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Efficient scheme for preparation of the multi-atom W state via cavity quantum electrodynamics

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We present an efficient scheme for preparation of the multi-atom W state via cavity quantum electrodynamics. Involved in this scheme are n identical two-level atoms and a single-mode cavity field. Discussion indicates that this scheme can be realized easily by current technologies.

Keywords: entangled states, W state, cavity QED
PACC: 4250, 0365

1. Introduction

Quantum entanglement is at the very heart of quantum physics. Because of some essential properties, entangled states are widely used in the testing for correlations violating Bell's inequalities,[1] quantum key distribution[2] and quantum teleportation.[3–5] It has been shown that multi-particle entangled states have more advantages than the two-particle entangled states in their applications in quantum cloning,[6] teleportation[7] and dense coding.[8] So the preparation and manipulation of multi-particle entangled states have attracted much attention. Many people have proposed a number of different schemes using the cavity quantum electrodynamics (QED),[9] ion trap,[10] or NMR.[11,12] But in most previous schemes the cavity and ion motion were used as memories. Thus we must do some measurements to acquire certain states, while in the ion traps it is difficult to gain the joint ground state of the ion motion and the heating of the ions. In the cavity QED, Zheng and Guo[13] have proposed a scheme for the generation of two-atom maximally entangled states. Osnahai *et al*[14] realized their scheme in experiment very quickly. Recently, using the same model Guo *et al*[15] have presented a scheme to generate tripartite and multi-particle entangled states. In their paper, when the particle number $n > 4$, they must measure the nth atom to obtain the (n-1)-particle maximally entangled W state. Guo and Zhang [16] have also proposed a novel scheme to prepare the tripartite W state by letting atoms pass through the cavity one by one. In this paper, we present a new scheme to obtain the multi-atom W state. We let several atoms interact with the cavity simultaneously to reduce the total interaction time and need not do any measurement. So the requirement on this scheme is greatly relaxed.

2. The preparation of the n-atom W state

From the papers of Zheng and Guo[13] and Guo *et al*,[15] we assume that n identical two-level atoms simultaneously interact with a single-mode cavity and there is no energy exchange between the atomic system and the cavity. The effective Hamiltonian in the interaction picture is

$$H = \lambda\left[\sum_{i,j=1}^{n}(S_j^+ S_i^- a a^+ - S_j^- S_i^+ a^+ a)\right], \qquad (1)$$

where $\lambda = g^2/\delta$, g is the atom–cavity coupling strength, while δ is the detuning between the atomic transition frequency and the cavity frequency; $g \gg \delta$, $S_j^+ = |e_j\rangle\langle g_j|$ and $S_j^- = |g_j\rangle\langle e_j|$, $|e_j\rangle$ and $|g_j\rangle$ are the excited and ground states of the jth atom, and a^+ and a are respectively the creation and annihilation operators for the cavity mode.

If the cavity field is initially in the vacuum state,

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the Hamiltonian reduces to

$$H = \lambda\Big(\sum_{j=1}^{n} |e_j\rangle\langle e_j| + \sum_{i,j=1,i\neq j}^{n} S_j^+ S_i^-\Big). \quad (2)$$

Assume the atoms are initially in the states $|e_1 g_2\rangle$. Then the state evolution of the system is

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}\lambda t}(\cos\lambda t\,|e_1 g_2\rangle - \mathrm{i}\sin\lambda t\,|g_1 e_2\rangle). \quad (3)$$

With the choice of $\lambda t = \frac{\pi}{4}$, we can obtain (in fact as long as $\lambda t = \frac{\pi}{4} + 2k\pi$ and k is a integer, Eq.(3) can be written as Eq.(4), so we can realize it in experiment by choosing appropriate k. In this paper, we ignore the problem about this aspect)

$$|\Psi\rangle = \mathrm{e}^{-\mathrm{i}\frac{3\pi}{4}}\frac{1}{\sqrt{2}}(\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}|e_1 g_2\rangle + |g_1 e_2\rangle). \quad (4)$$

