitter, after being transformed by the beam splitter, the variance of photons in the output field is given by

$$\begin{aligned}\mathrm{var}(\hat{N}_c) =& t^4\mathrm{var}(\hat{N}_a) + r^4\mathrm{var}(\hat{N}_b) + r^2t^2\mathrm{var}(\hat{I}) \\ &+ t^2r^2\left[\mathrm{cov}(\hat{N}_a,\hat{N}_b) + \mathrm{cov}(\hat{N}_b,\hat{N}_a)\right] \\ &+ rt^3\left[\mathrm{cov}(\hat{N}_a,\hat{I}) + \mathrm{cov}(\hat{I},\hat{N}_a)\right] \\ &+ r^3t\left[\mathrm{cov}(\hat{N}_b,\hat{I}) + \mathrm{cov}(\hat{I},\hat{N}_b)\right], \qquad (13)\end{aligned}$$

where we have used the operator identity $\mathrm{var}(\hat{A}+\hat{B}) = \mathrm{var}(\hat{A})+\mathrm{var}(\hat{B})+\mathrm{cov}(\hat{A},\hat{B})+\mathrm{cov}(\hat{B},\hat{A})$, and the quantum covariances $\mathrm{cov}(\hat{A},\hat{B}) = \langle\hat{A}\hat{B}\rangle - \langle\hat{A}\rangle\langle\hat{B}\rangle$ which vanishes for the number-operator due to the independent input fields. The first two terms of Eq.(13) arise from the quantum statistical fluctuations inherent in the fields prior to photomixing. The third term represents fluctuations generated by the interference operator. It is worthy to mention that it contains the two-photon interference effect. The last two terms express fluctuations related to the number operators and interference operators. The last three terms of Eq.(13) result only from the beam splitter. If the quantum variance is smaller than the mean photon number, we call such a field sub-Poissonian and it is an example of nonclassical fields. In experiments, the statistic behaviour of the output fields is often characterized by the *Mandel's Q* parameter[42]

$$Q = \frac{\mathrm{var}(\hat{N}_c) - \langle\hat{N}_c\rangle}{\langle\hat{N}_c\rangle}, \qquad (14)$$

which can be calculated in a straightforward manner using Eqs.(12)–(14). If $Q = 0$, the field is called Poissonian. If $Q < 0$ and $Q > 0$, it is called sub-Poissonian and super-Poissonian, respectively.

We plot the mesh in Fig.4 which shows how the Q-parameter depends on the parameters of the input fields and the beam splitter. Figure 4(a) shows that for certain M and N, the output field is sub-Poissonian when the value of the binomial parameter q or p is not very large, while it is super-Poissonian when the values of both q and p are large. On the other hand, it can be seen that the classicality of the output field increases with the increase of M or N. We also note that the output field is always sub-Poissonian for vacuum and binomial state inputs, regardless of the values of M and N. In Fig.4(b) it is not difficult to see that for given parameters of the joint input state (cf the title), the output field is super-Poissonian for ϕ around π when R lies within the range of 0.1–0.8, but it is sub-Poissonian out of the range; in particular, it is sub-Poissonian when $\phi = 0$ or 2π despite the value of R.

