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Atomic dynamics in the mode–mode competition system*

Wu Qin(吴 琴) and Fang Mao-Fa(方卯发)†

College of Physics and Information Science, Hunan Normal University, Changsha 410081, *China*

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The atomic dynamical properties in the system with competing k-photon and l-photon transitions are studied fully by means of quantum theory. We discuss the influences of the mode–mode competition, the relative competing strengths of the atom and the two-mode field, and the initial state of the system on the atomic dynamics. We show that the presence of the mode–mode competition can result in quite a periodical collapses-revivals of the atomic inversion and the increase of the initial photons of the system can lead to the collapse–revival phenomenon and prolong the revival time of the atomic inversion.

Keywords: two-mode competition model, atomic inversion, collapse and revival
PACC: 4250

1. Introduction

Since Jaynes and Cummings first put forward what has become the Jaynes–Cummings model (JCM),[1] there has arisen a sizeable scientific industry[2−8] devoted to exploiting and extending this work for its interesting nonclassical effects, such as collapses and revivals of Rabi oscillations,[9] squeezing[10,11] and anti-bunching effects.[12,13] By now multi-photon processes, multi-atom processes, multi-level atoms and multi-mode quantized fields analogues have appeared.[14−20] It is worth noticing that in such models one assumes that a given mode acts only on one optical transition, i.e. the two modes do not act simultaneously on one transition. Semiclassically one knows that if an atom interacts with different fields, several nonlinear processes can have effect at the same time, and a large body of literature has been devoted to the interaction of a bichromatic field with a two-level atom. The bichromatic fields lead to many new dynamical features as a result of mode–mode competition.[21,22] There are two well-known examples of such competing effects in nonlinear optics: (i) competition between the three-photon absorption and the absorption of a single photon of three times the frequency;[23] (ii) competition between Stokes and anti-Stokes processes.[24] In Ref.[25], Jyotsna and Agarwal studied the competing three-photon and one-photon transitions in cavity quantum electrodynamics, and showed that the quantum effects are dominant in the mode–mode competition model. However, the atomic-dynamical property in the two-mode competition model is rarely reported. On the other hand, quantum collapses and revivals of atomic inversion provide a means for measuring the quantum statistics of the field. So it is significant to study the atomic dynamics in mode–mode competition model. In this paper, we generalize the competing three-photon and one-photon model in Ref.[25], and study the competing k-photon and l-photon processes ($k \neq l$) model fully by means of quantum theory. We investigate the influences of the relative competing strengths of the atom and the two-mode field and the initial state of the system on the quantum collapses and revivals of the atomic inversion in mode–mode competition model.

2. Model and solution

We consider the interaction between a two-level atom with states $|+\rangle$ and $|-\rangle$ separated by the frequency ω_0, and a two-mode field with frequencies ω_a

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†Corresponding author. E-mail: mffang@hunnu.edu.cn

and ω_b. We assume that one of the modes, a-mode, interacts with the atom via a k-photon process and the other mode, b-mode, interacts via a l-photon process. For simplicity, we further assume that the two modes are in perfect resonance with the atomic transition, i.e. $\omega_0 = k\omega_a = l\omega_b$. In the rotating-wave approximation the whole Hamiltonian of the system can be written as

$$H = \omega_a S_z + \omega_a a^+ a + \omega_b b^+ b + \lambda(S_+ a^k + S_- a^{+k}) + g(S_+ b^l + S_- b^{+l}), \quad (1)$$

where the field modes a and b are described by the annihilation and creation operators a, a^+ and b, b^+, respectively. $S_\pm$ are the atomic pseudospin operators. λ and g are the coupling constants for the k-photon process and for the l-photon process, respectively. In the interaction picture, the Hamiltonian takes the form

$$H_{\rm I} = \lambda(S_+ a^k + S_- a^{+k}) + g(S_+ b^l + S_- b^{+l}). \quad (2)$$

In order to understand the basic behaviour of the system, we first consider a simple case. We assume that initially the a-mode is in a Fock state with just three photons which is the minimum number of photons required for the three-photon absorption; the b-mode is in the vacuum state and the atom is in the ground state $|-\rangle$. Thus, the initial state of the system is given by $|\psi(0)\rangle = |3,0,-\rangle$. When the atom absorbs the three photons from a-mode, it goes to the excited state and the system becomes $|0,0,+\rangle$. Now the atom can return to the ground state by two different paths. One is by the emission of three photons into the a-mode with the system reverting to the state $|3,0,-\rangle$. The other way is through the emission of one photon into the b-mode with the system going to the state $|0,1,-\rangle$. The interference between the two pathways, $|0,0,+\rangle \longrightarrow |3,0,-\rangle$ and $|0,0,+\rangle \longrightarrow |0,1,-\rangle$, leads to great difference in the dynamical behaviour of the system.[25]

