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Physical mechanism of the chaotic detection of the unknown frequency of weak harmonic signal and effects of damping ratio on the detection results*

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In the zero-order approximation, we use the perturbation method of parameter with small magnitude to prove that the harmonic frequency in the solution of the equation is close to that of the driving force when the chaotic system from Duffing–Holmes equation stays in the stable periodic state, which is the physical mechanism of the detection of the unknown frequency of weak harmonic signal using the chaotic theory. The result of the simulation experiment shows that the method proposed in this paper, by which one can determine the frequency of the stable system from the number of circulation change of the phase state directionally across a fixed phase state point $(x, \dot{x})$ in fixed simulation time period, is successful. Analyzing the effects of the damping ratio on the chaotic detection result, one can see that for different frequency ranges it is necessary to carefully choose corresponding damping ratio α.

Keywords: weak harmonic signal, unknown frequency, chaotic detection, damping ratio
PACC: 0545

1. Introduction

Weak signal detection based on chaotic theory has always been a subject of advanced research in the application of the chaotic technique.[1,2] It is an important contribution[3,4] to detect weak signals using phase varying action of dynamical motion of the chaotic system. This method broke through the limitation[5,6] that it can only be applied to chaotic noise background. Recently, researches have shown that it can strongly suppress the noise with zero mean value. Some Chinese workers have made progress in the chaos-based weak harmonic signal amplitude detection and weak square wave signal amplitude detection under the background of strong noise, in which the threshold of the signal-to-noise ratio is approximately –60dB,[7−9] which is unattained by any other detection method as yet. At present, most researchers only apply the chaotic theory to decide whether weak signals with known frequency exist or not, and detect its amplitude when it exists.

There is only one detection method using the array of oscillators based on the principle of frequency-locking[10] that is applicable to many actual harmonic signals with unknown frequency. However, the process of this method is complex. In this paper, we discuss the physical mechanism of detecting the unknown frequency of weak harmonic signals and the effect of the damping ratio on the results is analyzed by means of simulation experiment. Some features of importance are pointed out in conclusion.

2. Principle of detecting weak harmonic signal with unknown frequency based on the chaotic theory

We take the Duffing–Holmes equation as the mathematical model for detecting the unknown fre-

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quency of weak harmonic signal:

$$\ddot{x} + \alpha\dot{x} + kx + \mu x^3 = F\cos(\omega t). \tag{1}$$

This equation describes a nonlinear dynamical system, where α is the damping ratio, $(kx + \mu x^3)$ is the nonlinear restoring force, and $F\cos(\omega t)$ is the periodic driving force, and which is the harmonic signal to be detected. We set $\alpha = 0.5$, and consider F as a variable. With F increasing, the dynamical behaviour of the system changes from chaotic oscillation to the periodic state. When F is large enough, the system is totally controlled by the external driving force of the linear oscillator and enters into the periodic state.[9] When detecting the weak harmonic signal $F\cos(\omega t)$, we put the harmonic signal as the periodic driving force applied to the system. By increasing F, the system turns into the periodic state, i.e. the dynamical behaviour of the system becomes periodic. Studying the motion period of the system's phase state $(x, \dot{x})$, it is found that the motion period is equal to the reciprocal of the frequency of signal to be detected (i.e. $T_0 = \frac{1}{f} = \frac{2\pi}{\omega}$), for which we introduce the mathematical interpretation as follows.

