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# Transport characteristics of Rabi oscillations in the two-band superlattices*

Duan Su-Qing(段素青)†, Wu Bin-Yi(吴彬毅),
Wang Zhi-Gang(王志刚), and Zhao Xian-Geng(赵宪庚)

*Institute of Applied Physics and Computational Mathematics, Beijing* 100088, *China*

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We investigated the transport characteristics of Rabi oscillations, by using numerical methods, within a two-band tight-binding model driven by dc–ac electric fields. We found that Rabi oscillations make the long-time average current a sharply change, i.e. the current have resonant peaks appearing. Rabi oscillations are destroyed by dephasing; these peaks are the signatures of Rabi oscillations on the current response. The strong interband coupling will change the conditions of Rabi oscillations and, correspondingly, changes the places where the current resonant peaks appear.

**Keywords:** Rabi oscillations, interband coupling, current resonant peaks, quasi-energy spectrum
**PACC:** 7335, 7340L

## 1. Introduction

The influence of external electric fields on semiconductor superlattices has been extensively studied in recent years. Many phenomena have been predicted theoretically and/or observed experimentally, including Bloch oscillation,[1] dynamical localization,[2−4] band collapse,[5−6] band suppression,[7] fractional Wannier–Stark Ladders,[8] and multi-photon absorption.[9] A particular interest is to investigate the coherent oscillations and the current response between Bloch bands based on two-band binding model[10−13] which contains the essential physics of interband transitions, and can be compared in practice with realistic situation where only a single pair of bands is important.

We studied Rabi oscillations between dissipative Bloch bands.[14] We found that Rabi oscillations are destroyed by scattering from lattice imperfections. We also the current response between dissipative Bloch bands.[15] But a thorough study of the transport characteristics of Rabi oscillations, to our best knowledge, has not been conducted. In this paper, we study the transport characteristics of Rabi oscillations in the superlattice within a two-band tight-binding model driven by an external field $E = E_0 + E_1 \cos \omega t$. We calculate the electric current response versus the ratio of the band centre separation $\Delta$ and the laser frequency $\omega$. There are some current resonant peaks appearing at the avoided crossings between the Wannier–Stark ladders or the fractional Wannier–Stark ladders when $\Delta$ and $\omega$ satisfy some relations; under the same relations Rabi oscillations occur. When the interband coupling is very weak, Rabi oscillations happen at the positions of $\Delta/\omega = m$ and $\Delta/\omega_0 = n$; here $m$ and $n$ are integers, $\omega_0$ is the Bloch frequency and equals $edE_0$; here $e$ and $d$ are the particle charge and the lattice constant, respectively. When the interband coupling is strong, Rabi oscillations occur at the positions which have a deviation from those of $\Delta/\omega = m$ and $\Delta/\omega_0 = n$. The appearance of the current resonant peaks have an analogous rule to Rabi oscillations. When the interband coupling is very weak, the current resonant peaks occur at the positions of $\Delta/\omega = m$ and $\Delta/\omega_0 = n$; when the interband coupling is strong, the current resonant peaks occur at the positions which have a deviation from those of $\Delta/\omega = m$ and $\Delta/\omega_0 = n$.

## 2. Model

We use the standard tight-binding model of a two-

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†Corresponding author. E-mail: Duan_suqing@mail.iapcm.ac.cn

band system in a dc–ac field $E(t) = E_0 + E_1 \cos \omega t$. The Hamiltonian can be written as

$$\begin{aligned} H(t) = & \sum_n [(\Delta_a + neE(t)d)a_n^\dagger a_n \\ & + (\Delta_b + neE(t)d)b_n^\dagger b_n \\ & - (W_a/4)(a_{n+1}^\dagger a_n + \text{h.c.}) \\ & + (W_b/4)(b_{n+1}^\dagger b_n + \text{H.c.}) \\ & + eE(t)R(a_n^\dagger b_n + b_n^\dagger a_n)]. \end{aligned} \tag{1}$$

Here the subscript $n$ labels the lattice sites, and letters $a$ and $b$ respectively denote the lower and upper minibands. The first two terms correspond to the site energies of Wannier states at the presence of the electric field, and $W_{a,b}$ are the widths of the isolated ($E(t) = 0$) minibands induced by nearest-neighbour hopping: $\epsilon_{1,2}(k) = \Delta_{a,b} \mp (W_{a,b}/2)\cos(kd)$. The last term is on-site electric dipole coupling between minibands; $eR$ is the corresponding dipole moment.

