Vol 13 No 9, September 2004
1009-1963/2004/13(09)/1391-05

Chinese Physics

© 2004 Chin. Phys. Soc.
and IOP Publishing Ltd

Synchronization of chaotic systems based on adaptive observer design*

Hua Chang-Chun(华长春)† and Guan Xin-Ping(关新平)

Institute of Electrical Engineering, Yanshan University, Qinhuangdao 066004, *China*

(Received 21 July 2003; revised manuscript received 16 February 2004)

The synchronizing problem of a chaotic system is investigated based on the observer design. The nonlinear section is assumed to satisfy the Lipschitz condition. Firstly, the normal observer is designed based on the known Lipschitz constant and the results are given in linear matrix inequality (LMI) form. Then a fairly simple adaptive observer is designed with the Lipschitz constant unknown. Simulations on synchronizing the Lorenz system are investigated and the results show the validity and feasibility of our main results.

Keywords: chaos synchronization, adaptive method, linear matrix inequality
PACC: 0545, 0547

1. Introduction

Chaos, as a complicated nonlinear phenomenon in dynamic systems, has aroused much interest in the last two decades. Chaos has many applications in many areas, such as signal generator design, secure communication, etc. On the other hand, chaos is undesirable in many cases for causing irregular oscillations. Many research efforts have focused on the chaos control[1,2] and chaos synchronization[3−18] in physical chaotic systems.

Synchronization in chaotic systems has received much attention since 1990.[3] In the last few years, synchronization of chaotic systems has been extensively investigated. Based on the nonlinear H_∞ theory, Suykens *et al*[4] proposed the master–slave H_∞ synchronization for chaotic Lurie systems. Based on the backstepping method, Wang and Ge[5] considered the synchronization of a class of nonlinear systems in the strict-feedback form. Guan and Hua[6] and Yue and Jun[18] investigated the synchronization problem using an adaptive fuzzy method. Based on the observer method, the chaotic synchronization problem has been intensively investigated in Refs.[9–17].

As we know, synchronization is used to design a system whose states synchronize with the states of a former system in limited time. In the existing literature, the synchronization problem is often investigated under the condition that the states of the former system are completely known, such as in Refs.[6,7,9–16]. But the states are often partly known in practical systems. Therefore investigating the synchronization problem of chaotic systems under the latter condition will be more significant.

In this paper, we investigate the synchronization problem of chaotic systems based on the observer design. Different from the literature, we only require that part of the states can be obtained. For the nonlinear section, the Lipschitz condition is assumed. Firstly, we design the normal observer with the Lipschitz constant known and the sufficient condition for observer to exist is given in the form of linear matrix inequality (LMI). Then with the Lipschitz constant unknown we introduce a fairly simple adaptive observer. Computer simulations are performed to verify the validity of our main results.

This paper is organized as follows. The problem is formulated in Section 2. Main results are given in Section 3, including normal observer design and adaptive observer design. Simulation examples are provided to illustrate the performance of the proposed methods in Section 4. Some concluding remarks are made in Section 5.

*Project supported by the National Natural Science Foundation of China (Grant No 60274023).
†E-mail: cch@ysu.edu.cn

2. System formulation and assumptions

Consider the following general chaotic system as the drive system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bf(x), \\ y = Cx, \end{cases} \tag{1}$$

where $x \in R^n$ is the system state, $y \in R^r$ is the output including the part states, $f(x)$ is nonlinear function vector, and A, B and C are known matrices with proper dimensions.

As we know, many chaotic systems investigated are in the form (1), such as the Lorenz system, Chen system and Arneodo system. In this paper the problem of synchronizing chaotic system (1) is investigated based on observer design.

For system (1), we assume that the following conditions hold.

Assumption 1 Nonlinear function $f(x)$ satisfies the Lipschitz condition: that is, there always exists a positive scalar β such that

$$\|f(x_1) - f(x_2)\| \leq \beta \|x_1 - x_2\|. \tag{2}$$

Assumption 2 There exists matrix H and positive matrices P and Q satisfying the following equalities:

$$P(A - HC) + (A - HC)^{\mathrm{T}} P = -Q, \tag{3}$$

$$B^{\mathrm{T}} P = C. \tag{4}$$

Remark 1 Since the states of chaotic system are bounded, Assumption 1 is easily satisfied. Assumption (2) is a strictly positive real condition. If the following system:

$$\begin{aligned} \dot{x} =& (A - HC)x + Bu, \\ y =& Cx, \end{aligned}$$

is strictly positive real, Assumption 2 will hold.