We adopt the method of perturbation and introduce an infinitesimal value ε to the Eq.(1). Then we solve Eq.(1) and consider the nonlinear restoring force and the driving force as the perturbation to the system. The Eq.(1) is written as

$$\ddot{x} + \omega^2 x = \varepsilon[-\xi\omega\dot{x} + \omega^2(ax - bx^3) + p\cos(\Omega t)]. \tag{2}$$

Suppose $\omega^2(1 - \varepsilon a) = k = \omega_0^2$, $\omega^2\varepsilon b = \mu$, $\varepsilon\xi\omega = \alpha$ and $\varepsilon P = F$, then the equivalence of Eq.(2) is $\ddot{x} + \alpha\dot{x} + kx + \mu x^3 = F\cos(\Omega t)$, where ω can be regarded as the frequency of the system without perturbation.[11]

In order to find out the periodic solution when the ω is supposed to be the vibrating frequency of the system, we introduce a new variable τ to replace the former time variable t:

$$\Omega t = \tau + \varphi,$$

where φ is the phase to be determined. Changing the derivatives of Eq.(2) to that with respect to the new variable τ, Eq.(2) becomes

$$\begin{aligned}\Omega^2\ddot{x} + \omega^2 x =& \varepsilon[-\xi\Omega\omega\dot{x} + \omega^2(ax - bx^3) \\ &+ P\cos(\tau + \varphi)].\end{aligned} \tag{3}$$

As the periodic solution, one has $x(\tau + 2\pi) = x(\tau)$. Since φ is to be determined, we suppose $x(0) = 0$. The series expansion of Eq.(3) for ε can be expressed as

$$x(\tau) = x_0(\tau) + x_1(\tau)\varepsilon + x_2(\tau)\varepsilon^2 + \cdots, \tag{4}$$

$$\varphi = \varphi_0 + \varphi_1\varepsilon + \varphi_2\varepsilon^2 + \cdots. \tag{5}$$

From $x(\tau + 2\pi) = x(\tau)$, $x(0) = 0$, one has

$$x_i(\tau + 2\pi) = x_i(\tau), \quad i = 0, 1, 2, 3, \cdots \tag{6}$$

$$x_i(0) = 0, \quad i = 0, 1, 2, 3, \cdots \tag{7}$$

Substituting Eq.(4) and Eq.(5) into Eq.(3), one has

$$\begin{aligned}&\Omega^2(x_0 + x_1\varepsilon + x_2\varepsilon^2 + \cdots)'' \\ &+ \omega^2(x_0 + x_1\varepsilon + x_2\varepsilon^2 + \cdots) \\ =&\varepsilon\{-\xi\Omega\omega(x_0 + x_1\varepsilon + x_2\varepsilon^2 + \cdots)' \\ &+ \omega^2[a(x_0 + x_1\varepsilon + x_2\varepsilon^2 + \cdots) \\ &- b(x_0 + x_1\varepsilon + x_2\varepsilon^2 + \cdots)^3] \\ &+ P\cos(\tau + \varphi_0 + \varphi_1\varepsilon + \varphi_2\varepsilon^2 + \cdots)\}.\end{aligned}$$

Suppose that the power for ε in both sides is equal, one has

$$\begin{aligned}\Omega^2 x_0'' + \omega^2 x_0 =& 0, \\ \Omega^2 x_1'' + \omega^2 x_1 =& -\xi\Omega\omega x_0' + a\omega^2 x_0 - b\omega^2 x_0^3 \\ &+ P\cos(\tau + \varphi_0), \\ \Omega^2 x_2'' + \omega^2 x_2 =& -\xi\Omega\omega x_1' + a\omega^2 x_1 - 3b\omega^2 x_0^2 x_1 \\ &- P\varphi_1\sin(\tau + \varphi_0), \\ &\cdots\cdots\end{aligned} \tag{8}$$

Thus, x_i is determined step by step. From the first formula in Eq.(8) and initial condition Eq.(7), one obtains the zero-order approximation solution:

$$x_0(\tau) = A_0\cos\frac{\omega}{\Omega}\tau, \tag{9}$$

where A_0 is a constant. The equation which determines A_0 can be written in the form

$$\alpha^2\Omega^2 + (\Omega^2 - \omega_0^2 - \frac{3}{4}\mu A_0^2)^2 = \frac{F^2}{A_0^2}. \tag{10}$$