Assume the first atom is in the excited state $|e\rangle$, and the other $n-1$ atoms are in the ground state $|g\rangle$. Let the n atoms simultaneously pass through the cavity, and then the evolution of the state is

$$\begin{aligned} W_n =& \frac{\mathrm{e}^{-\mathrm{i}n\lambda t}+n-1}{n}|e_1 g_2\cdots g_n\rangle \\ &+ \frac{\mathrm{e}^{-\mathrm{i}n\lambda t}-1}{n}(|e_2 g_1 g_3\cdots g_n\rangle + \cdots \\ &+ |e_n g_1\cdots g_{n-1}\rangle). \end{aligned} \quad (5)$$

Only in the case of $n \leq 4$ can $\left|\frac{\mathrm{e}^{-\mathrm{i}n\lambda t}+n-1}{n}\right|$ equal $\left|\frac{\mathrm{e}^{-\mathrm{i}n\lambda t}-1}{n}\right|$, which represents the maximally entangled W states. By taking certain measurement at some time t, Guo *et al* gained the other $n-1$ atoms in the maximally entangled states with the probability of $\left|\frac{\sqrt{n-1}}{n}(\mathrm{e}^{-\mathrm{i}n\lambda t}-1)\right|^2$.

Next we propose a different scheme to prepare an n-atom W state. When n=3 or 4 we can have a maximally entangled W state in one step.

Atoms 1, 2, 3 are initially in the state $|e_1 g_2 g_3\rangle$ and interact with the cavity together. From Eq.(5), the state evolution of the system can be written as

$$\begin{aligned} W_3 =& \frac{\mathrm{e}^{-\mathrm{i}3\lambda t}+2}{3}|e_1 g_2 g_3\rangle \\ &+ \frac{\mathrm{e}^{-\mathrm{i}3\lambda t}-1}{3}(|e_2 g_1 g_3\rangle + |e_3 g_1 g_2\rangle). \end{aligned} \quad (6)$$

In the case of $\lambda t = \frac{2\pi}{9}$, we obtain

$$W_3 = \mathrm{e}^{-\mathrm{i}\frac{5\pi}{6}}\times\frac{1}{\sqrt{3}}(\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}|e_1 g_2 g_3\rangle + |e_2 g_1 g_3\rangle + |e_3 g_1 g_2\rangle). \quad (7)$$

If n=4, the initial state of the atoms 1, 2, 3, 4 is $|e_1 g_2 g_3 g_4\rangle$, then

$$\begin{aligned} W_4 =& \frac{\mathrm{e}^{-\mathrm{i}4\lambda t}+3}{4}|e_1 g_2 g_3 g_4\rangle \\ &+ \frac{\mathrm{e}^{-\mathrm{i}4\lambda t}-1}{4}(|e_2 g_1 g_3 g_4\rangle \\ &+ |e_3 g_1 g_2 g_4\rangle + |e_4 g_1 g_2 g_3\rangle). \end{aligned} \quad (8)$$

When $\lambda t = \frac{\pi}{4}$, we can obtain a four-atom W state,

$$\begin{aligned} W_4 =& \mathrm{e}^{-\mathrm{i}\pi}\frac{1}{\sqrt{4}}(\mathrm{e}^{\mathrm{i}\pi}|e_1 g_2 g_3 g_4\rangle + |e_2 g_1 g_3 g_4\rangle \\ &+ |e_3 g_1 g_2 g_4\rangle + |e_4 g_1 g_2 g_3\rangle). \end{aligned} \quad (9)$$

Now we consider the case of $n \geq 5$. We assume $n = 3k+m+1$ ($k \geq 1, m \geq 0$, k and m are integers). Based on this assumption, we can first separate $3k$+1 atoms into k parts, and each of them involves atoms 1 and other 3 atoms such as 1, 2, 3, 4 and 1, 5, 6, 7, while atom 1 and the other m atoms comprise the last part. So the number of the atoms involved in the last part is determined by m (depicted in Fig.1). Further assume that all atoms except atom 1 are initially in the ground states $|g_2\cdots g_n\rangle$.

