Vol 13 No 9, September 2004
1009-1963/2004/13(09)/1447-07

Chinese Physics

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Effects of correlated perturbations on dark soliton propagation and interaction*

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(Received 9 January 2004; revised manuscript received 16 March 2004)

Correlated perturbations are considered in a dark soliton system, and their effects on soliton propagation and interaction are investigated numerically. These perturbations result in large sidebands, lead to submergence of dark soliton, and enhance the interaction. The correlation amplifies these effects and shortens the distance until submergence. The comparison of the distinction is made between the degradations of these effects on dark soliton and the corresponding bright soliton. It is found that these effects on dark soliton are less than those on bright soliton. Finally the nonlinear gain is introduced to suppress efficiently these effects.

Keywords: stochastic perturbation, correlation, submergence of dark soliton, control
PACC: 4260, 4281D

1. Introduction

Optical solitons in fibres are formed by a balance of group dispersion and Kerr non-linearity. Numerical simulation and experiments have demonstrated that solitons can propagate an extended distance without distortion, so they may become the ideal message carrier in long-distance communication.[1] Dark solitons propagate in the normal dispersion region. They have robust features such as low intrinsic transmission loss, resistance to perturbations, and weak interaction between neighbouring solitons. The amplified spontaneous emission (ASE) noise in a dark soliton system is only half of that corresponding to the bright counterpart. The techniques for generating and detecting dark soliton pulses have been developed.[2]

There are many perturbations in a practical soliton system, such as stochastic fibre dispersion and stochastic refractive index along transmission line, these perturbations lead to the fluctuation in amplitude or pulse-width, and deform the shape of soliton. Their effects cannot be neglected in soliton communication system, such as degrading the stability of coherently amplified solitons, increasing the time jitters in arrival, and reducing the system capacity.[3–7]

Though various perturbations are present simultaneously in some stochastic process, perturbations are assumed to have different origins and are treated as independent stochastic variables in most previous investigations. However, in certain situation perturbations may have a common origin and thus may be correlated with each other as well. Experimental measurements and theoretical analyses of a soliton system can show that the system contains many stochastic perturbations,[8] and some are of the common origin, so they are correlated. When correlated stochastic perturbations are simultaneously present, their effects on dark soliton systems should contain their individual contributions and the correlated part.

In this paper, effects of correlated perturbations and the distinction between their degradations on dark soliton and the corresponding bright solitons are investigated by numerical simulation. Some new con-

*Project supported by the National High Technology Development Programme of China (Grant No 2002AA312050), and the Important Program of Education Department of Hubei Province, China (Grant No 2002Z00005).

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clusions on dark soliton propagation and interaction are obtained.

2. Theoretical formulae

In a real transmission system, the envelope of the field becomes inhomogeneous because of possible variations, such as stochastic fibre dispersion and stochastic refractive index, and can be described by a normalized non-linear Schrödinger equation,[8]

$$\mathrm{i}\frac{\partial u}{\partial Z}-\frac{1}{2}\frac{\partial^2 u}{\partial T^2}+|u|^2 u=\eta_1(Z)\frac{\partial^2 u}{\partial T^2}+\eta_2(Z)\,|u|^2 u, \quad (1)$$

where u is the dimensionless envelop of the field, $Z=z/z_{\mathrm{d}}$, $T=t/\tau_0$, τ_0 and z_{d} being the pulse-width and dispersion length respectively. $\eta_1(Z)$ and $\eta_2(Z)$ are perturbations caused by stochastic dispersion and nonlinearity, respectively. The statistical properties of these perturbations are characterized by

$$\begin{aligned}&\langle\eta_i(Z)\rangle=0, \qquad i=1,2\\ &\langle\eta_i(Z)\eta_i(Z')\rangle=D_i\delta(Z-Z'),\end{aligned} \quad (2)$$

where D_i $(i=1,2)$ is the standard deviation of every perturbation.

In a practical soliton system, there are coloured perturbations. The coloured perturbation is of the same origin as other perturbations, and the correlation is strong when it is present simultaneously with other perturbations of the same origin. When a perturbation and other strong coloured perturbations are simultaneously present in a soliton system, their effects on dark soliton propagation and interaction should contain their individual contributions and the correlated parts.

