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Influence of atomic motion on the population and dipole squeezing of a cascade three-level atom in cavity field

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The dynamical property of a cascade three-level atom is investigated in the condition of atomic motion. The influence of atomic motion on the population and dipole squeezing is discussed. The results show that atomic motion makes the amplitude of atomic population be steady and increasing the parameter εp which denotes the atomic motion and the structure of field mode can shorten the period of collapse–revivals. By choosing an appropriate parameter εp, we can obtain a dipole squeezed atom of long standing.

Keywords: atomic population, dipole squeezing effect, atomic motion, cascade three-level atom
PACC: 4250

1. Introduction

The Jaynes–Cummings model (JCM) is an exactly solvable theoretical model describing the interaction between an atom and a quantum photon field. In this model, the atom is regarded as motionless and the quantum field as homogeneous. In recent years, research on the JCM has exposed many nonclassical effects such as collapse and revivals of Rabi oscillations, squeezing, and quantum correlation.[1−6] However, with the development in the technologies of laser cooling and atomic trapping, it is necessary to consider the atomic motion in preparing the cool or supercool atom. Therefore, people have generalized the JCM to one including the atomic motion.[7−11]

The phenomenon of periodical collapse–revivals of atomic population and the dipole squeezing are important quantum effects in the interaction of atom with field. The influence of Kerr medium on population of the three-level atom has been studied.[12] The dipole squeezing and dipole amplitude-squared squeezing of a three-level atom have also been studied.[13−15] But the influences of atomic motion have not been considered in these researches. In the researches including atomic motion, the interaction between a two-level atom and a single-mode field in the atomic motion has been studied.[16,17] As for the influences of atomic motion on squeezing and atomic population for a three-level atom, little can been found in the literature to date.

In this paper, we investigate the influence of atomic motion on dipole squeezing and atomic population in the interaction between a cascade three-level moving atom and a single-mode field by means of the technique of numerical analysis. In Sec.2, we set up the theoretical model and get the state vector of the system at an arbitrary moment t. In Secs.3 and 4, the influences of atomic motion and the structure of field mode on population and dipole squeezing effect are discussed, respectively. Finally, we draw conclusions from the above discussions.

2. Theoretical model and state vector

We consider a system consisting of a cascade three-level atom interacting with a single-mode field. Figure 1 is the model of a cascade three-level atom, in which the transition frequencies between $|2\rangle$ and $|1\rangle$ and $|3\rangle$ and $|2\rangle$ have the same value ω, and the transition between $|3\rangle$ and $|1\rangle$ is electrical dipole forbidden. We shall consider only the resonance condition between the atom and field, that is $\omega = \omega_3 - \omega_2 = \omega_2 - \omega_1$.

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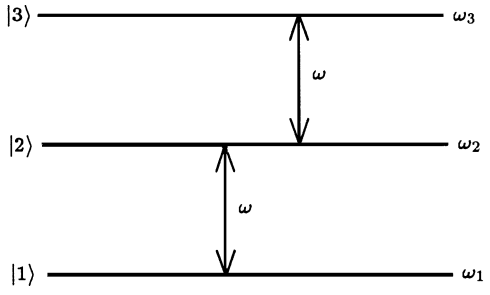


Fig.1. Model of a cascade three-level atom.

Supposing the atom moves along the axis z of the cavity and interacts with the photon field, and the cavity mode has been defined as $\mathrm{TEM}_{\mathrm{mnp}}$ mode. The atomic motion can be expressed as

$$f(z) \to f(vt) = \sin\left(\frac{p\pi vt}{L}\right), \tag{1}$$

where v stands for the velocity of the atom and p is the half-wave number of the field mode in the cavity with length L.

The Hamiltonian of the system can be written as

$$\begin{aligned} H =& \sum_{i=1}^{3} \omega_i C_i^{\dagger} C_i + \omega a^{\dagger} a + g f(z) \\ & \times (a C_3^{\dagger} C_2 + a^{\dagger} C_2^{\dagger} C_3 + a C_2^{\dagger} C_1 + a^{\dagger} C_1^{\dagger} C_2), \\ & (\hbar = 1), \end{aligned} \tag{2}$$

where $a^{\dagger}$ and a denotes the creation and annihilation operators of the photon field, respectively, ω_i the eigen-energy of level $|i\rangle$ of the atom, g the coupling constant between the field mode and the atom, and $C_i^{\dagger}$ and $C_i (i = 1, 2, 3)$ the Fermi creation and annihilation operators of the ith level of the atom which follows the rules

$$C_i C_j^{\dagger} = 0 \qquad (i \neq j), \tag{3}$$

$$C_i C_i^{\dagger} C_j = C_i. \tag{4}$$

Meanwhile, we take the value of v as $v = \varepsilon g L/\pi$ (ε a real parameter denoting the atomic motion) which is achievable in experiment.