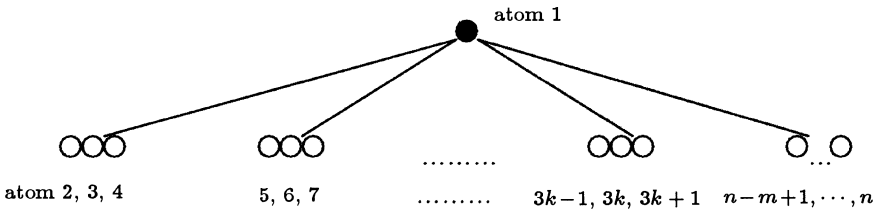


Fig.1. n atoms are separated into $k+1$ parts.

Then let every part seriatim pass through the cavity. We describe the process (depicted in Fig.2) of preparing n-atom W states:

First let atoms 1, 2, 3, 4 simultaneously enter the cavity. After the atoms passing through the cavity, we can obtain Eq.(8). If we choose $\lambda t_1 = \frac{1}{4}\arccos\frac{n-8}{n}$, atoms 1, 2, 3, 4 are in the entangled state:

$$|\Psi_1\rangle = \frac{1}{\sqrt{n}}[\sqrt{n-3}\mathrm{e}^{\mathrm{i}\theta_1'}|e_1g_2g_3g_4\rangle + \mathrm{e}^{\mathrm{i}\theta_1}(|e_2g_1g_3g_4\rangle + |e_3g_1g_2g_4\rangle + |e_4g_1g_2g_3\rangle)], \tag{10}$$

where θ_1' and θ_1 are determined by λt_1.

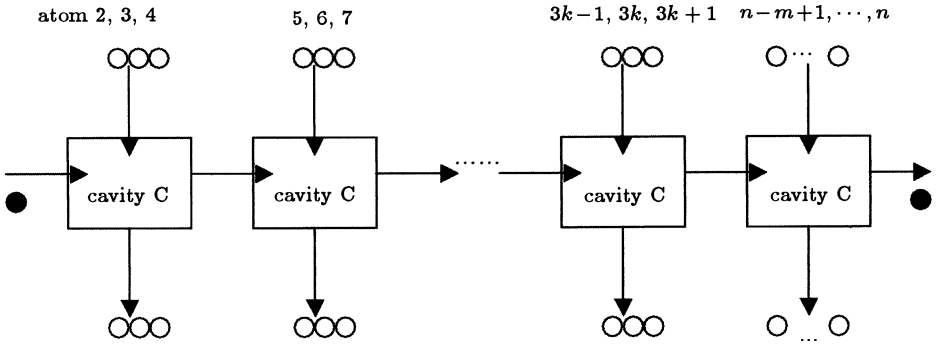


Fig.2. Preparation of n-atom W states.

Because the effective Hamiltonian has no effect on the state $|g_1g_2\cdots g_n\rangle$, when we let the atoms 1, 5, 6, 7 simultaneously interact with the cavity and make the interaction time satisfy $\lambda t_2 = \frac{1}{4}\arccos\frac{n-11}{n-3}$, the state of the whole system $|\Psi_1\rangle|g_5g_6g_7\rangle$ becomes

$$\begin{aligned}|\Psi_2\rangle =& \frac{1}{\sqrt{n}}[\sqrt{n-6}\mathrm{e}^{\mathrm{i}\theta_2'}|e_1g_2\cdots g_7\rangle \\ &+ \mathrm{e}^{\mathrm{i}\theta_1}(|e_2g_1g_3\cdots g_7\rangle + |e_3g_1g_2g_4\cdots g_7\rangle \\ &+ |e_4g_1g_2g_3g_5g_6g_7\rangle) \\ &+ \mathrm{e}^{\mathrm{i}\theta_2}(|e_5g_1\cdots g_4g_6g_7\rangle + |e_6g_1\cdots g_5g_7\rangle \\ &+ |e_7g_1\cdots g_6\rangle)],\end{aligned} \tag{11}$$

where θ_2', θ_1, and θ_2 are determined by λt_1 and λt_2.

Similarly, atoms $8, 9, 10, \cdots 3k-1, 3k, 3k+1$ undergo the same process, successively.