It should be pointed out that multiplicative perturbations are all assumed as independent white Gaussian perturbations in previous models, such as in Ref.[8]. However, more realistic physical models for soliton systems require to consider the case of coloured perturbations, especially when these perturbations are of a common origin, for the presence of correlation may change the dynamics of the soliton system. For example, coefficients of dispersion and nonlinearity are given by $\beta_2=\frac{1}{c}\left(2\frac{\mathrm{d}n}{\mathrm{d}\omega}+\omega\frac{\mathrm{d}^2 n}{\mathrm{d}\omega^2}\right)$ and $\gamma=\frac{n\omega}{cA_{\mathrm{eff}}}$ in a fibre.[1] If there is an origin of perturbation taking place in the stochastic variation of the refractive index along the transmission line, namely $\Delta n=\delta(z)$, then the dispersion and the nonlinearity perturbations along the transmission line can be given as $\Delta\beta_2=\Delta\left[\frac{1}{c}\left(2\frac{\mathrm{d}n}{\mathrm{d}\omega}+\omega\frac{\mathrm{d}^2 n}{\mathrm{d}\omega^2}\right)\right]=C_1\delta(z)$ and $\Delta\gamma=\Delta\left[\frac{n\omega}{cA_{\mathrm{eff}}}\right]=C_2\delta(z)$ due to the stochastic variation of the refractive index, where $\delta(z)$ is the stochastic variable and satisfies the equation $\langle\delta(z)\rangle=0$, $\langle\delta(z)\delta(z')\rangle=C\delta(z-z')$; C_1, C_2 and C are constant against the propagation distance z. Furthermore, when the normalized equation is introduced, perturbations caused by the stochastic dispersion and the stochastic nonlinearity in Eq.(1) can be written as $\eta_1(Z)=\sqrt{D_1/C}\delta(Z)$ and $\eta_2(Z)=\sqrt{D_2/C}\delta(Z)$. Especially, when they are partially of a common origin, the strength of correlation is introduced, and Eq.(2) and the equation below are adopted to demonstrate their correlation

$$\begin{aligned}\langle\eta_1(Z)\eta_2(Z')\rangle&=\langle\eta_2(Z)\eta_1(Z')\rangle\\ &=k\sqrt{D_1D_2}\delta(Z-Z'), \quad |k|\le 1\end{aligned} \quad (3)$$

(most previous investigations are based on this assumption[9]), where k is the strength of correlation between two stochastic perturbations, the correlation is maximum corresponding to k=1, and perturbations are independent of each other as k=0.

3. Effects of correlated perturbations on dark soliton propagation

Numerical simulations of pulse propagation are performed in the optical fibre link. When we take the pulse-width to be 10ps, the dispersion distance is about 122km corresponding to the path-average dispersion $\bar{d}$=1.27ps^2/km. The random modulation with Gaussian random deviation is used in calculating the magnitude of dispersion and nonlinearity. Uniformly distributed random numbers from the intervals θ_1,θ_2,$\theta\in(0,1)$ are used, and random numbers are produced by the built-in generator Random (x) from the Matlab. Transformation of the uniformly distributed random numbers from the intervals θ_1, θ_2 ,θ into Gaussian random numbers with properties in Eq.(3) is performed using the following relations:[10]

$$\begin{aligned}\eta_1=&\sqrt{2D_1}\ \times\ [\sqrt{1-k^2}\sin(2\pi\theta_1)\times\sqrt{-2\ln\theta_1}\\ &+k\sin(2\pi\theta)\times\sqrt{-2\ln\theta}],\\ \eta_2=&\sqrt{2D_2}\ \times\ [\sqrt{1-k^2}\sin(2\pi\theta_2)\times\sqrt{-2\ln\theta_2}\\ &+k\sin(2\pi\theta)\times\sqrt{-2\ln\theta}].\end{aligned} \quad (4)$$

As usual, an average of a few different sequences of random numbers has been used in the below numerical simulations. We find that perturbations caused by

stochastic dispersion and stochastic nonlinearity lead to submergence of dark soliton, and degrade the stability of dark soliton propagation. Figure 1 shows the propagation distance until dark soliton submergence versus correlation strength for different standard deviations, and for simplicity, we assume the standard deviation of every perturbation is the same, namely $D_i = D$. Figure 2 demonstrates the normalized intensity of soliton versus the propagation distance for different correlation strengths, and the standard deviation of every perturbation is $D_i = D$=0.5 (i=1, 2). Initially the input pulse is tanh(T) in every figure. These perturbations result in large sidebands, lead to submergence of dark soliton, and the distance until dark soliton submergence is related to the deviations of perturbations. The correlation amplifies these effects, and shortens the distance until submergence. Moreover, there is resonance between two perturbations in the system, their effect is the largest due to the maximum correlation ($k = 1$) and the corresponding distance until submergence is the shortest.

