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Adiabatic population transfer in effective three-level systems driven by laser beams*

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The population transfer in effective three-state systems driven by laser beams has been studied based on the theory of Lewis–Riesenfeld Hermitian invariants in the full- and partial-adiabatic approximations. A strict formulation of adiabatic conditions is given, and a new adiabatic condition for inducing a complete population transfer is found.

Keywords: adiabatic population transfer, Lewis–Riesenfeld Hermitian invariant, π-pulse method, stimulated Raman adiabatic passage methods

PACC: 4250, 3280B, 0365

1. Introduction

The time-evolution of dynamical systems with an explicitly time-dependent Hamiltonian has attracted considerable attention because of its various applications. Lewis and Riesenfeld (LR) started the investigation of the dynamics and quantization of time-dependent systems with the method of Hermitian invariants.[1,2] We study the time-evolution of an effective three-state system in which the Hamiltonian can be expressed in terms of the generators of the SU(2) algebra. Using LR theory, we have obtained exact solutions of the system, then we recover both the π-pulse method[3] based on the time-evolution operator of the system and the STIRAP method[4,5] in terms of instantaneous eigenstates with the help of time-dependent gauge transformation.[6] Moreover, adiabatic conditions of these two schemes which have been widely investigated and generalized in many papers[7−17] are discussed. Finally, a strict formulation of adiabatic conditions is given, and a new adiabatic condition for inducing a complete population transfer is found.

2. Model and theoretical formalism

We consider an atomic or molecular system driven by two or more lasers in which the major atomic transitions take place among only three of the many states available. For a strong field the semiclassical theory can be used and the time-dependent Schrödinger equation of the system (in units of $\hbar = 1$) $\mathrm{i}\dfrac{\mathrm{d}}{\mathrm{d}t}\Psi(t) = \hat{H}(t)\Psi(t)$ is reduced to a simple form of a three-level system with effective time-dependent Hamilton operator as follows:[18]

$$\hat{H}(t) = \Omega_1(t)\hat{K}_1 + \Omega_2(t)\hat{K}_2 + \Omega_3(t)\hat{K}_3, \qquad (1)$$

and

$$\hat{K}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\hat{K}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

$$\hat{K}_3 = \begin{bmatrix} 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 \end{bmatrix}, \qquad (2)$$

where the coupling Rabi frequencies $\Omega_{1,2,3}$ are generally time-dependent real quantities with the slowly varying electric-field envelopes of the applied lasers, and the vanishing of the diagonal elements implies that frequencies of the lasers are in resonance with

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those of the atomic transitions.[19] Level 1 is assumed to be the ground state, and levels 2 and 3 are respectively labelled in such a way that the electric-dipole transitions between levels 1 and 2 and between levels 2 and 3 are permitted by the electric-dipole selection rule, so that $\Omega_1(t)$ and $\Omega_2(t)$ generally do not vanish. The presence of nonzero $\Omega_3(t)$ represents some effective coupling between levels 1 and 3. We rewrite the Hamiltonian in the 'standard' form of the $SU(2)$ model[18] as

$$\tilde{H}(t)=\frac{\Omega_0(t)}{2}[\hat{K}_+\mathrm{e}^{\mathrm{i}\varphi(t)}+\hat{K}_-\mathrm{e}^{-\mathrm{i}\varphi(t)}]+\Omega_3(t)\hat{K}_0, \quad (3)$$

where

$$\hat{K}_+=\sqrt{2}\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix},$$

$$\hat{K}_-=\sqrt{2}\begin{bmatrix}0&0&0\\1&0&0\\0&1&0\end{bmatrix},$$

$$\hat{K}_0=\begin{bmatrix}1&0&0\\0&0&0\\0&0&-1\end{bmatrix} \quad (4)$$

and $\Omega_1(t)=\Omega_0(t)\cos\varphi(t)$, $\Omega_2(t)=-\Omega_0(t)\sin\varphi(t)$, and equivalently the angle

$$\varphi(t)=-\arctan\frac{\Omega_2(t)}{\Omega_1(t)} \quad (5)$$

plays an important role in the scheme to obtain the adiabatic population transfer besides the norm of $\Omega_1(t)$ and $\Omega_2(t)$, i.e. $\Omega_0(t)=\sqrt{\Omega_1^2(t)+\Omega_2^2(t)}$.