For system (1), we will design the following observer (a response system):

$$\dot{\widehat{x}}(t) = A\widehat{x}(t) + Bf(\widehat{x}) + u_1 + Bu_2, \tag{5}$$

where H is a matrix satisfying Assumption (2).

From (1) and (5), we obtain the error system

$$\begin{aligned} \dot{e} =& (A - HC)e + B(f(x) - f(\widehat{x})) \\ & - (u_1 + H(C\widehat{x} - y)) - Bu_2, \end{aligned} \tag{6}$$

where $e = x - \widehat{x}$ is the error.

Our aim is to make x-system and $\widehat{x}$-system synchronize: that is, to render error $e \to 0$ as time $t \to \infty$. In this paper, we will consider the following two cases. One case is that the Lipschitz constant β is known: in this case we will design u_1 with $u_2 = 0$ to render error system asymptotically stable. The other case is that β is not known: in this case we will design u_1 and u_2 with the help of the adaptive law.

Remark 2 In some quoted literature investigating the synchronization problem of chaotic systems based on observer, the controller is related to all the states of the drive system. The aim of observer design is to reconstruct the state of the former system by using the output of the system which contains the part state. In this paper, synchronizing a chaotic system is investigated in the two cases.

3. Main results

In this section, we will investigate system (1) under Assumptions 1 and 2.

Theorem 1 For system (1) satisfying Assumptions 1 and 2, the response system (5) with the controller

$$u_1 = -H(C\widehat{x} - y), \quad u_2 = 0$$

will synchronize the drive system (1) where $H = P^{-1}K$, while the positive matrix P and matrix K satisfy the following LMI:

$$\begin{bmatrix} PA - KC + A^{\mathrm{T}}P - C^{\mathrm{T}}K^{\mathrm{T}} + \varepsilon\beta^2 I & PB \\ B^{\mathrm{T}}P & -\varepsilon I \end{bmatrix} < 0, \tag{7}$$

where ε is a positive scalar.

Proof For error system (6) we define the following Lyapunov function:

$$V = e^{\mathrm{T}} P e. \tag{8}$$

Taking the derivative of (8), we have

$$\begin{aligned} \dot{V} =& e^{\mathrm{T}}[P(A - HC) + (A - HC)^{\mathrm{T}}P]e \\ & + 2e^{\mathrm{T}}PB(f(x) - f(\widehat{x})). \end{aligned} \tag{9}$$

By using Assumption (1), the following inequality holds:

$$2e^{\mathrm{T}}PB(f(x) - f(\widehat{x})) \leq \frac{1}{\varepsilon} e^{\mathrm{T}} PBB^{\mathrm{T}} Pe + \varepsilon\beta^2 e^{\mathrm{T}} e. \tag{10}$$

Substituting (10) into (9) and let $H = P^{-1}K$, we ob-

tain

$$\begin{aligned}\dot{V} \leq & e^{\mathrm{T}}[P(A-HC)+(A-HC)^{\mathrm{T}}P]e \\ & + \frac{1}{\varepsilon}e^{\mathrm{T}}PBB^{\mathrm{T}}Pe + \varepsilon\beta^2 e^{\mathrm{T}}e \\ = & e^{\mathrm{T}}\Big[PA + A^{\mathrm{T}}P - KC - C^{\mathrm{T}}K^{\mathrm{T}} \\ & + \frac{1}{\varepsilon}PBB^{\mathrm{T}}P + \varepsilon\beta^2 I\Big]e; \end{aligned} \tag{11}$$

if

$$\begin{aligned} & PA + A^{\mathrm{T}}P - KC - C^{\mathrm{T}}K^{\mathrm{T}} \\ & + \frac{1}{\varepsilon}PBB^{\mathrm{T}}P + \varepsilon\beta^2 I < 0, \end{aligned} \tag{12}$$

the derivative of V is negative. Since (12) is equivalent to (7), we obtain $\dot{V} < 0$. Based on Lyapunov stability theory, the error system is asymptotically stable. That is to say, the response system synchronizes with the drive system.

In Theorem 1, the sufficient condition for the controller to exist is given in the form of LMI, which can be solved by Matlab tool box directly.

In most cases estimating the Lipschitz constant β is not easy. So we will investigate the synchronizing problem of system (1) and system (5) with β unknown in the following, and corresponding adaptive feedback controller will be proposed. In addition, we will give a more simple adaptive observer.