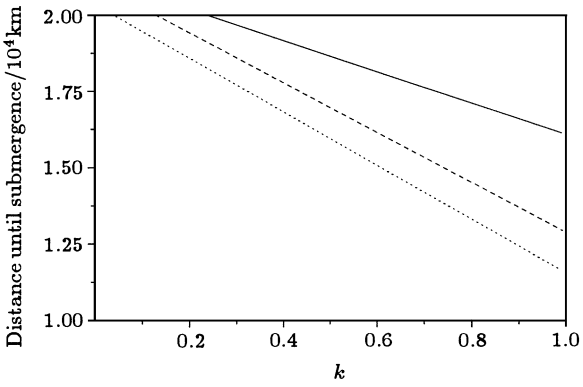


Fig.1. The distance until dark soliton submergence versus correlation strength for different standard deviations. Solid line: D=0.05, dashed line: D=0.1, dotted line: D=0.5.

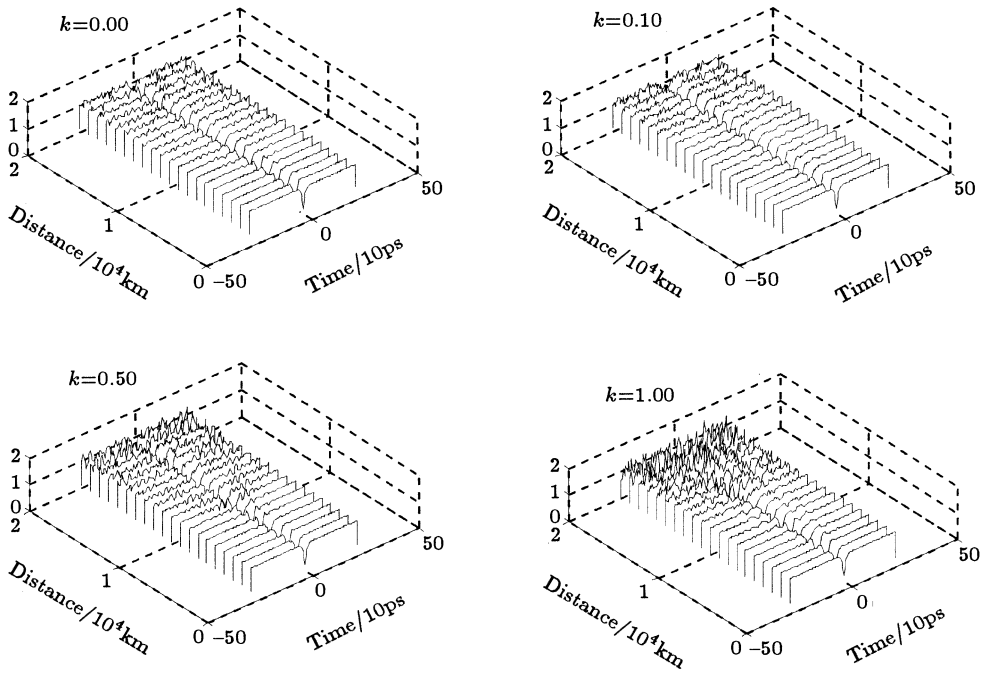


Fig.2. The normalized intensity of single dark soliton versus the propagation distance for different correlation strengths.

For the comparison of the difference between the degradations of these effects on dark soliton and the corresponding bright soliton, these effects on bright soliton propagation are investigated numerically. The initial input pulse is sech(T), pulse-width of the bright soliton is 10ps, and the path-average dispersion is $-1.27\text{ps}^2/\text{km}$. Figure 3 demonstrates the normalized intensity of soliton versus the propagation distance for different correlation strengths, while the standard deviation is the same as in Fig.2. Perturbations result in the widening of pulse-width, lead to splitting and disintegration of bright solitons, and the distance until the bright soliton disintegration is related to the deviations of perturbations and the correlation strength.

The correlation reinforces their effects, quickens the disintegration, and shortens the distance until disintegration. Also, there is a resonance corresponding to their effect which is the largest. We can see that these effects on dark soliton propagation are less than those on bright soliton, and the distance until dark soliton submergence is longer than that until bright soliton disintegration under the same deviations of perturbations. For example, under the correlation strength $k = 1$ and the standard deviation $D = 0.5$, the distance until dark soliton submergence is about 11200km, and the distance until bright soliton disintegration is about 3800km.