In order to make Eq.(9) meet the periodic condition, it is required that $\Omega = \omega$. Eq.(9) indicates that the zero-order approximation solution is equivalent to the resonance. Going on with the rest of formulas of Eq.(8), one has

$$x_1 = -\frac{bA_0^3}{32}\cos 3\tau + A_1\cos\tau,$$

$$A_1 = \frac{3b^2A_0^5/128}{\frac{9}{4}bA_0^2 - a + \tan\varphi_0\cdot\xi},$$

$$\varphi_1 = \frac{\xi A_1\omega^2}{P\cos\varphi_0};$$

$$x_2 = A_2\cos\tau + \frac{3b\xi}{256}A_0^3\sin 3\tau + \left(\frac{abA_0^3}{256} - \frac{3bA_0^2A_1}{32} - \frac{3b^2A_0^5}{512}\right)\cos 3\tau - \frac{b^2A_0^5}{1024}\cos 5\tau,$$

$$A_2 = -\frac{\varphi_1^2 p\sin\varphi_0}{2\xi\omega^2} + \frac{\varphi_2 p\cos\varphi_0}{\xi\omega^2} - \frac{9bA_0^5}{1024},$$

$$\varphi_2 = \frac{\omega^2}{p\sin\varphi_0 - \frac{p}{\xi}\cos\varphi_0\left(a + \frac{9A_0^2}{4}\right)} \times \left[\left(a + \frac{9A_0^2}{4}\right)\left(-\frac{\varphi_1^2 p\sin\varphi_0}{2\xi\omega^2} - \frac{9bA_0^5}{1024}\right) - \frac{\varphi_1^2 p\cos\varphi_0}{2\omega^2} + \frac{3}{4}A_0A_1^2 + \frac{b^2A_0^7}{2048} - \frac{A_1A_0^4b}{64}\right].$$

Under the conditions $x \approx x_0 + \varepsilon x_1 + \varepsilon^2 x_2$ and $\varphi \approx \varphi_0 + \varepsilon\varphi_1 + \varepsilon^2\varphi_2$, one has

$$x \approx (A_0 + \varepsilon A_1 + \varepsilon^2 A_2)\cos\tau - \frac{\varepsilon b}{32}A_0^3\cos 3\tau + \frac{\varepsilon^2 3b\xi}{256}A_0^3\sin 3\tau + \varepsilon^2\left(\frac{abA_0^3}{256} - \frac{3bA_0^2A_1}{32} - \frac{3b^2A_0^5}{512}\right)\cos 3\tau - \frac{\varepsilon^2 b^2A_0^5}{1024}\cos 5\tau. \quad (11)$$

Discussion:

(1) For Eq.(10), the relational curves of A_0 vis Ω^2 is given in Fig.1. The figure consists of three parts: the curve $\mu = 0$ refers to linear equation, whose extremal point is just the resonant point; the corresponding curves $\mu > 0$ and $\mu < 0$ have instability zones at different positions. The shaded area in the figure shows the range of the instability zone of curve $\mu > 0$.

(2) A_0 is dependent on μ, Ω and α: The point $\Omega = \omega_0^2$ is the feature control point of the corresponding curves ($\mu > 0$ and $\mu < 0$), while l_1 and l_2 are the feature control lines.

(3) Synthesizing Fig.1 and Eq.(11), we reach the conclusion that

$$x = A_0\cos(\omega t + \varphi) + o(\varepsilon). \quad (12)$$

This indicates that in the zero order approximation although x varies with the amplitude A_0 and phase φ, its vibration frequency is very close to that of the driving force (i.e. $\Omega \approx \omega$).