3. Solutions based on LR theory

According to LR theory,[1,2] the construction of the invariant Hermitian operator is given for $SU(1,1)$ and $SU(2)$ systems,[6] and it is shown that invariant operator $\hat{I}$ can be obtained from the Hermitian operator $\hat{K}_0$ with a time-dependent unitary transformation $\hat{I}(t)=\hat{R}(t)\hat{K}_0\hat{R}^\dagger(t)$, where $\hat{R}(t)=\exp\left[\frac{\gamma(t)}{2}\left(\hat{K}_+\mathrm{e}^{-\mathrm{i}\beta(t)}-\hat{K}_-\mathrm{e}^{\mathrm{i}\beta(t)}\right)\right]$. The auxiliary parameters $\gamma(t)$ and $\beta(t)$ are related to $\Omega_0(t),\varphi(t)$ and $\Omega_3(t)$ by the following auxiliary equations:

$$\dot{\gamma}(t)=\Omega_0(t)\sin(\varphi(t)+\beta(t)), \quad (6)$$

$$\dot{\beta}(t)=\Omega_3(t)+\Omega_0(t)\cot\gamma(t)\cos(\varphi(t)+\beta(t)) \quad (7)$$

derived from the equation $\frac{\mathrm{d}\hat{I}}{\mathrm{d}t}=0$. It is easy to find a new Hamiltonian operator using time-dependent gauge transformation[20] with the help of unitary operator $\hat{R}$, and the result is

$$\hat{H}'=\hat{R}^\dagger(t)\tilde{H}(t)\hat{R}(t)-\mathrm{i}\hat{R}^\dagger(t)\frac{\partial}{\partial t}\hat{R}(t) \quad (8)$$

$$=[\Omega_3(t)+2\Omega(t)]\hat{K}_0, \quad (9)$$

which is equivalent to $\tilde{H}(t)$, where

$$\Omega(t)=\left(\dot{\beta}(t)-\Omega_3(t)\right)\sin^2\frac{\gamma(t)}{2}-\frac{\Omega_0(t)}{2}\sin\gamma(t)\cos(\beta(t)+\varphi(t)). \quad (10)$$

The time-dependent Schrödinger equation is covariant

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\Psi'(t)\rangle=\hat{H}'(t)|\Psi'(t)\rangle, \quad (11)$$

where $|\Psi'(t)\rangle=\hat{R}^\dagger(t)\left|\tilde{\Psi}(t)\right\rangle=\hat{R}^\dagger(t)\hat{M}^\dagger|\Psi(t)\rangle$, where $\hat{M}=\begin{bmatrix}\frac{1}{\sqrt{2}}&0&\frac{1}{\sqrt{2}}\\0&1&0\\\frac{\mathrm{i}}{\sqrt{2}}&0&-\frac{\mathrm{i}}{\sqrt{2}}\end{bmatrix}$.

3.1. Time evolution of quantum states

Assume that $|n\rangle$, which is time-independent, is the eigenstate of $\hat{K}_0$ with eigenvalue K_n. The eigenstates of $\hat{I}(t)$ with the same eigenvalue K_n are obviously given by $|n,t\rangle=\hat{R}(t)|n\rangle$. According to Ref.[6], the general solution of the Schrödinger equation is obtained in a compact form in terms of the time-evolution operator $\left|\tilde{\Psi}(t)\right\rangle=\hat{U}(t,0)\left|\tilde{\Psi}(0)\right\rangle$, where the time-evolution operator is regarded as

$$\hat{U}(t,0)=\hat{R}(t)\mathrm{e}^{-\mathrm{i}\epsilon(t)\hat{K}_0}\hat{R}^\dagger(0), \quad (12)$$

and $\epsilon(t)=\int_0^t\mathrm{d}t'[\Omega_3(t')+2\Omega(t')]$. We assume that only level 1 is initially populated, i.e. $|\Psi(0)\rangle=|1\rangle$, and the initial condition in $K_{\pm,0}$ representation is expressed as $\tilde{\Psi}(0)=\hat{M}^\dagger\Psi(0)=\frac{1}{\sqrt{2}}(\ 1\quad 0\quad 1\ )^{\mathrm{T}}$. Then we obtain