We next consider the more general case in which the a-mode is in a Fock state $|kn\rangle$, the b-mode in vacuum state and the atom in the ground state $|-\rangle$ at time $t = 0$. Thus the initial state of the system is

$$|\Psi(0)\rangle = |kn,0,-\rangle. \quad (3)$$

The state vector at any time for each value of n is a linear combination of all the possible states the system can exist in. The number of such possible states is $(2n+1)$. The state vector of the system at time t for a particular n can be expressed in terms of the complete set of the possible states in interaction picture as

$$|\Psi_{\rm I}(t)\rangle = \sum_{m=0}^{n} A_m(t)|k(n-m), lm, -\rangle + \sum_{m=1}^{n} B_m(t)|k(n-m), l(m-1), +\rangle. \quad (4)$$

Through solving Schrödinger equation in the interaction picture

$$\mathrm{i}\frac{\partial|\Psi_{\rm I}(t)\rangle}{\partial t} = H_{\rm I}|\Psi_{\rm I}(t)\rangle, \quad (5)$$

and we can obtain the equations for the complex amplitudes

$$\dot{A}_p(t) = -\mathrm{i}\lambda B_{p+1}(t)\Big\{k(n-p)\big[k(n-p)-1\big]\big[k(n-p)-2\big]\ldots\big[k(n-p)-(k-1)\big]\Big\}^{1/2} - \mathrm{i}gB_p(t)\Big\{lp\ (lp-1)(lp-2)\ldots\big[lp-(l-1)\big]\Big\}^{1/2}, \quad (6)$$

$$\dot{B}_p(t) = -\mathrm{i}\lambda A_{p-1}(t)\Big\{\big[k(n-p)+1\big]\big[k(n-p)+2\big]\ldots\big[k(n-p)+k\big]\Big\}^{1/2} - \mathrm{i}gA_p(t)\Big\{lp\ (lp-1)(lp-2)\ldots\big[lp-(l-1)\big]\Big\}^{1/2}. \quad (7)$$

Here p varies from 0 to n. At time $t = 0$, $A_0 = 1$ and $A_p = 0$, $B_p = 0$ where $p = 1, 2, \ldots, n$. It is difficult to obtain the analytical solutions to the coupled Eqs.(6) and (7), so we use fourth-order Runge–Kutta method to solve these equations with the initial conditions.

3. Atomic dynamics

The atomic inversion $\langle S_z(t)\rangle$, i.e. the difference between the probabilities to find the atom in the excited state and the ground state, is given by

$$\langle S_z(t)\rangle = \sum_{m=1}^{n} |B_m(t)|^2 - \sum_{m=0}^{n} |A_m(t)|^2. \quad (8)$$

In what follows, we shall explore the dynamics of the atom in terms of the quantum collapses and revivals of the atomic inversion $\langle S_z(t)\rangle$.

First, we consider the case where there is no mode–mode competition prior to discussing the influence of the mode–mode competition on the atomic

dynamics. In the absence of mode–mode competition, i.e. $\lambda = 0$ or $g = 0$, the Hamiltonian of the system corresponds to the l-photon or k-photon transition in the standard JCM. For instance, when $\lambda = 0$, the l-photon transition exists only and the initial state of the system is given by $|0, -\rangle$. Obviously, in this case the atom is always in the ground state $|-\rangle$ and the atomic inversion $\langle S_z(t)\rangle$ maintains the value -1, while when $g = 0$, the k-photon transition occurs only and the initial state of the system is given by $|kn, -\rangle$. In this case, using the Schrödinger equation in the interaction picture we can write the state vector of the system at time t as

$$|\Psi_{\rm I}(t)\rangle = -\,{\rm i}\sin\left\{\lambda\sqrt{kn(kn-1)(kn-2)...\big[kn-(k-1)\big]}\;t\right\}|k(n-1),+\rangle + \cos\left\{\lambda\sqrt{kn(kn-1)(kn-2)...\big[kn-(k-1)\big]}\;t\right\}|kn,-\rangle. \tag{9}$$

The atomic inversion $\langle S_z(t)\rangle$ satisfies

$$\langle S_z(t)\rangle = -\frac{1}{2}\cos\left\{2\lambda\sqrt{kn(kn-1)(kn-2)...\big[kn-(k-1)\big]}\;t\right\}, \tag{10}$$

so it shows simple cosinoidal behaviour with the Rabi frequency

$$\Omega = 2\lambda\sqrt{kn(kn-1)(kn-2)...\big[kn-(k-1)\big]}. \tag{11}$$

It is clear that in the absence of mode–mode competition ($\lambda = 0$ or $g = 0$) the atomic inversion exhibits no collapse–revival phenomenon.