After atoms $1, 3k-1, 3k, 3k+1$ passing through the cavity, the interaction time t satisfies $\lambda t_k = \frac{1}{4}\arccos\frac{n-3k-5}{n-3k+3}$, then

$$\begin{aligned}|\Psi_k\rangle =& \frac{1}{\sqrt{n}}[\sqrt{n-3k}\mathrm{e}^{\mathrm{i}\theta_k'}|e_1g_2\cdots g_{3k+1}\rangle \\ &+ \mathrm{e}^{\mathrm{i}\theta_1}(|e_2g_1g_3\cdots g_{3k+1}\rangle \\ &+ |e_3g_1g_2g_4\cdots g_{3k+1}\rangle \\ &+ |e_4g_1g_2g_3g_5\cdots g_{3k+1}\rangle) \\ &+ \mathrm{e}^{\mathrm{i}\theta_2}(|e_5g_1\cdots g_4g_6\cdots g_{3k+1}\rangle \\ &+ |e_6g_1\cdots g_5g_7\cdots g_{3k+1}\rangle \\ &+ |e_7g_1\cdots g_6g_8\cdots g_{3k+1}\rangle) \\ &+ \mathrm{e}^{\mathrm{i}\theta_3}(|e_8g_1\cdots g_7g_9\cdots g_{3k+1}\rangle \\ &+ |e_9g_1\cdots g_8g_{10}\cdots g_{3k+1}\rangle \\ &+ |e_{10}g_1\cdots g_9g_{11}\cdots g_{3k+1}\rangle) \\ &+ \cdots \\ &+ \mathrm{e}^{\mathrm{i}\theta_k}(|e_{3k-1}g_1\cdots g_{3k-2}g_{3k}g_{3k+1}\rangle \\ &+ |e_{3k}g_1\cdots g_{3k-1}g_{3k+1}\rangle \\ &+ |e_{3k+1}g_1\cdots g_{3k}\rangle)].\end{aligned} \tag{12}$$

Finally we consider m=0, 1, or 2.

In the case of m=0, $n = 3k+1$. From the above procedure, in Eq.(12) we can directly obtain the n-atom maximally entangled W state:

$$\begin{aligned}W =& \frac{1}{\sqrt{n}}[\mathrm{e}^{\mathrm{i}\theta_k'}|e_1g_2\cdots g_n\rangle \\ &+ \mathrm{e}^{\mathrm{i}\theta_1}(|e_2g_1g_3\cdots g_n\rangle + |e_3g_1g_2g_4\cdots g_n\rangle + |e_4g_1g_2g_3g_5\cdots g_n\rangle) \\ &+ \mathrm{e}^{\mathrm{i}\theta_2}(|e_5g_1\cdots g_4g_6\cdots g_n\rangle + |e_6g_1\cdots g_5g_7\cdots g_n\rangle + |e_7g_1\cdots g_6g_8\cdots g_n\rangle) \\ &+ \mathrm{e}^{\mathrm{i}\theta_3}(|e_8g_1\cdots g_7g_9\cdots g_n\rangle + |e_9g_1\cdots g_8g_{10}\cdots g_n\rangle + |e_{10}g_1\cdots g_9g_{11}\cdots g_n\rangle) \\ &+ \cdots \\ &+ \mathrm{e}^{\mathrm{i}\theta_k}(|e_{n-2}g_1\cdots g_{n-3}g_{n-1}g_n\rangle + |e_{n-1}g_1\cdots g_{n-2}g_n\rangle + |e_ng_1\cdots g_{n-1}\rangle)].\end{aligned} \tag{13}$$

If m=1, we let atom 1 and the last atom n ($n = 3k+2$) pass through the cavity at one time.