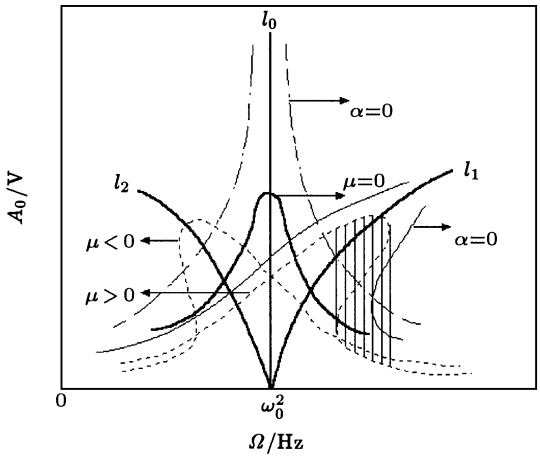


Fig.1. Relation between A_0 and Ω^2.

3. Simulation experiment

Using the graph of periodic orbit of the phase states in the simulation system, we can count the number of times that the orbit directionally crosses some fixed phase state point. By fixed simulation time, we can obtain the number of periods and determine the frequency when the system is in the stable state of the phase. Based on the theoretical analysis given above, when the system has entered the stable periodic state, the vibration with frequency ω and amplitude γ is predominant in the forced vibrations of the system. After the harmonic signal with unknown frequency and noise has entered into the chaotic system, although the S/N of the mixed signal is very low, yet the system can achieve a stable periodic state.[3] Then one can get the value of the unknown frequency for the signal to be detected. The data of the simulation experiment are given in Table 1.

Table 1. Result of the simulation experiment for the detected frequency.

ω/Hz	t_b	t_e	n	ω'/Hz	$\lvert\Delta\omega\rvert/(\times 10^4)$	$\frac{\lvert\Delta\omega\rvert}{\omega}/(\times 10^4)$
1	50	120	11.1428	1.0002	2	2
2	50	120	22.2860	2.0004	4	2
5	50	120	55.7162	5.0011	11	2.2
10	50	120	111.4164	10.0007	7	0.7
20	50	120	222.8275	20.0010	10	0.5
50	50	120	557.0504	50.0007	7	0.14
100	50	120	1114.100	100.0012	12	0.12
150	50	120	1671.110	150.0017	17	0.11
177	50	120	1971.930	177.0020	20	0.11
200	50	120	2228.200	200.0022	22	0.11

Comparing the data, we can find that the result of the simulation very closely approximates the initial frequency of the system.

4. Influence of damping ratio on the detection result

In the Duffing–Holmes equation Eq.(1) that we use to detect the frequency of the weak harmonic signal, there are three important parameters, namely the damping ratio α, the coefficient k of linear term and the coefficient μ of nonlinear term x^3, which affect the detected result entirely. Table 2 and Fig.2 show the variation range of $|\Delta\omega|$ caused by varying α.

Table 2. Simulation experiment results of frequency ω and damping ratio α.

ω/Hz	α=0.3	0.5	0.8	1	1.2	1.5
1	ω'=0.912Hz	0.908	0.999	0.979	0.993	0.984
2	1.818	1.996	1.996	1.980	1.989	1.982
3	2.995	2.992	2.992	2.976	2.986	2.978
5	4.988	4.984	4.984	4.966	4.978	4.971
7	6.993	6.989	6.989	6.970	6.982	6.974
10	9.924	10.003	10.016	10.001	10.006	9.996
20	19.932	20.019	20.019	20.000	20.012	20.005
30	29.990	29.986	29.986	29.967	29.980	29.971
40	40.012	40.005	40.002	39.993	39.999	39.996
50	50.003	49.999	49.999	49.981	49.992	49.985
60	59.938	59.926	60.011	59.998	60.007	60.003
70	69.928	69.923	70.013	69.998	70.008	69.998
80	79.939	79.928	80.012	79.995	80.006	79.995
90	90.036	90.029	90.029	90.016	90.023	90.016
100	99.995	99.995	99.995	99.974	99.988	99.981
120	119.924	120.013	120.013	119.988	120.005	119.996
140	139.846	139.936	140.026	140.006	140.016	140.016
160	160.022	160.011	160.011	159.999	159.999	160.011
177	177.016	176.953	177.030	177.018	177.018	177.018
180	179.930	180.020	180.020	179.994	180.007	180.007
200	199.750	200.020	200.024	200.009	200.024	200.009