$$\left|\tilde{\Psi}(t)\right\rangle=\tilde{C}_1(t)|1\rangle+\tilde{C}_2(t)|2\rangle+\tilde{C}_3(t)|3\rangle, \quad (13)$$

where

$$\tilde{C}_1(t)=\frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\int_0^t\dot{\beta}(t')\mathrm{d}t'}\left[\mathrm{e}^{-\frac{\mathrm{i}}{2}\delta(t)}\cos\frac{\gamma(t)}{2}\cos\frac{\gamma(0)}{2}+\mathrm{e}^{\frac{\mathrm{i}}{2}\delta(t)}\sin\frac{\gamma(t)}{2}\sin\frac{\gamma(0)}{2}\right]^2+\frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\beta(0)-\mathrm{i}\beta(t)}\left[\mathrm{e}^{-\frac{\mathrm{i}}{2}\delta(t)}\cos\frac{\gamma(t)}{2}\sin\frac{\gamma(0)}{2}-\mathrm{e}^{\frac{\mathrm{i}}{2}\delta(t)}\sin\frac{\gamma(t)}{2}\cos\frac{\gamma(0)}{2}\right]^2,$$

$$\begin{aligned}\tilde{C}_2(t) =& \mathrm{i}\Big\{\sin\beta(0)\sin\gamma(0)\cos\gamma(t)\\ &+\sin[\delta(t)-\beta(0)]\sin\gamma(t)\cos^2\frac{\gamma(0)}{2}\\ &+\sin[\delta(t)+\beta(0)]\sin\gamma(t)\sin^2\frac{\gamma(0)}{2}\Big\},\\ \tilde{C}_3(t) =& \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\int_0^t\dot{\beta}(t')\mathrm{d}t'}\Big[\mathrm{e}^{\frac{\mathrm{i}}{2}\delta(t)}\cos\frac{\gamma(t)}{2}\cos\frac{\gamma(0)}{2}\\ &+\mathrm{e}^{-\frac{\mathrm{i}}{2}\delta(t)}\sin\frac{\gamma(t)}{2}\sin\frac{\gamma(0)}{2}\Big]^2\\ &+\frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\beta(0)+\mathrm{i}\beta(t)}\Big[\mathrm{e}^{\frac{\mathrm{i}}{2}\delta(t)}\cos\frac{\gamma(t)}{2}\sin\frac{\gamma(0)}{2}\\ &-\mathrm{e}^{-\frac{\mathrm{i}}{2}\delta(t)}\sin\frac{\gamma(t)}{2}\cos\frac{\gamma(0)}{2}\Big]^2, \qquad (14)\end{aligned}$$

and

$$\begin{aligned}\delta(t) &= \int_0^t[\Omega_3(t')+2\Omega(t')-\dot{\beta}(t')]\mathrm{d}t'\\ &= \int_0^t[(\Omega_3(t')-\dot{\beta}(t'))\cos\gamma(t')\\ &\quad -\Omega_0(t')\sin\gamma(t')\cos(\beta(t')+\varphi(t'))]\mathrm{d}t' \qquad (15)\end{aligned}$$

is the pulse area which is a key quantity in the π-pulse scheme. It is easy to verify from Eqs.(14), (15) that the initial condition is satisfied. When the parameters in Eqs.(14) are adjusted such that $\tilde{C}_1(t) = -\dfrac{\mathrm{i}}{\sqrt{2}}$, $\tilde{C}_2(t) = 0$, $\tilde{C}_3(t) = \dfrac{\mathrm{i}}{\sqrt{2}}$, we then achieve the complete population transfer $|\Psi(t)\rangle = |3\rangle$ because of

$$\Psi(t) = M\tilde{\Psi}(t) = (\ 0 \quad 0 \quad 1\ )^{\mathrm{T}}. \qquad (16)$$