The dynamics of the system is determined by the density matrix, which is defined by

$$\rho(t) = \sum_{ijmn} \rho_{mn}^{ij} \xi_m^{i\dagger} \xi_n^j, \tag{2}$$

where $i$, $j$ are band indices and equal to 1 or 2; $\xi_m^{1\dagger}$ ($\xi_m^1$) and $\xi_m^{2\dagger}$ ($\xi_m^2$) designate $a_m^\dagger$ ($a_m$) and $b_m^\dagger$ ($b_m$), respectively. It is easy to transform the density matrix to the wavevector space by Fourier transforming. We will be interested in the wavevector diagonal part of the Fourier component $\rho_{kq}^{ij} = \sum_{mn} \rho_{mn}^{ij}(t)\exp(-\mathrm{i}km + \mathrm{i}qn)$, i.e. $\rho_{kk}^{ij} \equiv \rho^{ij}(k,t)$. At the case of finite temperature $T$, the wavevector and band diagonal occupation number relax to the thermal equilibrium value when tending to the long time limit,

$$\rho_{kk}^{ii} \to \rho_T^i \equiv e^{-\beta\epsilon_i(k)} / \sum \mathrm{e}^{-\beta\epsilon_j(q)}, \tag{3}$$

where $\beta = 1/k_\mathrm{B}T$. The sum in the partition function is over $j = 1$, 2 and over all wavevectors $q$. Within a constant relaxation rate approximation, the density matrix $\rho(t)$ satisfies the following stochastic Liouville equation (SLE) (through this paper we set $\hbar = 1$):

$$\mathrm{i}\frac{\partial \rho(t)}{\partial t} = [H, \rho(t)] - \mathrm{i}\Gamma[\rho(t) - \rho_T]. \tag{4}$$

Here $\Gamma[\rho(t) - \rho_T]$ describes the relaxation of the off-diagonal elements of $\rho$ through dephasing,

$$\Gamma[\rho(t) - \rho_T] = \sum_{ij} (\rho^{ij}(k,t) - \delta_{ij}\rho_T^i(k))\alpha_{ij}\xi^{i\dagger}(k)\xi^j(k). \tag{5}$$

The utility of this simple form of the SLE in the single band case was discussed by Dunlap and Kenkre.[16] The parameters $\alpha_{ij}$ measure the loss of phase coherence between sites. It is convenient to introduce the linear combinations of matrix elements: $\rho_+ = \rho^{11}(k,t) + \rho^{22}(k,t)$, $\rho_- = \rho^{11}(k,t) - \rho^{22}(k,t)$, $\rho_\pm = \rho^{12}(k,t) + \rho^{21}(k,t)$, $\rho_\mp = \mathrm{i}[\rho^{21}(k,t) - \rho^{12}(k,t)]$. For simplicity, we take $\alpha_{ij} = \alpha$ ($i,j = 1$, or 2) to reduce the number of parameters in the theory. Then the SLE (Eq.4) has the explicit components

$$\begin{aligned} & \frac{\partial}{\partial t}\rho_+(k,t) - edE(t)\frac{\partial}{\partial k}\rho_+(k,t) \\ = & -\alpha[\rho_+(k,t) - \rho_T^+(k)], \end{aligned} \tag{6}$$

$$\begin{aligned} & \frac{\partial}{\partial t}\rho_-(k,t) - edE(t)\frac{\partial}{\partial k}\rho_-(k,t) \\ = & -2eE(t)R\rho_\mp(k,t) - \alpha[\rho_-(k,t) - \rho_T^-(k)], \end{aligned} \tag{7}$$

$$\begin{aligned} & \frac{\partial}{\partial t}\rho_\pm(k,t) - edE(t)\frac{\partial}{\partial k}\rho_\pm(k,t) \\ = & (\Delta - W\cos kd)\rho_\mp(k,t) - \alpha\rho_\pm(k,t), \end{aligned} \tag{8}$$

$$\begin{aligned} & \frac{\partial}{\partial t}\rho_\mp(k,t) - edE(t)\frac{\partial}{\partial k}\rho_\mp(k,t) \\ = & -(\Delta - W\cos kd)\rho_\pm(k,t) \\ & + 2eE(t)R\rho_-(k,t) - \alpha\rho_\mp(k,t). \end{aligned} \tag{9}$$

Here we set $\Delta \equiv \Delta_a - \Delta_b$ and $W = (W_a + W_b)/2$.