Next, we show that the mode–mode competition utterly changes the dynamical behaviour of atomic inversion $\langle S_z(t)\rangle$. We first consider the case of competing two-photon and one-photon transition model. We set the initial photon number of the field $n = 150$.

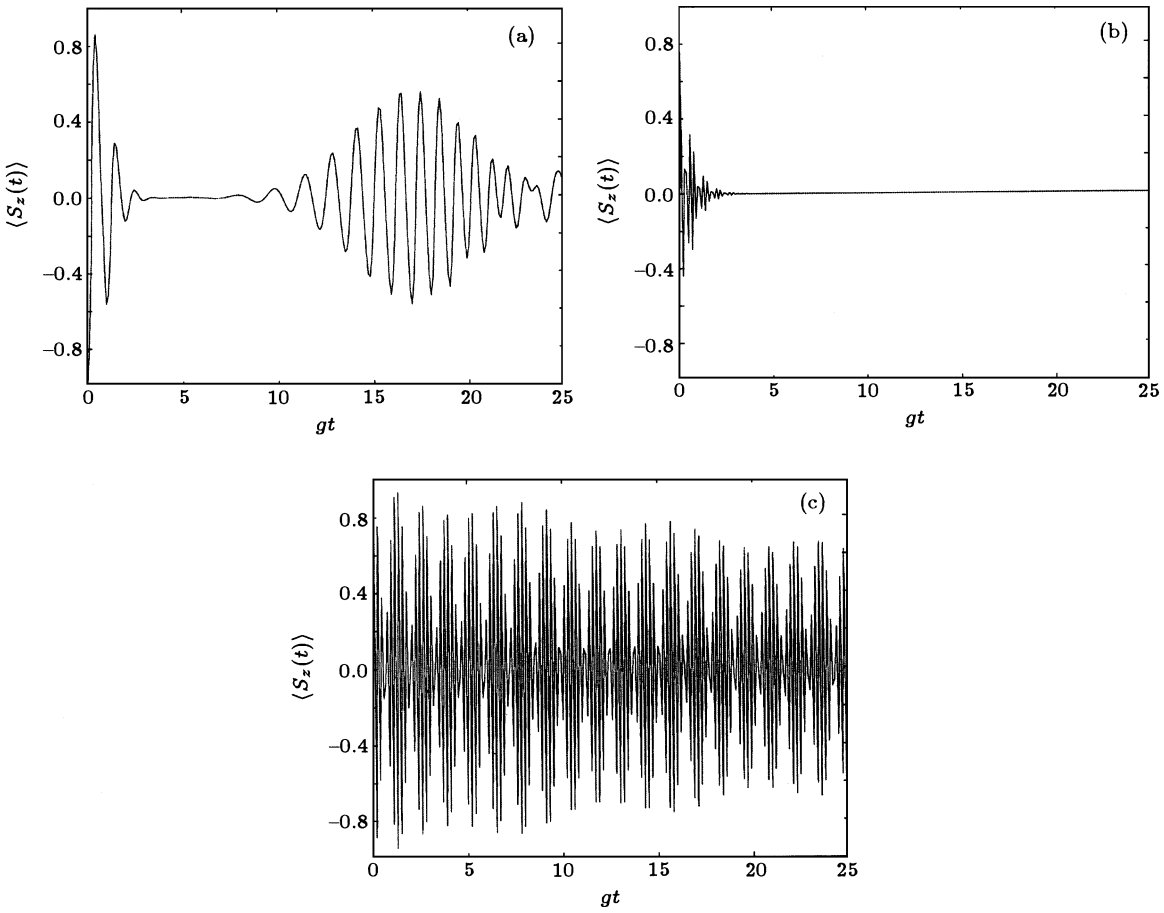


Fig.1. Time evolution of the atomic inversion $\langle S_z(t)\rangle$ for k=2, l=1, n=150 (a) λ/g=0.01, (b) g/λ=0.01, (c) λ/g=1.