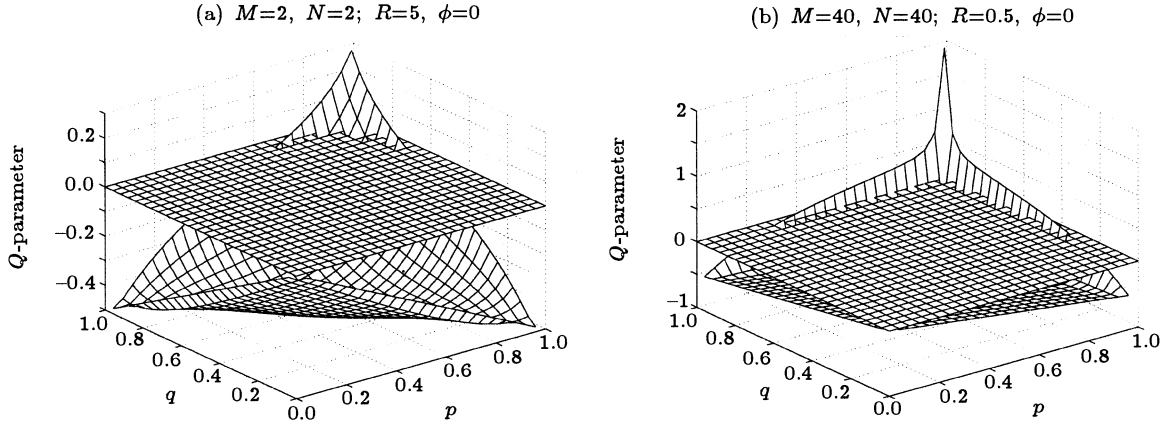


Fig.4. Q-parameter (a) as a function of the binomial parameters q and p for $M = 2$, $N = 4$, $R = 0.5$, and $\phi = 0$, and (b) against the beam splitter parameters ϕ and R for $q = 0.5$, $M = 2$, $p = 0.5$, and $N = 4$.

4.2. Quadrature squeezing

The squeezing characteristic of the quantum variance of output fields is another important representation of the quantum properties of the output fields. Two Hermite operators $\hat{X}_1$ and $\hat{X}_2$ are defined as[43]

$$\hat{X}_1 = \frac{1}{2}\left(\hat{c} + \hat{c}^+\right), \qquad (15)$$

$$\hat{X}_2 = \frac{1}{2\mathrm{i}}\left(\hat{c} - \hat{c}^+\right) \qquad (16)$$

for the output mode c. It is easily verified that $[\hat{X}_1, \hat{X}_2] = \mathrm{i}/2$, hence $\hat{X}_1$ and $\hat{X}_2$ are both quadrature conjugate variables. According to the uncertainty relation

$$\mathrm{var}(\hat{X}_1)\mathrm{var}(\hat{X}_2) \geq \frac{1}{16} \tag{17}$$

for the two quadratures, with the aid of Eqs.(13), (15), and (16), we can straightforwardly give

$$\begin{aligned}\mathrm{var}(\hat{X}_1) =& \frac{1}{4} + \frac{1}{2}t^2qM + \frac{1}{2}r^2pN - t^2qM\left[\sum_{m=0}^{M-1} F_q^{M-1}(m)F_q^M(m)\right]^2 - r^2pN\left[\cos\phi\sum_{n=0}^{N-1} F_p^{N-1}(n)F_p^N(n)\right]^2 \\ &+ \frac{1}{2}t^2q\sqrt{M(M-1)}\sum_{m=0}^{M-2} F_q^{M-2}(m)F_q^M(m) + \frac{1}{2}r^2p\sqrt{N(N-1)}\cos(2\phi)\sum_{n=0}^{N-2} F_p^{N-2}(n)F_p^N(n), \end{aligned} \tag{18}$$

$$\begin{aligned}\mathrm{var}(\hat{X}_2) =& \frac{1}{4} + \frac{1}{2}t^2qM + \frac{1}{2}r^2pN - r^2pN\left[\sin\phi\sum_{n=0}^{N-1} F_p^{N-1}(n)F_p^N(n)\right]^2 \\ &- \frac{1}{2}t^2q\sqrt{M(M-1)}\sum_{m=0}^{M-2} F_q^{M-2}(m)F_q^M(m) - \frac{1}{2}r^2p\sqrt{N(N-1)}\cos(2\phi)\sum_{n=0}^{N-2} F_p^{N-2}(n)F_p^N(n), \end{aligned} \tag{19}$$

for the joint input state $|q, M; p, N\rangle$. The first term of Eqs.(18) and (19) is due to vacuum fluctuation. The second term comes from the input fields. The remaining terms result from the interferences. If $\mathrm{var}(\hat{X}_i) < 1/4$ $(i = 1, 2)$, we can say that the ith quadrature component $\hat{X}_i$ in the two-mode output field is squeezed. An ideal squeezing can be obtained when $\mathrm{var}(\hat{X}_i) < 1/4$ and $\mathrm{var}(\hat{X}_1)\mathrm{var}(\hat{X}_2) = 1/16$.