Theorem 2 For system (5) with controller

$$\begin{aligned} u_1 = & 0, u_2 = \frac{1}{2}\overline{\theta}(y - c\widehat{x}), \\ \dot{\overline{\theta}} = & l\|y - c\widehat{x}\|^2, \end{aligned} \tag{13}$$

where l is a positive scalar, the constructed observer will also succeed in synchronizing the drive system (1).

Proof From (1), (5) and (13) we obtain the following error system:

$$\dot{e} = (A - LC)e + B(f(x) - f(\widehat{x})) - Bu_2 + LCe. \tag{14}$$

Choose the Lyapunov function

$$V = e^{\mathrm{T}}Pe + \frac{1}{2}l^{-1}\widetilde{\theta}^2,$$

where $\widetilde{\theta} = \theta - \overline{\theta}$. Taking the time derivative along (14), one can obtain

$$\begin{aligned} \dot{V} = & e^{\mathrm{T}}[P(A-LC)+(A-LC)^{\mathrm{T}}P]e + l^{-1}\widetilde{\theta}\dot{\widetilde{\theta}} \\ & + 2e^{\mathrm{T}}PB(f(x) - f(\widehat{x})) \\ & - 2e^{\mathrm{T}}PBu_2 + 2e^{\mathrm{T}}PLCe. \end{aligned} \tag{15}$$

Using Assumptions 1 and 2, we have

$$\begin{aligned} \dot{V} = & -e^{\mathrm{T}}Qe + l^{-1}\widetilde{\theta}\dot{\widetilde{\theta}} + 2e^{\mathrm{T}}PB(f(x) - f(\widehat{x})) \\ & - 2e^{\mathrm{T}}PBu_2 + 2e^{\mathrm{T}}PLB^{\mathrm{T}}Pe \\ \leq & -e^{\mathrm{T}}Qe + l^{-1}\widetilde{\theta}\dot{\widetilde{\theta}} + \frac{\beta^2}{\varepsilon}\|B^{\mathrm{T}}Pe\|^2 + \varepsilon e^{\mathrm{T}}e \\ & - 2e^{\mathrm{T}}PBu_2 + \frac{1}{\mu}\|B^{\mathrm{T}}Pe\|^2 + \mu\|e^{\mathrm{T}}PL\|^2. \end{aligned} \tag{16}$$

Letting $\theta = \dfrac{\beta^2}{\varepsilon} + \dfrac{1}{\mu}$ and using the following inequality:

$$\mu\|e^{\mathrm{T}}PL\|^2 \leq \mu\lambda_{\max}(PLL^{\mathrm{T}}P)e^{\mathrm{T}}e, \tag{17}$$

where $\lambda_{\max}(PLL^{\mathrm{T}}P)$ is the maximum eigenvalue of matrix $PLL^{\mathrm{T}}P$, one can obtain

$$\begin{aligned} \dot{V} \leq & -e^{\mathrm{T}}(Q - \varepsilon I - \mu\lambda_{\max}(PLL^{\mathrm{T}}P)I)e \\ & + \widetilde{\theta}\|B^{\mathrm{T}}Pe\|^2 \\ & + l^{-1}\widetilde{\theta}\dot{\widetilde{\theta}}. \end{aligned} \tag{18}$$

Substituting adaptive law (13) into (18), we obtain

$$\dot{V} \leq -e^{\mathrm{T}}(Q - \varepsilon I - \mu\lambda_{\max}(PLL^{\mathrm{T}}P)I)e. \tag{19}$$

The parameters ε and μ can be selected small enough to let

$$Q - \varepsilon I - \mu\lambda_{\max}(PLL^{\mathrm{T}}P)I > 0. \tag{20}$$

One can obtain that the closed-loop error system is asymptotically stable. So the slave system succeeds in synchronizing with the drive system.

The idea of synchronization based on observer design can be applied to controlling two coupled systems. If the two systems have the same structure with different initial values, we can apply the controller designed above. Particularly, the precise mathematical model needs not be known when using the adaptive controller.

4. Simulation investigation

In this section simulation on synchronizing Chen's chaotic system will be investigated.