With the choice of $\lambda t_{k+1} = \frac{\pi}{4}$, for Eqs.(4) and (12), we can also obtain the n-atom W state:

$$\begin{aligned} W =& \frac{1}{\sqrt{n}}[\mathrm{e}^{\mathrm{i}\theta'_{k+1}}\,|e_1 g_2 \cdots g_n\rangle \\ &+ \mathrm{e}^{\mathrm{i}\theta_1}(|e_2 g_1 g_3 \cdots g_n\rangle + |e_3 g_1 g_2 g_4 \cdots g_n\rangle + |e_4 g_1 g_2 g_3 g_5 \cdots g_n\rangle) \\ &+ \mathrm{e}^{\mathrm{i}\theta_2}(|e_5 g_1 \cdots g_4 g_6 \cdots g_n\rangle + |e_6 g_1 \cdots g_5 g_7 \cdots g_n\rangle + |e_7 g_1 \cdots g_6 g_8 \cdots g_n\rangle) \\ &+ \mathrm{e}^{\mathrm{i}\theta_3}(|e_8 g_1 \cdots g_7 g_9 \cdots g_n\rangle + |e_9 g_1 \cdots g_8 g_{10} \cdots g_n\rangle + |e_{10} g_1 \cdots g_9 g_{11} \cdots g_n\rangle) \\ &+ \cdots \\ &+ \mathrm{e}^{\mathrm{i}\theta_k}(|e_{n-3} g_1 \cdots g_{n-4} g_{n-2} g_{n-1} g_n\rangle + |e_{n-2} g_1 \cdots g_{n-3} g_{n-1} g_n\rangle + |e_{n-1} g_1 \cdots g_{n-2} g_n\rangle) \\ &+ \mathrm{e}^{\mathrm{i}\theta_{k+1}}\,|e_n g_1 \cdots g_{n-1}\rangle]. \end{aligned} \tag{14}$$

When m=2, $n = 3k + 3$, we finally let atoms 1, $n-1$, n interact with the cavity simultaneously and $\lambda t_{k+1} = \frac{2\pi}{9}$, then the n-atom W state is written as

$$\begin{aligned} W =& \frac{1}{\sqrt{n}}[\mathrm{e}^{\mathrm{i}\theta'_{k+1}}\,|e_1 g_2 \cdots g_n\rangle \\ &+ \mathrm{e}^{\mathrm{i}\theta_1}(|e_2 g_1 g_3 \cdots g_n\rangle + |e_3 g_1 g_2 g_4 \cdots g_n\rangle + |e_4 g_1 g_2 g_3 g_5 \cdots g_n\rangle) \\ &+ \mathrm{e}^{\mathrm{i}\theta_2}(|e_5 g_1 \cdots g_4 g_6 \cdots g_n\rangle + |e_6 g_1 \cdots g_5 g_7 \cdots g_n\rangle + |e_7 g_1 \cdots g_6 g_8 \cdots g_n\rangle) \\ &+ \mathrm{e}^{\mathrm{i}\theta_3}(|e_8 g_1 \cdots g_7 g_9 \cdots g_n\rangle + |e_9 g_1 \cdots g_8 g_{10} \cdots g_n\rangle + |e_{10} g_1 \cdots g_9 g_{11} \cdots g_n\rangle) \\ &+ \cdots \\ &+ \mathrm{e}^{\mathrm{i}\theta_k}(|e_{n-4} g_1 \cdots g_{n-5} g_{n-3} \cdots g_n\rangle + |e_{n-3} g_1 \cdots g_{n-4} g_{n-2} g_{n-1} g_n\rangle + |e_{n-2} g_1 \cdots g_{n-3} g_{n-1} g_n\rangle) \\ &+ \mathrm{e}^{\mathrm{i}\theta_{k+1}}(|e_{n-1} g_1 \cdots g_{n-2} g_n\rangle + |e_n g_1 \cdots g_{n-1}\rangle)], \end{aligned} \tag{15}$$

where all θ involved are determined by every λt.

3. Conclusion

In summary, we have proposed a new scheme to prepare the n-atom W state. Compared with the previous schemes there are some distinct advantages of the scheme in this paper. It only needs n-identical two-level atoms and a large-detuned single-mode cavity field that is initially in the vacuum state. There is no energy exchange between the atomic system and the cavity. Nor is there any quantum measurement needed during the whole process. Moreover, we let several atoms interact with the cavity simultaneously, so the total interaction time is greatly reduced. We believe that compared with previous schemes, this scheme is more amenable to realization in experiment.

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