We can calculate the interesting quantity, the current $j(t)$ along the superlattice direction. The instantaneous current is given by the sum over wavevectors of the relevant electron velocity $v_i(k) = (W_i d/2)\sin(kd)$ times the number density at that wavevector, $n_0\rho_i(k,t)$, where $n_0$ is the number of carriers per unit area in each cell of the superlattice. Then we have the current

$$\begin{aligned} j(t) = & \int_0^{2\pi} \frac{\mathrm{d}k}{8\pi}[(W_a - W_b)\rho_+(k,t) \\ & + 2W\rho_-(k,t)]n_0 \sin(kd). \end{aligned} \tag{10}$$

Of particular interest is the steady state long time average of this current

$$< j >= \lim_{T_0 \to \infty} \frac{1}{T_0} \int_0^{T_0} \mathrm{d}t j(t). \tag{11}$$

To clearly study the transport characteristics of Rabi oscillations of the two-band superlattices, we need to know the long-time average current $< j >$. Solving Eqs.(6)–(11) by using the numerical method, we can obtain the dependence of the long-time average current $< j >$ on the dimensionless band centre separation $DW \equiv \Delta/\omega$. In fact, $\Delta$ is a structure parameter of the superlattices which cannot be conveniently changed, so experimentally one can change $DW$ by changing the ac field frequency $\omega$.

## 3. Numerical results and discussions

Figures 1(a) and 1(b) show the dependence of the long-time average current $< j >$ on the dimensionless band centre separation $DW$. The superlattice parameters are $W_a = W_b = W = 0.8\omega$, $\omega_0/\omega = 1/3$, $\alpha/\omega = 0.067$, $R/d = 0.09$. The ac field strengths $edE_1/\omega$ equal to 0.1 and 0.5, respectively for Figs.1(a) and (b). The temperature parameter $\beta\omega$ can be conveniently selected (e.g. $\beta\omega = 10.0$). In both figures, we can find that the current resonant peaks appear not only at $DW = n$ ($n$ is an integer), but also at $DW = n/3$. These just coincide with the conditions of Rabi oscillations. Physically speaking, the quasi-energies will split into a series of ladders, i.e. Wannier–Stark ladders (the separation between the nearest ladders is $\omega$) and fractional Wannier–Stark ladders (the separation between the nearest ladders is $\omega_0$) under the external field $E(t) = E_0 + E_1 \cos \omega t$. When the band centre separation $\Delta$ equals the separation $m\omega + n\omega_0$ ($m$, $n$ are integers) of two arbitrary ladders, the conditions of Rabi oscillations are satisfied. Rabi oscillations lead to a shorter time for the arrival to the equal distribution of electrons between the two Bloch bands.[13]