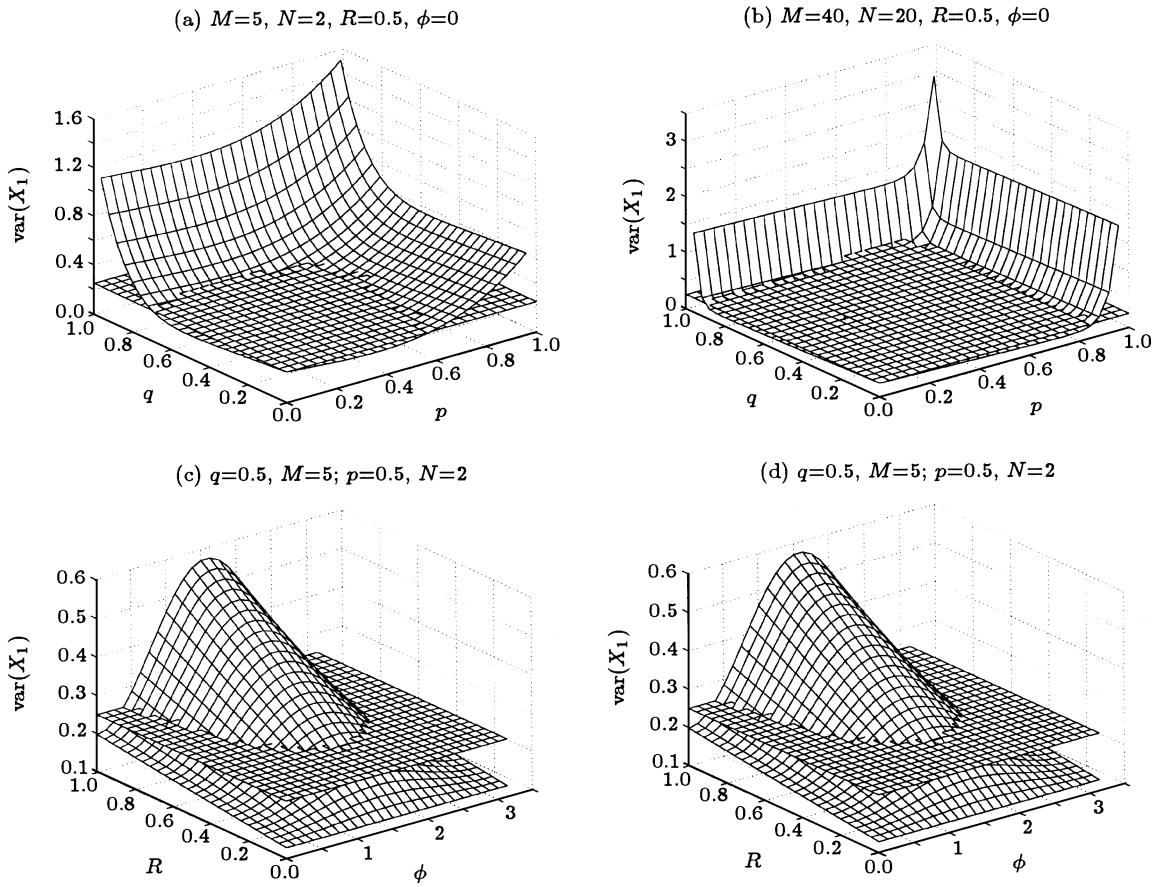


Fig.5. Effect of parameters of both binomial states and beam splitter on squeezing properties is plotted using the variance $\mathrm{var}(X_1)$ against the binomial parameters q and p for (a) $M = 5$, $N = 2$, $R = 0.5$, and $\phi = 0$, (b) $M = 40$, $N = 20$, $R = 0.5$, and $\phi = 0$, (c) against the beam splitter parameters ϕ and R for $q = 0.5$, $M = 5$, $p = 0.5$, and $N = 2$. And (d) gives the mesh of the product of variances of two quadratures $\hat{X}_1$ and $\hat{X}_2$ against the binomial parameters q and p for $M = 5$, $N = 2$, $R = 0.5$, and $\phi = 0$.

From Figs.5(a) and (b) we observe that the squeezing is always obtained for small values of q and p regardless of the values of M and N (except for zero). The best squeezing may be achieved for the values of q and p around 0.5, and the squeezing range corresponding to the values of q and p increases with increasing M and N. From Fig.5(c) we can see that the squeezing decreases with increasing R and has much to do with ϕ. For given parameters (cf the title), the squeezing is maximized when $\phi = 0,\ \pi$, while it is minimized when $\phi = \pi/2$. From Fig.5(d), with the help of Fig.5(a), it is surprising to find that an ideal squeezing may be obtained for the 50:50 beam splitter when $q < 0.5$ and $p < 0.5$. In a word, the squeezing properties of the output fields vary greatly after interacting of the input binomial states with the beam splitter.

5. Distinguishability between joint input and joint output states

The distinguishability between two quantum states is crucial to the theory of quantum information processing. How similar are the input state and the output state? A useful criterion of their distinguishability is the quantum error probability which is a typical 'measurement + guess' mode.[44,45] Suppose a given pure quantum system is secretly prepared either in the state $|a\rangle$ or in the state $|b\rangle$ with the prior probabilities p_a and p_b, respectively. Then the minimum probability of error to discriminate two pure states is given by[45]