Consider Chen's chaotic system

$$\begin{aligned} \dot{x} = & Ax + Bf(x), \\ y = & Cx, \end{aligned}$$

where

$$A = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & 0 \\ 0 & 0 & -3 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$f(x) = \begin{bmatrix} -x_1 x_3 \\ x_1 x_2 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Since the state is bounded, we can estimate the Lipschitz constant $\beta = 10$. Based on Theorem 1, the observer is designed as

$$\dot{\widehat{x}} = (A - HC)\widehat{x} + Bf(\widehat{x}) + Ly,$$

where

$$H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

It is easy to see that when $P = I, \varepsilon = 1$, LMI (7) is satisfied. The chaotic behaviour of Chen's system is shown in Fig.1. The synchronizing results are shown in Figs.2–4. Here e_1, e_2 and e_3 are three states of error e, namely $e = [e_1, e_2, e_3]$. The controller is added to system in 15s. From the error curves, we can see that the error system is stabilized quickly by the added controller.

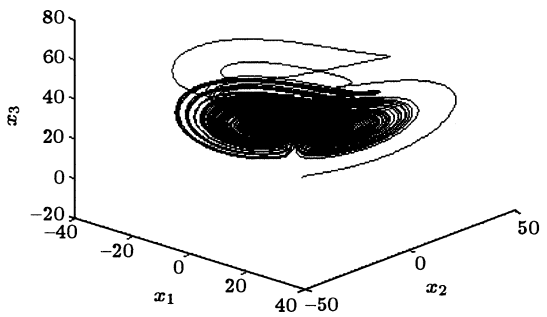


Fig.1. Chaotic behaviour of Chen's chaotic system.

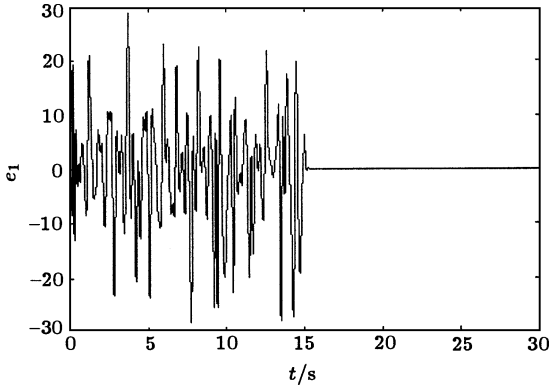


Fig.2. Error curve of e_1 of synchronizing results.

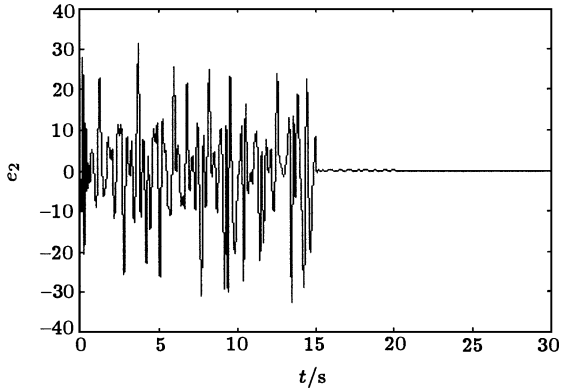


Fig.3. Error curve of e_2 of synchronizing results.

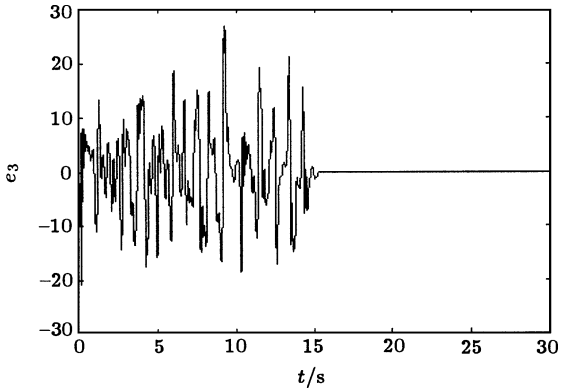


Fig.4. Error curve of e_3 of synchronizing results.

We will verify the validity of Theorem 2 in the following. The following observer is designed:

$$\dot{\widehat{x}} = A\widehat{x} + Bf(\widehat{x}) + Bu_2, \tag{21}$$

where the adaptive controller

$$u_2 = \frac{1}{2}\overline{\theta}(y - c\widehat{x}),$$
$$\dot{\overline{\theta}} = \|y - c\widehat{x}\|^2. \tag{22}$$

The simulation results are shown in Figs.5–7 (the controller is also added to the system in 15s).