Correspondingly, there will be a sharp change in the curve $< j >$ versus $DW$. Thus, the current resonant peaks in Figs.1(a) and 1(b) are the manifestation of Rabi oscillations. As the ac field strength is larger, the current peaks are more extrusive, which can be clearly seen from Figs.1(a) and 1(b). This implies that the experimenter can study the Wannier–Stark ladders and the fractional Wannier–Stark ladders by studying the current response. The current resonant peaks at $DW =$ integers can be thought of as Rabi oscillations happening between Wannier–Stark ladders. The current resonant peaks at $DW =$ fractions can be surely regarded as though there are Rabi oscillations happening between fractional Wannier–Stark ladders. When the interband coupling $R/d$ is strong (the other parameters are fixed), we can find that the current resonant peaks will have a deviation from the position of $\Delta = m\omega$ and $\Delta = n\omega_0$. In Figs.2(a) and 2(b), we draw the long-time average current versus the dimensionless band centre separation $DW$ with the interband coupling $R/d = 0.09$ and $R/d = 0.7$, respectively. The other parameters are chosen as $W_a = W_b = W = 0.8\omega$, $\omega_0/\omega = 1/2$, $edE_1/\omega = 0.5$, $\alpha/\omega = 0.067$ in both figures. The temperature parameter $\beta\omega$ can be conveniently selected (e.g. $\beta\omega = 10.0$). From these figures we can see many resonant peaks which are the signatures of Rabi oscillations on the current response, though Rabi oscillations are destroyed by the dephasing.[14] One can obtain the information of Rabi oscillations by studying the current response. When $R/d$ is very small (Fig.2(a)), the current peaks appear at $DW = n/2$ (where $n$ is an integer). But when the interband coupling becomes strong (Fig.2(b)), the current peaks will have a deviation from $DW = n/2$. This can be understood as the band coupling destroying the conditions of Rabi oscillations. When the interband coupling is very weak, the two-band model can be regard as two independent single-band models. The conditions of Rabi oscillations are $\Delta = m\omega$ and $\Delta = n\omega_0$. When the interband coupling is strong, it will destroy the conditions of avoided crossings of quasi-energies as it coincides with those of Rabi oscillations. Thus the current resonant peaks will have a deviation from the position of $\Delta = m\omega$ and $\Delta = n\omega_0$.

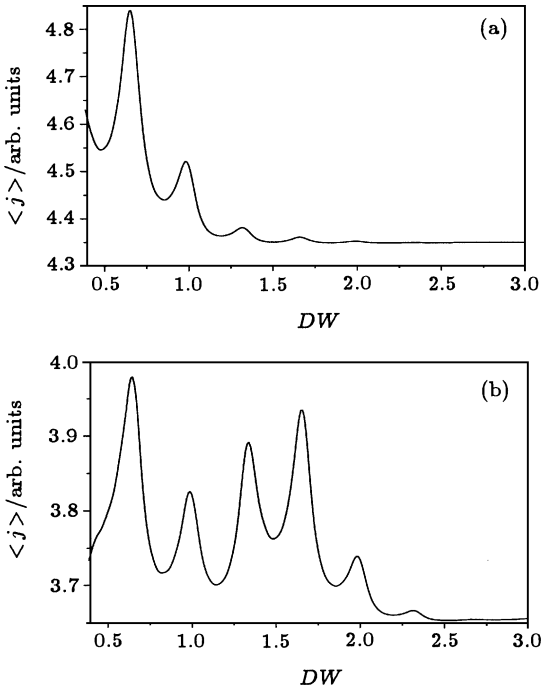



**Fig.1.** Long-time average current $< j >$ versus $DW$. The superlattice parameters are $W_a = W_b = W = 0.8\omega$, $\omega_0/\omega = 1/3$, $\alpha/\omega = 0.067$, $R/d = 0.09$. The ac field strengths are $edE_1/\omega$ (a) 0.1 and (b) 0.5, respectively.

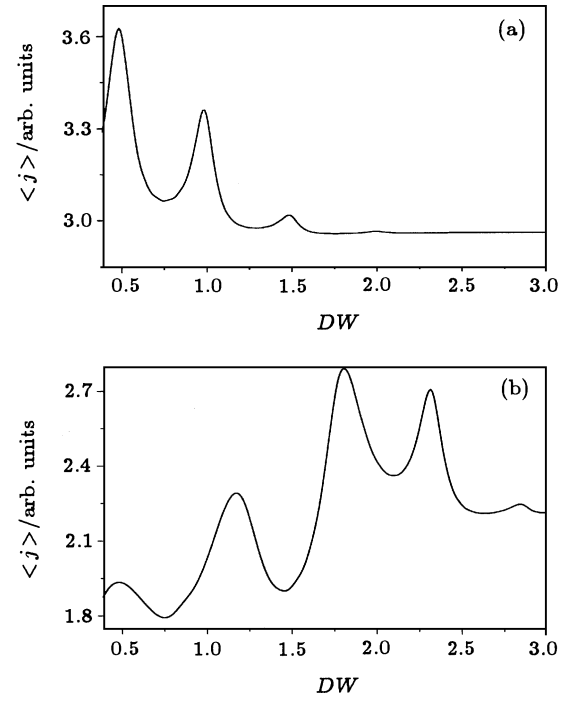



**Fig.2.** Long-time average current $< j >$ versus $DW$. The parameters are $W_a = W_b = W = 0.8\omega$, $\omega_0/\omega = 1/2$, $\alpha/\omega = 0.067$, $edE_1/\omega = 0.5$. The interband coupling $R/d$ is (a) 0.09 and (b) 0.7, respectively.