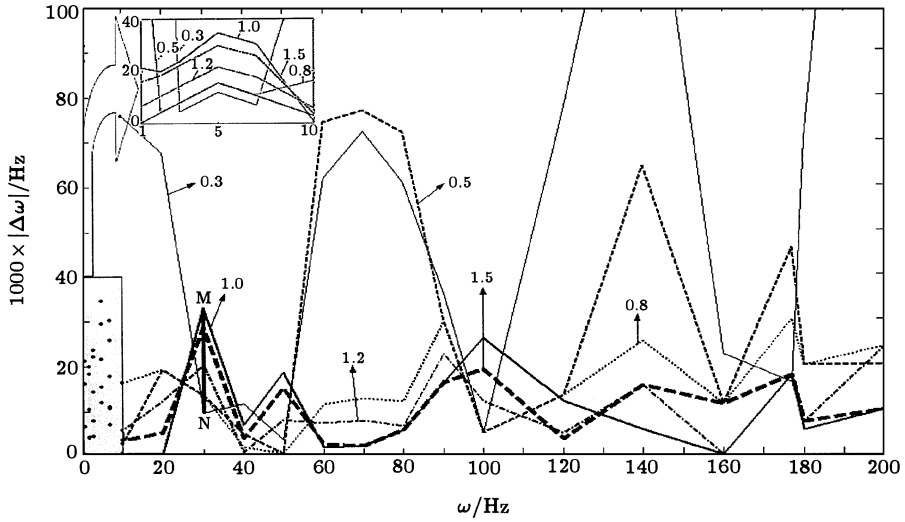


Fig.2. Relation curves between damping ratio and error of detected frequency.

Furthermore, analysing Table 2 and Fig.2, one can see:

(1) For different damping ratios (0.3, 0.5, 0.8, 1.0, 1.2, 1.5), $|\Delta\omega|$ varies diversely. What is common is that the curves of $|\Delta\omega|$ have the characteristics of multi-peak. One can further see that $|\Delta\omega|$ corresponding to $\alpha = 0.3$ changes in a violent manner.

(2) For a frequency ω to be detected, it is possible that there exists a varying band of $|\Delta\omega|$ for different damping ratio α. For example, for ω=30Hz the thickened line segment MN in Fig.2 shows the varying band of $|\Delta\omega|$. The $|\Delta\omega|$ corresponding to α=2.2 equals to 0.0244, and falls in the scope of the thickened line segment MN.

(3) The inset in Fig.2 ($\omega \leq$ 10Hz) shows that the variation in the range of 1 to 10Hz is similar to that in the whole range (from 1 to 200Hz) covered by Fig.2. It can be inferred that in any other narrow frequency band things are likely to be similar as in the above case. This kind of characteristics is potentially the expression of the 'self-organization'.

(4) When actually detecting a weak harmonic signal, one should choose the stable computing system with small $|\Delta\omega|$. For example, for the band $0 < \omega \leq$ 100Hz, one had better choose $\alpha = 0.8$. In addition, for single frequency and narrow band of frequency one should carefully examine the changing ways of $|\Delta\omega|$ due to α.

5. Conclusion

a) For the chaotic detection of the frequency of a weak harmonic signal, the existence of solution is proven by the theoretical analysis. Moreover, the frequency to be detected is close to that of the driving force. The simulation experiment shows that counting the number of times the orbit directionally crosses some fixed phase-space point in a fixed time period can approximately determine the frequency to be detected. Furthermore, the main reason of the appearance of $|\Delta\omega|$ is that during the counting there may appear a missed time difference which is smaller than a period.

b) The choice of damping ratio in the chaotic system of frequency detection affects the detection result. For different detection range of frequency we should choose corresponding α. In addition, the detection of unknown frequency doesn't require the known amplitude of harmonic signal. After detecting the frequency, one can use the method in Ref.[9] to detect the unknown amplitude of the harmonic signal.

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