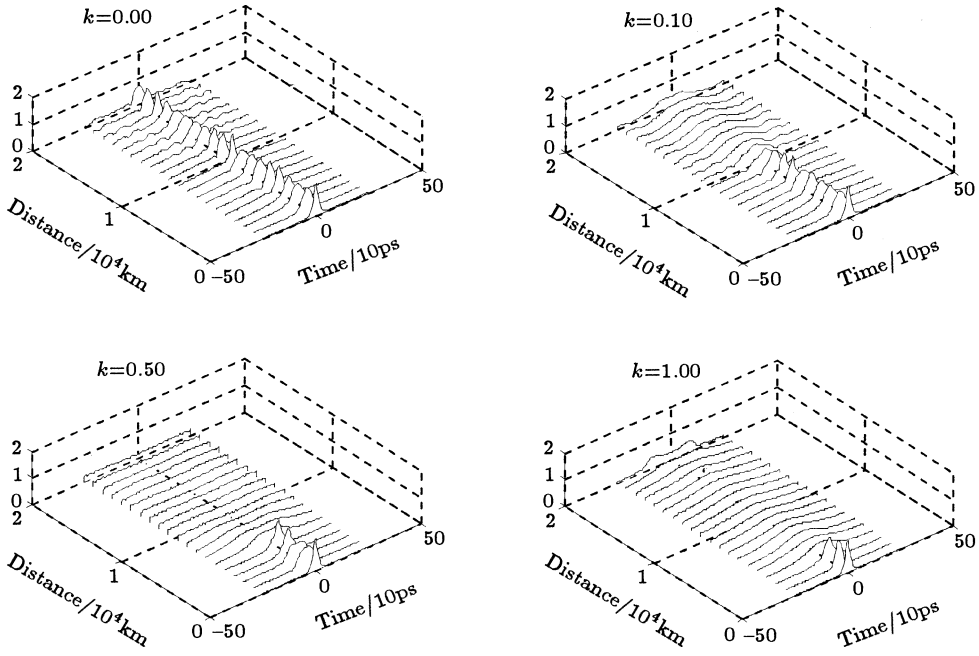


Fig.3. The normalized intensity of single bright soliton versus the propagation distance for different correlation strengths.

4. Effects of correlated perturbations on dark soliton interaction

Interaction between neighbouring dark solitons may induce the crosstalk between two communication channels and the time jitter, thus increasing the bit-error-rate of the transmission system. When initial separation between solitons is larger than five times the soliton width in a usual soliton transmission system, the interaction can be prevented efficiently.[1,2] But in a dark soliton system with perturbations or correlated perturbations, the interaction exhibits very different behaviours.

Figure 4 shows the normalized intensity of two dark solitons versus the propagation distance for different correlation strengths; the standard deviation of every perturbation is $D_i = D = 0.5$ (i=1, 2). The initial input dark soliton pulses are $\tanh(T - \Delta/2)$ for $T > 0$ and $\tanh(T + \Delta/2)$ for $T \leq 0$, and the separation between dark solitons is $\Delta = 10$ (about six times as large as the initial width of soliton). We can see that the perturbations enhance the interaction even if the separation is large enough, and the reason is that perturbations lead to large sidebands which affect neighbouring dark solitons, and then enhance their interaction. We find that the correlation plays important roles in the interaction, and also in the system where there is a resonance at which the effect on the interaction is the largest.