The time evolution of the atomic inversion $\langle S_z(t)\rangle$ for the different coupling strengths between the atom and the two-mode field is plotted in Fig.1. When $\lambda/g = 0.01$, namely the two-photon transition is much weaker than the one-photon transition, the mode–mode competition results in quite a periodical collapse–revival of the atomic inversion, as shown in Fig.1(a). When the two-photon transition is much stronger than one-photon transition in the mode–mode competition, i.e. $g/\lambda = 0.01$, in Fig.1(b), it can be seen that the atomic inversion $\langle S_z(t)\rangle$ collapses to zero quickly as the time passes and the atom shows a long-time scale collapse, which means that the atom is always in a equi-probability coherent superposition of the excited and the ground states. However, when the relative competition strengths of the atom and the two-mode field are evenly matched, say $\lambda/g = 1$ (in Fig.1(c)), we can see that the atomic inversion $\langle S_z(t)\rangle$ exhibits high-frequency oscillations with the amplitude decreasing gradually and no periodical collapse–revival phenomenon shows up in the time evolution of the atom inversion.

Figure 2 presents the time evolution of the atomic inversion $\langle S_z(t)\rangle$ in the competing three-photon and one-photon transition model when $n = 30$ and other parameters are the same as in Fig.1, while the time evolution of the atomic inversion $\langle S_z(t)\rangle$ of the competing four-photon and two-photon transition model is displayed in Fig.3. Comparing Figs.2 and 3 with Fig.1, we can see the similarity of the atomic dynamical properties between the three different mode-competition models: (i) the periodical collapse-and-revival feature of the atom occurs only when the coupling strength of the more-photon transition is much weaker than one of the less-photon transition (for instance $\lambda/g = 0.01$); (ii) the atomic inversion $\langle S_z(t)\rangle$ collapses to zero quickly when the k-photon transition is rather dominant in the mode–mode competition, e.g., $g/\lambda = 0.01$; (iii) no collapses and revivals show up in the time evolution of the atomic inversion $\langle S_z(t)\rangle$ when the two competing transition strengths match evenly. It is evident from these figures that the quantum dynamics of the atom is highly dependent on the presence of the mode–mode competition and the relative coupling strength between the atom and the two-mode field.