$$P_{\rm e} = \frac{1}{2}\left\{1 - \left[1 - 4p_a p_b |\langle a|b\rangle|^2\right]^{\frac{1}{2}}\right\}. \tag{20}$$

For the joint binomial input state ($|q, M; p, N\rangle$), with the help of Eqs.(7) and (20), we obtain

$$P_{\rm e} = \frac{1}{2}\left\{1 - \left[1 - 4p_{ab}p_{cd}\left|\sum_{m=0}^{M}\sum_{n=0}^{N}\sum_{j=0}^{m}\sum_{k=0}^{n} F_q^M(m)F_q^M(j+k)F_p^N(n)F_p^N(m+n-j-k)G(m,n,j,k)\right|^2\right]^{\frac{1}{2}}\right\}. \tag{21}$$

Here p_{ab} and $p_{cd} = 1 - p_{ab}$ express the a priori probabilities of the joint input state and the joint output state, respectively. If the minimum probability of error between the joint input state and the joint output state tends to zero, we can say that they are completely orthogonal and distinguishable from each other. The smaller the minimum probability of error ($P_{\rm e} > 0$) is, the greater the distinguishability is.

Figure 6 gives the probability of error against the binomial parameters q and p (see Fig.6(a)) and the beam splitter parameters R and ϕ (see Fig.6(b)). From Fig.6(a) we find that when $M = 10$, $N = 2$, $R = 0.5$ and $\phi = 0$, the larger the value of q or p is, the stronger the distinguishability between the joint input and the joint output states is, thus the input and the output states become orthogonal for large value of q or p. In the same way, the analogous result can be obtained for M and N. Figure 6(b) shows that when the parameters of the joint input state are given, the distinguishability between the joint input and the joint output states increases with increasing R. It is a