3.2. Instantaneous eigenstates

To avoid any dynamical phase,[21] we make the state vector a null-eigenvalue state. This idea can be realized in the gauge of diagonalized Hamiltonian $\hat{H}'(t)$, the basis vectors of which are $|\Psi'_+(t)\rangle$, $|\Psi'_0(t)\rangle$ and $|\Psi'_-(t)\rangle$ corresponding to the eigenvalues $\{E_+(t), E_0(t), E_-(t)\}$ respectively, where

$$\begin{aligned}E_+(t) &= \Omega_3(t)+2\Omega(t),\\ E_0(t) &= 0,\\ E_-(t) &= -[\Omega_3(t)+2\Omega(t)], \qquad (17)\end{aligned}$$

$|\Psi'_0(t)\rangle$ is the null-eigenvalue state desired, which satisfies $\hat{H}'(t)|\Psi'_0(t)\rangle = 0$.

We initially prepare the system as null-eigenvalue state $|\Psi'_0(0)\rangle$ and notice that $|\tilde{\Psi}_0(0)\rangle = \hat{R}(0)|\Psi_0(0)\rangle$, and the null-eigenvalue instantaneous eigenstate is (notice that the null-eigenvalue $\hat{K}_0|\Psi'_0\rangle = 0$)

$$\begin{aligned}|\Psi_0(t)\rangle =& \hat{M}\hat{U}(t,0)\hat{R}(0)|\Psi'_0(0)\rangle\\ =& \hat{M}\hat{R}(t)|\Psi'_0(0)\rangle, \qquad (18)\end{aligned}$$

which is expressed as

$$\begin{aligned}|\Psi_0(t)\rangle =& \mathrm{i}\sin\gamma(t)\sin\beta(t)|1\rangle+\cos\gamma(t)|2\rangle\\ &-\mathrm{i}\sin\gamma(t)\cos\beta(t)|3\rangle. \qquad (19)\end{aligned}$$

This is a state without a dynamic phase, and our task is to adjust time-dependent parameters to realize population transfer, i.e. to make

$$\begin{aligned}&|\langle 1|\Psi_0(t)\rangle| = 0\ ,\ |\langle 2|\Psi_0(t)\rangle| = 0,\\ &\text{and}\quad |\langle 3|\Psi_0(t)\rangle| = 1 \qquad (20)\end{aligned}$$

to achieve the complete population transfer.

4. Adiabatic approximation

We now discuss the explicit conditions for achieving a complete population transfer. We consider the adiabatic approximation where the terms containing time derivatives $\dot{\gamma}$, $\dot{\beta}$ in the auxiliary equations (6) and (7) can be neglected.

4.1. Full-adiabatic approximation

If the auxiliary parameters $\gamma(t)$ and $\beta(t)$ both vary with time slowly enough, i.e.

$$\dot{\gamma}(t) \simeq 0, \quad \text{and} \quad \dot{\beta}(t) \simeq 0, \qquad (21)$$

we have

$$\beta(t) = 2n\pi - \varphi(t), \tan\gamma(t) \simeq -\frac{\Omega_0(t)}{\Omega_3(t)}. \qquad (22)$$

Moreover, the values of Ω_0 and Ω_3 are adjusted to have

$$\gamma(t) = -\arctan\frac{\Omega_0(t)}{\Omega_3(t)} \simeq -\frac{\pi}{2}, \qquad (23)$$

which means that Ω_3 is negligibly small compared with the norm Ω_0 which is finite (in the partial-adiabatic approximation discussed later Ω_0 is regarded as a quantity with very high value).

4.1.1. modified π-pulse method

In the approximation shown in Eq.(21), the simplified form of $\tilde{C}_2(t)$ is regarded as

$$\tilde{C}_2(t) = -\mathrm{i}\cos\beta(0)\sin\delta_{\mathrm{fa}}(t), \qquad (24)$$

with the pulse area Eq.(15) reduced to

$$\delta_{\mathrm{fa}}(t) = \int_0^t\sqrt{\Omega_0^2(t')+\Omega_3^2(t')}\mathrm{d}t'. \qquad (25)$$