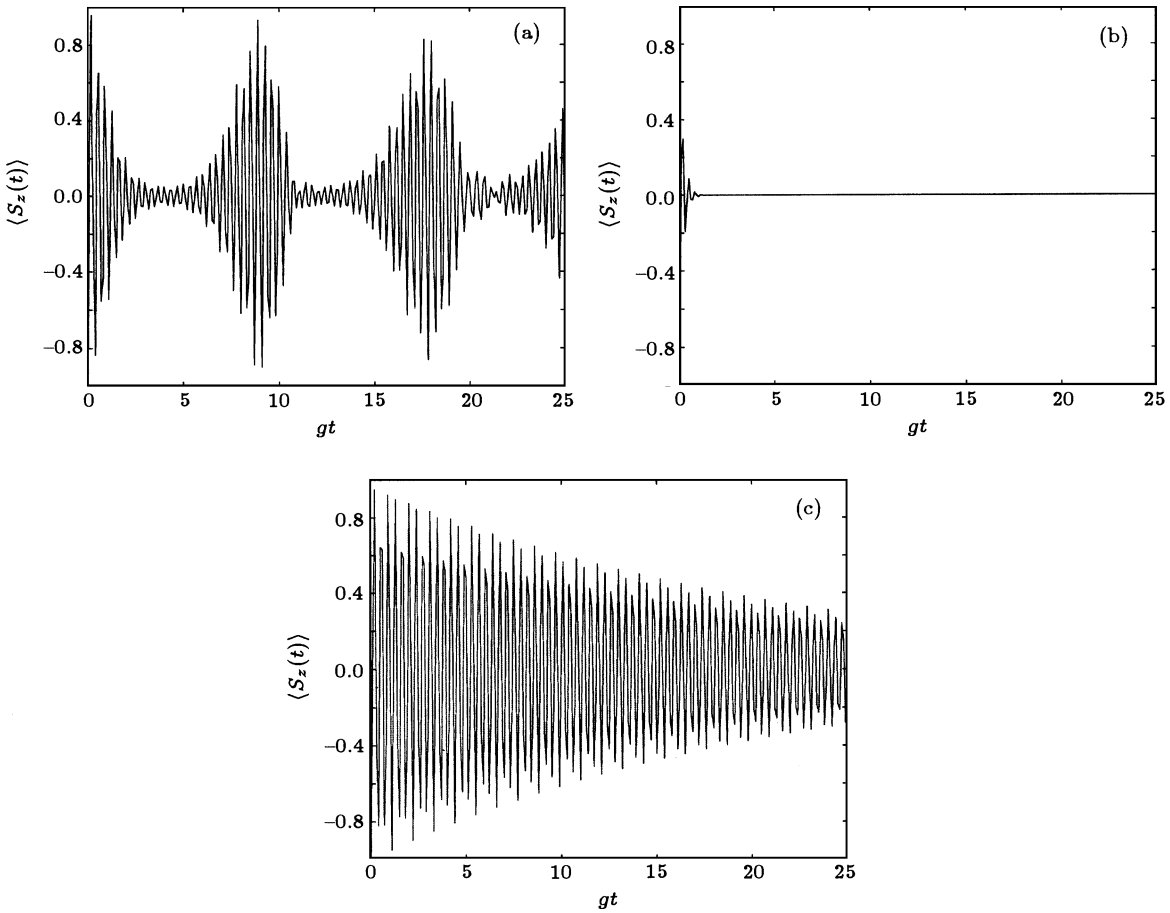


Fig.2. Time evolution of the atomic inversion $\langle S_z(t)\rangle$ for k=3, l=1, n=30 (a) λ/g=0.01, (b) g/λ=0.01, (c) λ/g=1.