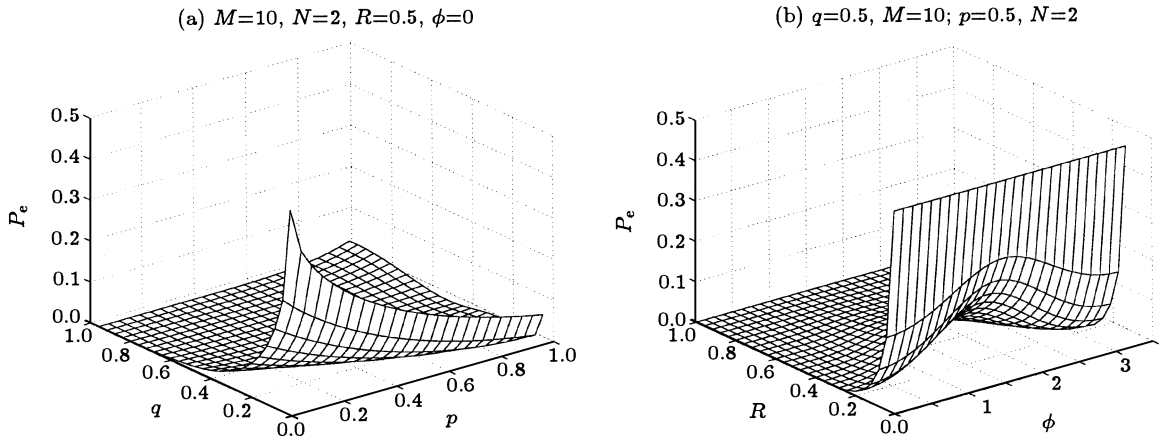


Fig.6. Minimum probability of error between joint input state and joint output state as a function of (a) the binomial parameters q and p for $M = 10,\ N = 2,\ R = 0.5$, and $\phi = 0$, (b) the beam splitter parameters ϕ and R for $q = 0.5,\ M = 10,\ p = 0.5$, and $N = 2$. Where we have supposed that the priori probabilities $p_{ab} = 0.5$ and $p_{cd} = 0.5$.

simple fact that it is most difficult to distinguish the joint output state from the joint input state for the beam splitter with full transition. The dependence of the distinguishability between the joint input and the joint output states on ϕ is very strong. For certain parameters q, M, p, N and R, the distinguishability is maximized when $\phi = 0$ or π, while it is minimized when $\phi = \pi/2$.

6. Conclusion

In summary, we have studied the properties of the entanglement, photon statistics distribution and distinguishability of the two-mode input binomial state after passing through the beam splitter. We have calculated the linear entropy of the entanglement of the output fields by using the reduced density operator of the output field. We found that the entanglement of the output fields depends on many factors such as the parameters of two binomial input states and the reflected coefficient of the beam splitter. When the joint input state is the product state of vacuum and binomial states, the best entanglement is always obtained for the 50:50 beam splitter together with the appropriate parameters of the binomial input state. When two identical binomial states ($M = N$) are incident on the 50:50 beam splitter, the entanglement of the output fields cannot be maximized for the large binomial parameters (q and p) due to the photo-pair effect. Generally, the dependence of entanglement of the output fields on the binomial parameters (q and p) is strongest. The large value of the binomial parameter (q or p) of at least one of the input fields may ensure the high entanglement of the output fields. Because binomial states have been obtained successfully in experiment,[16] it is possible to prepare various entangled states at the output ports of the beam splitter by using binomial state inputs, and the degree of the entanglement of the output fields is governed by the parameters of the beam splitter and the input fields. For given ranges of the parameters of the joint input state, the photon statistical properties of the output field from the beam splitter were investigated in terms of the photon-number variance, Q parameter, and quadrature squeezing of the output field. We found that the beam splitter operating on various binomial states can yield a great variety of nonclassical behaviour and the measure of the nonclassicalities of the output field depends on the parameters of the input fields and the beam splitter. The nonclassical output field can be obtained through adjusting these parameters. Finally, we have also discussed the distinguishability between joint input and joint output states, and the results show that the joint input state and the joint output state can be easily distinguished except for some special cases.

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