When $\delta_{\rm fa}(t) = n\pi$, we have $\tilde{C}_2(t) = 0$, and

$$\tilde{C}_1(t) = \frac{1}{2\sqrt{2}} e^{i\beta(0)-i\beta(t)} [1+\cos\delta_{\rm fa}(t)] - \frac{1}{2\sqrt{2}} e^{-i\beta(0)-i\beta(t)} [1-\cos\delta_{\rm fa}(t)], \quad (26)$$

$$\tilde{C}_3(t) = \frac{1}{2\sqrt{2}} e^{-i\beta(0)+i\beta(t)} [1+\cos\delta_{\rm fa}(t)] - \frac{1}{2\sqrt{2}} e^{i\beta(0)+i\beta(t)} [1-\cos\delta_{\rm fa}(t)]. \quad (27)$$

If the pulse area is adjusted such that $\delta_{\rm fa}(t) = 2n\pi$, we have

$$\tilde{C}_1(t) = \frac{1}{\sqrt{2}} e^{i\beta(0)-i\beta(t)}, \quad \tilde{C}_3(t) = \frac{1}{\sqrt{2}} e^{-i\beta(0)+i\beta(t)}. \quad (28)$$

We assume the orders of laser beams as follows: intuitive order ($\beta(t) - \beta(0) = -\frac{\pi}{2}$), whereas counterintuitive order ($\beta(t) - \beta(0) = \frac{\pi}{2}$),[22] then $\tilde{C}_1(t) = \frac{i}{\sqrt{2}}$, $\tilde{C}_3(t) = \frac{-i}{\sqrt{2}}$ for intuitive order; but $\tilde{C}_1(t) = \frac{-i}{\sqrt{2}}$, $\tilde{C}_3(t) = \frac{i}{\sqrt{2}}$ for counterintuitive order. In both cases the complete population transfer is achieved.

In the case of $\delta_{\rm fa}(t) = (2n+1)\pi$, the state coefficients are

$$\tilde{C}_1(t) = -\frac{1}{\sqrt{2}} e^{-i\beta(0)-i\beta(t)}, \quad \tilde{C}_3(t) = -\frac{1}{\sqrt{2}} e^{i\beta(0)+i\beta(t)}. \quad (29)$$

For both intuitive order with $\beta(t) = -\frac{\pi}{2}$, $\beta(0) = 0$, and counterintuitive order with $\beta(t) = 0$, $\beta(0) = -\frac{\pi}{2}$, we have $\tilde{C}_1(t) = -\frac{i}{\sqrt{2}}$, $\tilde{C}_3(t) = \frac{i}{\sqrt{2}}$.

Summing up all these situations, we obtain what we call the 'modified π-pulse method' which includes two conditions for achieving complete population transfer: (a) the pulse area should be controlled to have $\delta_{\rm fa}(t) = n\pi$, and (b)

$$\beta(t) - \beta(0) = \arctan\frac{\Omega_2(0)}{\Omega_1(0)} - \arctan\frac{\Omega_2(t)}{\Omega_1(t)} = \pm\frac{\pi}{2} \quad (30)$$

should be satisfied.

4.1.2. STIRAP method

Our starting point of the scheme-STIRAP is the null-eigenvalue instantaneous eigenstate Eq.(19) which, in the full-adiabatic approximation, is reduced to

$$|\Psi_0(t)\rangle = -i\sin\beta(t)|1\rangle + i\cos\beta(t)|3\rangle. \quad (31)$$

Assume the counterintuitive pulse sequence in this scheme to have the following forms, i.e.

$$\beta(0) = -\arctan\frac{\Omega_2(0)}{\Omega_1(0)} = -\frac{\pi}{2}$$

and

$$\beta(t) = -\arctan\frac{\Omega_2(t)}{\Omega_1(t)} \simeq 0, \quad (32)$$

which lead to obvious complete population transfer that at initial time ($t = 0$), $|\Psi_0(0)\rangle = i|1\rangle$; and at time t, $|\Psi_0(t)\rangle = i|3\rangle$. The population transfer can be asymptotically achieved if the ratio $\frac{\Omega_2(t)}{\Omega_1(t)}$ is very large initially and then becomes small enough after a long time adiabatic control.