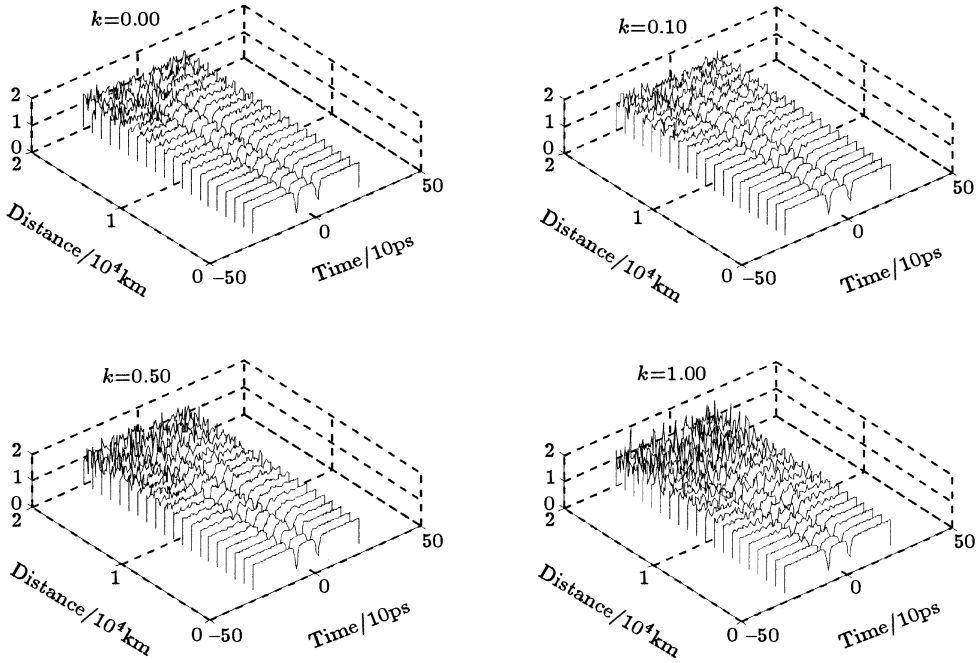


Fig.4. The normalized intensity of two dark solitons versus the propagation distance for different correlation strengths.

Figure 5 shows the normalized intensity of two bright solitons versus the propagation distance for different correlation strengths. The initial input pulses are $\mathrm{sech}(T-\Delta/2)+\mathrm{sech}(T+\Delta/2)$, the separation and the standard deviations are the same as in Fig.4. Perturbations and correlation also enhance their interaction, and by comparing Fig.4 with Fig.5, it is found that these effects on dark soliton interaction are less than those on bright soliton interaction. For example, with the correlation strength $k = 1$ and the standard deviation $D = 0.5$, the distance until distinguishing two dark solitons is about 7000km, and the distance until distinguishing two bright solitons is about 3500km.

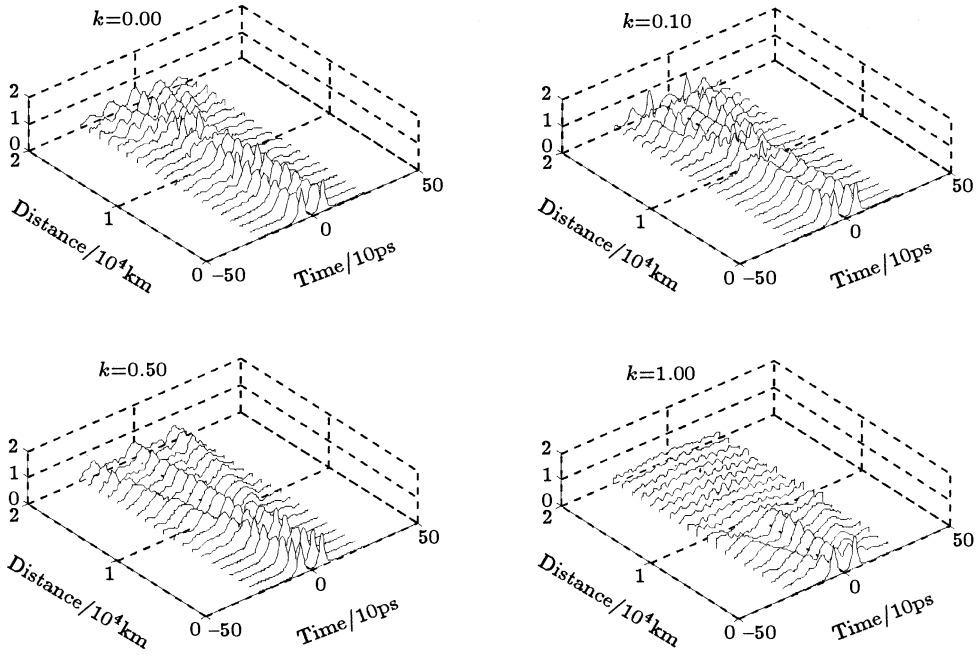


Fig.5. The normalized intensity of two bright solitons versus the propagation distance for different correlation strengths.