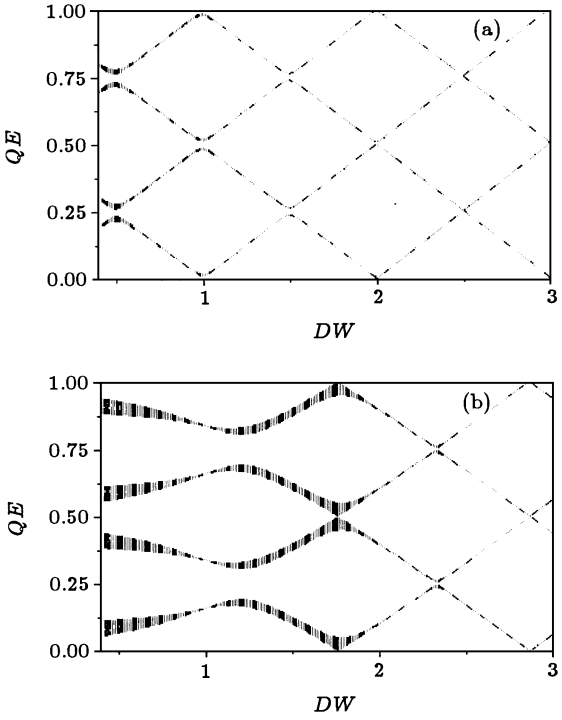



**Fig.3.** Dimensionless quasi-energy $QE$ versus dimensionless band centre separation $DW$. The parameters are $W_a = W_b = W = 0.8\omega$, $\omega_0/\omega = 1/2$, $\alpha/\omega = 0.067$, $edE_1/\omega = 0.5$. The interband coupling $R/d$ is (a) 0.09 and (b) 0.7, respectively. The subband width is indicated by the length of the short vertical line.

To clearly understand the above behaviour, we calculate the exact quasi-energy spectrum with interband coupling, corresponding to the parameters in Fig.2, but without scattering ($\alpha = 0$). The details of the method can be seen in Ref.[17]. Figure 3 shows the dependence of $QE$ ($\equiv \epsilon/\omega$) on dimensionless band centre separation $DW$. From the figure we can notice that in the quasi-energy Brillouin zone ($0 \leq \varepsilon \leq \omega$) quasi-energies will split into four ladders which indirectly proves that the fractional Wannier–Stark ladders exist. Due to the interband coupling the quasi-energies have little dispersion in the case of integral and fractional matching ratio. Thus the quasi-energy spectrum displays a series of subbands, whose widths are indicated by the length of the short vertical line. From Fig.3(a) we can clearly see when $R/d$ is small, the avoided crossings of quasi-energies appear at $DW = n/2$; but when $R/d = 0.7$ (see Fig.3(b)), the strong interband coupling makes the avoided crossings appear at places which have a deviation from $DW = n/2$.

Let us consider the experimental possibilities of observing the phenomena in this work. The superlattice parameters, such as $W$, $\Delta$, cannot be conveniently controlled in a given experiment. But one can vary the field parameters such as $E_0$, $E_1$ and $\omega$ to realize the predicted results.

# 4. Conclusions

In summary, within a two-band tight-binding model driven by dc–ac electric fields, we have studied the transport characteristics of Rabi oscillations between Bloch bands. Rabi oscillations can cause the current to have a sharp change. There are resonant peaks appearing in the curves of the long-time average current $< j >$ versus $DW$. These resonant peaks are the signatures of Rabi oscillations on the current response, though Rabi oscillations are destroyed by the dephasing. One can obtain the information of Rabi oscillations by studying the current response. When the interband coupling is small, Rabi oscillations and the current resonant peaks occur at $\Delta = m\omega$ or $\Delta = n\omega_0$ ($m$, $n$ are integers). When the interband coupling is strong to change the conditions of avoided crossings, Rabi oscillations and the current resonant peaks will appear at places which have a deviation from $\Delta = m\omega$ or $\Delta = n\omega_0$.

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