For both the above two schemes, the adiabatic condition Eq.(21) is required. We use

$$\dot{\beta}(t) = -\dot{\varphi}(t) = -\frac{d}{dt}\left(\arctan\frac{\Omega_2(t)}{\Omega_1(t)}\right) \simeq 0, \quad (33)$$

along with Eq.(23), which is exactly satisfied particularly for $\Omega_3(t) = 0$, to produce the transfer. In other words, the adiabatic time-evolution is required in the modified π-pulse method based on our exact solution, and the population transfer can be obtained only asymptotically, while the original π-pulse method,[3] in which the only condition is the pulse area being odd-multiple of π, does not hold true for an adiabatic process. The conditions in STIRAP scheme discussed in Ref.[7] (where $\Omega_3(t) = 0$) are given by $\frac{1}{2}\Omega_0 t \gg 1$, which can be approached by increasing Ω_0 in a finite time period t, or in other words, the complete population transfer can be achieved with strong enough pulses even for a short pulse duration. This is far different from our adiabatic conditions (30) and (33), which require a long interaction time no matter whether $\Omega_3(t)$ vanishes or not.

4.2. Partial-adiabatic approximation

We now demonstrate a new method to obtain population transfer in the framework of exact solution which has not been discussed in the literature. We consider a very high value of Ω_0 compared with a finite Ω_3 (different from the full-adiabatic case where Ω_3 is considered as being negligibly small) such that

Eq.(23) is still satisfied approximately. From Eq.(6) we have

$$\dot{\beta}(t) \simeq \Omega_3 \not\to 0 \tag{34}$$

while $\dot{\gamma}(t) \simeq 0$, so called the partial-adiabatic approximation.

4.2.1. Scheme A : π-pulse method

In the partial-adiabatic approximation, we acquire $\tilde{C}_2(t) = -\mathrm{i}\cos\beta(0)\sin\delta_{\mathrm{pa}}(t)$ with the pulse area in the partial-adiabatic approximation reduced to

$$\delta_{\mathrm{pa}}(t) = \int_0^t \Omega_0(t')\,\mathrm{d}t'. \tag{35}$$

When the norm Ω_0 is controlled to have pulse area $\delta_{\mathrm{pa}}(t) = n\pi$, we obtain $\tilde{C}_2(t) \equiv 0$. The condition for complete population transfer is formally the same as that in the full-adiabatic case with δ_{fa} replaced by δ_{pa}. The long time interaction required in the full-adiabatic case to have the finite change of phase angle, i.e. Eq.(30), is, however, not necessary in the partial-adiabatic approximation because of the non-adiabatic condition Eq.(34). The pulse area $\delta_{\mathrm{pa}}(t)$ in this case is totally independent of $\Omega_3(t)$.

4.2.2. Scheme B: STIRAP

With the condition Eq.(23), the state vector $|\Psi_0(t)\rangle$ in Eq.(19) has the same form as that in Eq.(31). The mechanism responsible for the complete population transfer in the partial-adiabatic approximation is similar to that in the full-adiabatic case, however without the long time requirement to achieve the population transfer Eq.(32). Particularly if Ω_3 is large enough the interaction time t could be shorter than spontaneous emission time, which is exactly the point interesting experimentalists.[23]

5. Conclusion

We have investigated the time evolution of a three-state system by using an exactly solvable time-dependent Hamiltonian given by a linear combination of $SU(2)$ generators with the help of the invariant Hermitian operator. Besides the same requirement $\gamma = -\arctan\dfrac{\Omega_0(t)}{\Omega_3(t)} \simeq -\dfrac{\pi}{2}$, the modified π-pulse and the STIRAP schemes can be realized under two different conditions, respectively (1) $\dot{\beta}(t) \simeq \Omega_3(t) \to 0$ for a long time; (2) $\dot{\beta}(t) \simeq \Omega_3 \not\to 0$ with a short pulse duration. The former procedure is experimentally invalid since slow population transfer conflicts with the requirement to move population more rapidly than spontaneous emission which depletes population.[23] The latter is a new route to realize population transfer experimentally for adjustment of Ω_3 and pulse duration to have the finite change of the phase angle Eq.(30) is an easy task. We expect the formalism based on the explicit time-evolution operator demonstrated in the present paper to find application in quantum computing. Further research along this direction is in progress.

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