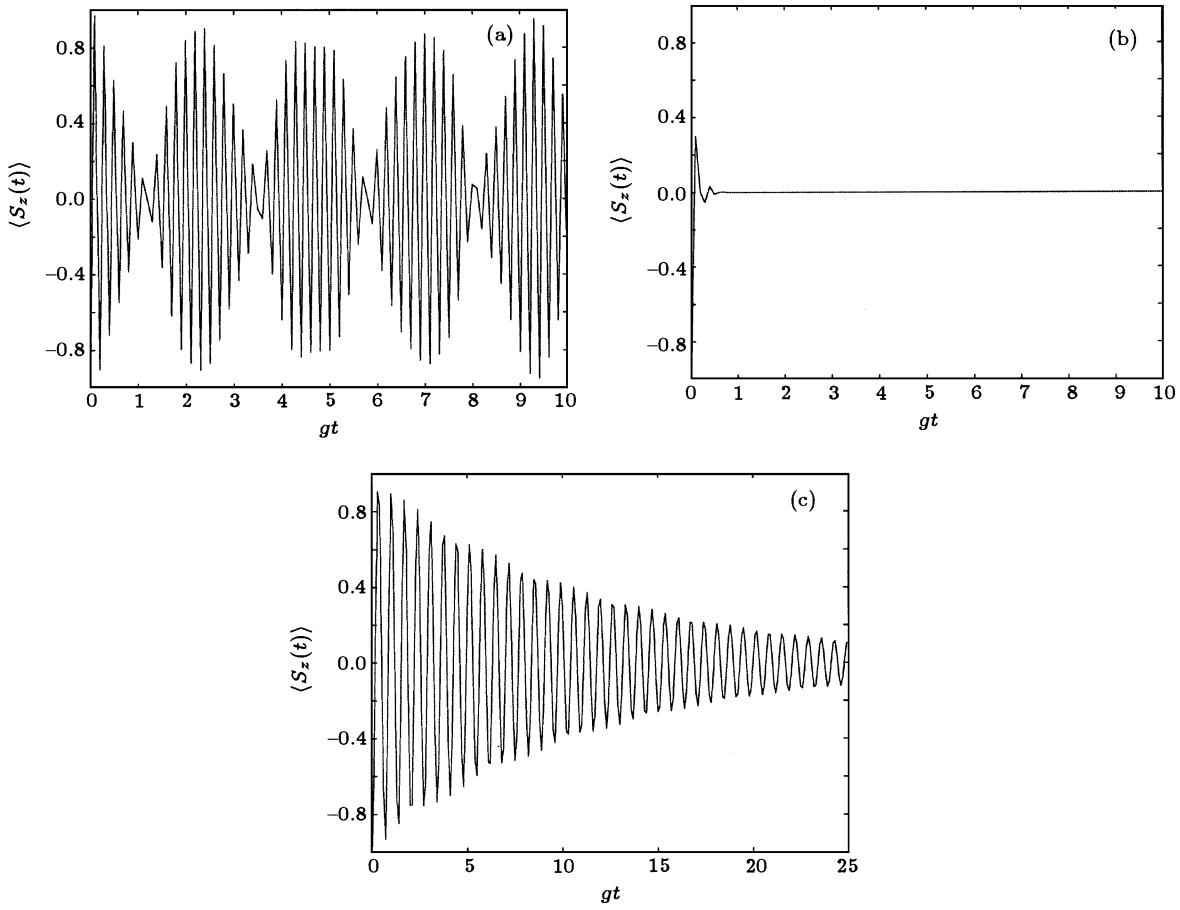


Fig.3. Time evolution of the atomic inversion $\langle S_z(t)\rangle$ for k=4, l=2, n=10 (a) λ/g=0.01, (b) g/λ=0.01, (c) λ/g=1.

The influence of the initial photon number of the system on the atomic inversion $\langle S_z(t)\rangle$ is displayed in Fig.4 for k=2, l=1, λ/g=0.01 and different values of n. When the initial photon number of the a-mode field is small, take n=1 for example (in Fig.4(a)), the initial state of the system is $|2,0,-\rangle$. Due to the much weaker coupling strength of the three-photon transition compared to a-mode field, the atom cannot be excited to the state $|+\rangle$, but is always in the ground state $|-\rangle$ in the overall time evolution process. When the initial photon number is larger, for instance n=150 or 200, see Figs.1(a) and 4(b), respectively, it is obvious that the atomic inversion exhibits collapses and revivals in the time evolution. Comparing Figs.1(a) and 4(b) with Fig.4(a), we can see that the increase of the initial photons of the system can lead to the collapse–revival phenomenon and prolong the revival time of the atomic inversion.

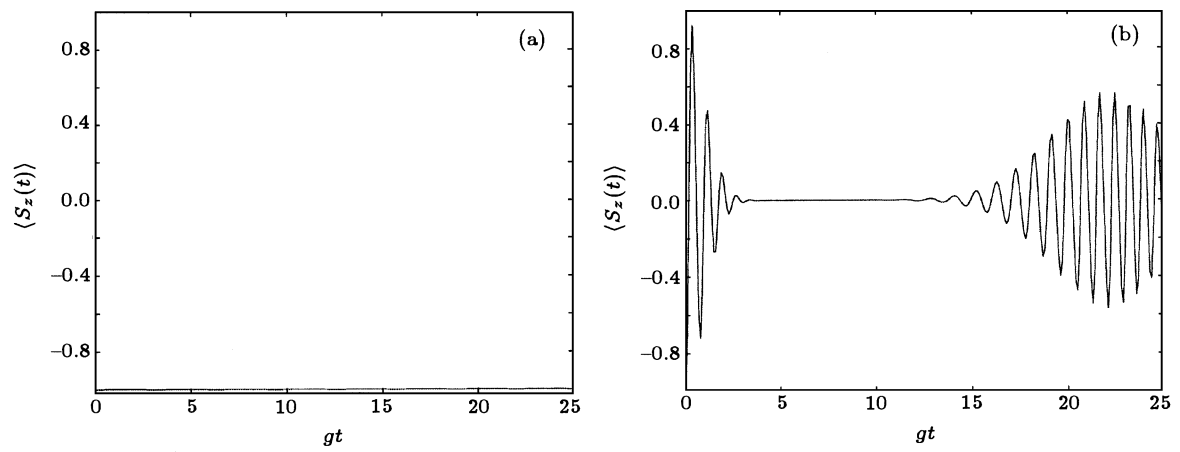


Fig.4. Time evolution of the atomic inversion $\langle S_z(t)\rangle$ for k=2, l=1, λ/g=0.01 (a) n=1, (b) n=200.

4. Conclusion

In this paper, we have extended the competing three-photon and one-photon model in Ref.[25] to the more general case of the competing k-photon and l-photon processes ($k \neq l$) model, studied the atomic dynamical properties of the generalized model fully by means of quantum theory. We found that the quantum dynamics of the atom is highly dependent on the relative coupling strengths between the atom and the two-mode field and the initial state of the system, and the presence of the mode–mode competition can utterly change the behaviour of atomic inversion $\langle S_z(t)\rangle$. Firstly, the periodical collapse and revival feature of the atomic inversion occurs only when the coupling strength of the more-photon transition is much weaker than one of the less-photon transitions. Secondly, the increase of the initial photon of the system can lead to the collapse–revival phenomenon and can prolong the revival time of the atomic inversion. Due to the general existence of the mode–mode competition in nonlinear optics, our results in this paper are of great significance for the study in this field.

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