5. Control for correlated perturbations

The nonlinear gain completely controls the velocity of dark solitons with higher-order terms, also controls the amplitude of dark solitons because of the fact that their velocity and amplitude are related to each other, so the instability of dark soliton transmission is suppressed. When a nonlinear gain is introduced to suppress these effects, the dark soliton pulses propagating in the fibre with the correlated perturbations can be described by the modified nonlinear Schrödinger equation

$$\begin{aligned} &\mathrm{i}\frac{\partial u}{\partial Z} - \frac{1}{2}\frac{\partial^2 u}{\partial T^2} + |u|^2 u \\ =&\eta_1(Z)\frac{\partial^2 u}{\partial T^2} + \eta_2(Z)|u|^2 u + i\gamma_1 u + i\gamma_2 |u|^2 u \\ &+ \mathrm{i}\gamma_3 |u|^4 u, \end{aligned} \tag{5}$$

where γ_1, γ_2, and γ_3 are the coefficients of the linear and the nonlinear gains respectively. These parameters are chosen based on the condition [11,12]

$$\gamma_1+\gamma_2+\gamma_3 = 0, \quad \gamma_2+2\,\gamma_3 < 0, \quad 5\gamma_2+6\gamma_3 > 0. \tag{6}$$

Figure 6 demonstrates the normalized intensity of a single dark soliton and two dark solitons versus the propagation distance with controllable nonlinear gain. The input pulses and the standard deviation are the same as in Figs.2 and 4. The coefficients of the linear and the nonlinear gains are $\gamma_1 = -0.15$, $\gamma_2 = 0.40$ and $\gamma_3 = -0.25$, respectively. We can see that when a control of nonlinear gain is applied to the transmission line, all ideal dark soliton properties are recovered as if these correlated perturbations had not existed. In other words, the use of the non-linear gain can efficiently suppress the effects on dark soliton propagation and interaction even if the propagation distance is long enough.

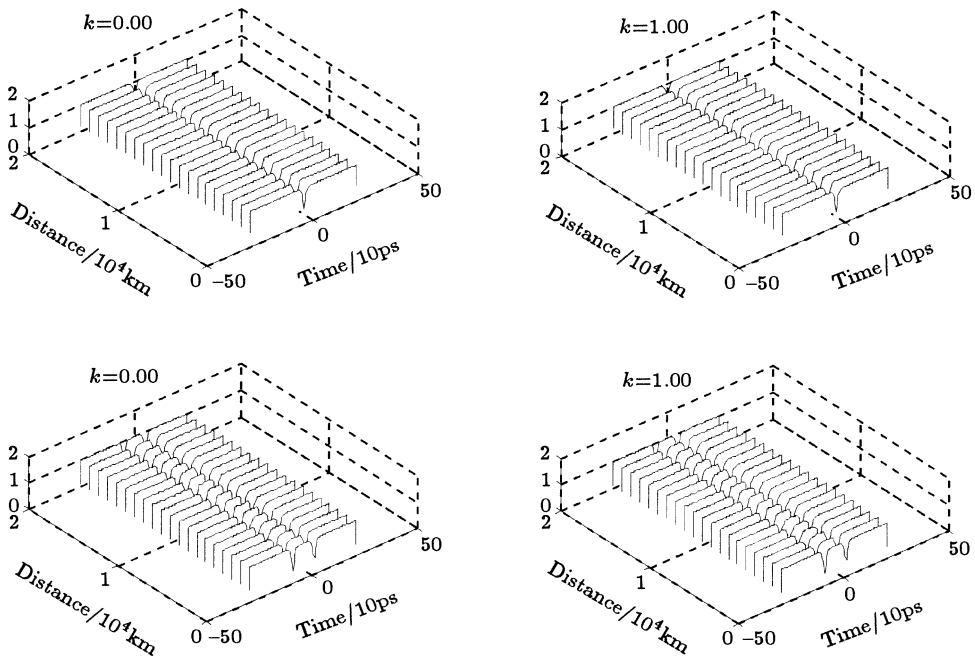


Fig.6. The normalized intensity of single dark soliton and two dark solitons versus the propagation distance with the controllable nonlinear gain.

6. Conclusion

Correlation between stochastic perturbations is introduced in a dark soliton system. It is found that the perturbations result in large sidebands, lead to submergence, and enhance the interaction. The correlation plays an important role, if there is a resonance between two correlated perturbations in the system, where their effects on soliton propagation and interaction are the largest, and the corresponding distance until submergence is the shortest. Comparison is made for the difference between the degradations of these effects on the dark soliton and the corresponding bright solitons. It is shown that these perturbations result in the widening of pulse-width, lead to splitting and disintegration of bright soliton, and the correlation enhances these effects. Their effects on dark soli-

ton are somewhat less than those on bright soliton, that is to say, the dark soliton has stronger resistance against correlated perturbations than the bright soliton. Finally the nonlinear gain is introduced to suppress efficiently these effects.

Acknowledgment

We thank Professor Y. Jia for his fruitful suggestions.

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