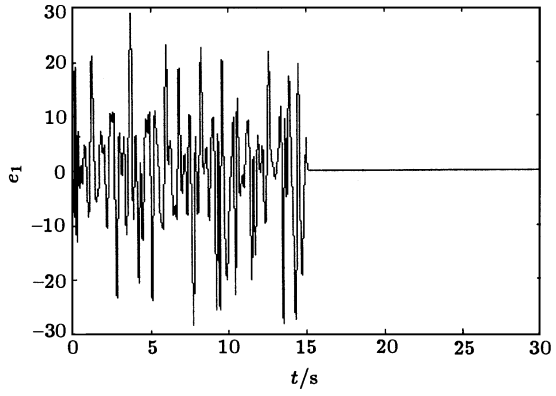


Fig.5. Curve of error e_1 of simulation results.

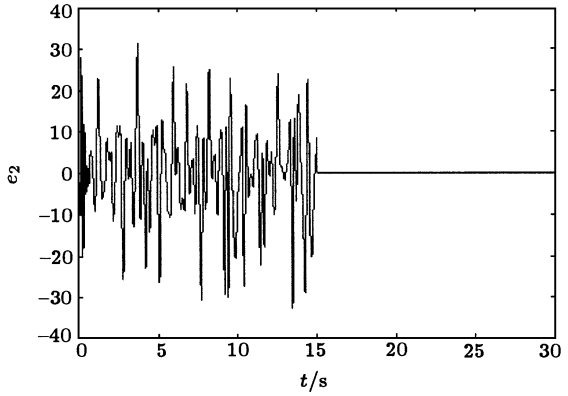


Fig.6. Curve of error e_2 of simulation results.

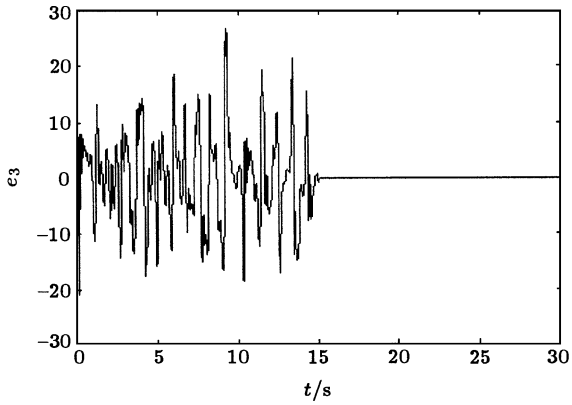


Fig.7. Curve of error e_3 of simulation results.

From the error curves of Figs.5–7, we can see the adaptive feedback controller renders the synchronization error system asymptotically stable.

5. Conclusion

In this paper, the synchronization problem of chaotic systems has been investigated. Normal observer and adaptive observer have been designed. The controller can render the error closed-loop system asymptotically stable. Simulations have shown that the observers and the controllers are valid and feasible.

References

[1] Chen G and Dong X 1998 *From Chaos to Order: Methodologies, Perspectives and Applications* (Singapore: World Scientific)
[2] Hua C C and Guan X P 2003 *Phys. Lett.* A **314** 72
[3] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* **64** 821
[4] Suykens J A K *et al* 1997 *IEEE Trans. Circuits Syst.* I **44** 891
[5] Wang C and Ge S S 2001 *Chaos, Solitons Fractals* **12** 1199
[6] Guan X P and Hua C C 2002 *Chin. Phys. Lett.* **19** 1031
[7] Morgul O and Solak E 1996 *Phys. Rev.* E 54 4803
[8] Liao T L and Huang N S 1999 *IEEE Trans. Circuits Syst.* I **46** 1144
[9] Jose A R *et al* 2001 *Phys. Lett.* A 289 193
[10] Liao T L and Lin S H 1999 *J. Franklin Inst.* **336** 925
[11] Li L X *et al* 2001 *Acta Phys. Sin.* **50** 629 (in Chinese)
[12] Jiang G P, Tang W K and Chen G R 2003 *Chaos, Solitons Fractals* **17** 925
[13] Li Z *et al* 2002 *Phys. Lett.* A **301** 224
[14] Chen S H and Lu J H 2002 *Phys. Lett.* A **299** 353
[15] Huijberts H J C 1997 *Int. J. Bifur. Chaos* **11** 1997
[16] Liu F *et al* 2002 *Acta Phys. Sin.* **51** 1954 (in Chinese)
[17] Guan X P, He Y H and Fan Z P 2003 *Acta Phys. Sin.* **52** 276 (in Chinese)
[18] Yue D and Jun Y M 2003 *Acta Phys. Sin.* **52** 292 (in Chinese)