The Hamiltonian of the system in the interactional picture in the rotating-wave approximation can be written as

$$H^{\mathrm{I}} = g \sin(\varepsilon p g t)(a C_3^{\dagger} C_2 + a^{\dagger} C_2^{\dagger} C_3 + a C_2^{\dagger} C_1 + a^{\dagger} C_1^{\dagger} C_2). \tag{5}$$

We suppose that the atom is initially in the upper state $|3\rangle$ and the field in the coherent state

$$|\psi_{\mathrm{f}}(0)\rangle = \sum_{n=0}^{\infty} F_n |n\rangle, \tag{6}$$

where

$$F_n = \mathrm{e}^{-\frac{|\alpha|^2}{2}} \frac{\alpha^n}{\sqrt{n!}}. \tag{7}$$

Then the initial state of the system can be expressed as

$$|\psi(0)\rangle = \sum_{n=0}^{\infty} F_n |n, 3\rangle. \tag{8}$$

The state vector of the system at an arbitrary moment t can be written as

$$\begin{aligned} |\psi(t)\rangle = \sum_{n=0}^{\infty} F_n \big[& a_n(t)|n, 3\rangle \\ & + b_{n+1}(t)|n+1, 2\rangle \\ & + c_{n+2}(t)|n+2, 1\rangle \big]. \end{aligned} \tag{9}$$

Substituting Eqs.(5) and (9) into the Schrödinger equation of interactional picture, we get the following set of differential equations:

$$\begin{aligned} & \mathrm{i}\dot{a}_n = g \sin(\varepsilon p g t)\sqrt{n+1}\, b_{n+1}, \\ & \mathrm{i}\dot{b}_{n+1} = g \sin(\varepsilon p g t)(\sqrt{n+1}\, a_n + \sqrt{n+2}\, c_{n+2}), \\ & \mathrm{i}\dot{c}_{n+2} = g \sin(\varepsilon p g t)\sqrt{n+2}\, b_{n+1}. \end{aligned} \tag{10}$$

Solving the above set of differential equations, we get the following results:

$$\begin{aligned} a_n(t) =& \frac{n+2}{2n+3} + \frac{n+1}{2n+3} \\ & \times \cos\left[\frac{\sqrt{2n+3}}{\varepsilon p}(1 - \cos \varepsilon p g t)\right], \end{aligned} \tag{11}$$

$$\begin{aligned} b_{n+1}(t) =& -\mathrm{i}\sqrt{\frac{n+1}{2n+3}} \\ & \times \sin\left[\frac{\sqrt{2n+3}}{\varepsilon p}(1 - \cos \varepsilon p g t)\right], \end{aligned} \tag{12}$$

$$\begin{aligned} c_{n+2}(t) =& \sqrt{\frac{(n+1)(n+2)}{2n+3}} \\ & \times \left\{-1 + \cos\left[\frac{\sqrt{2n+3}}{\varepsilon p}(1 - \cos \varepsilon p g t)\right]\right\}. \end{aligned} \tag{13}$$

Hence, the state vector of the system at moment t can be determined by substituting Eqs.(11)–(13) into Eq.(9), and we can discuss the influence of atomic motion on the dynamical property of the atom.

3. The evolution of atomic population

From Eq.(9), we obtain the atomic population while the atomic motion is considered as

$$W_3(t) = \langle C_3^\dagger C_3 \rangle = \sum_n |F_n a_n|^2, \tag{14}$$

$$W_2(t) = \langle C_2^\dagger C_2 \rangle = \sum_n |F_n b_{n+1}|^2, \tag{15}$$

$$W_1(t) = \langle C_1^\dagger C_1 \rangle = \sum_n |F_n c_{n+2}|^2, \tag{16}$$

and the normalizing condition is satisfied,

$$W_1(t) + W_2(t) + W_3(t) = 1. \tag{17}$$

The evolutions of $W_1(t)$, $W_2(t)$ and $W_3(t)$ are given in Fig.2 in which the atomic motion is not considered and in Fig.3 in which the atomic motion is considered.

To investigate the influence of the atomic motion on the population, we make a comparison between Figs.2 and 3.

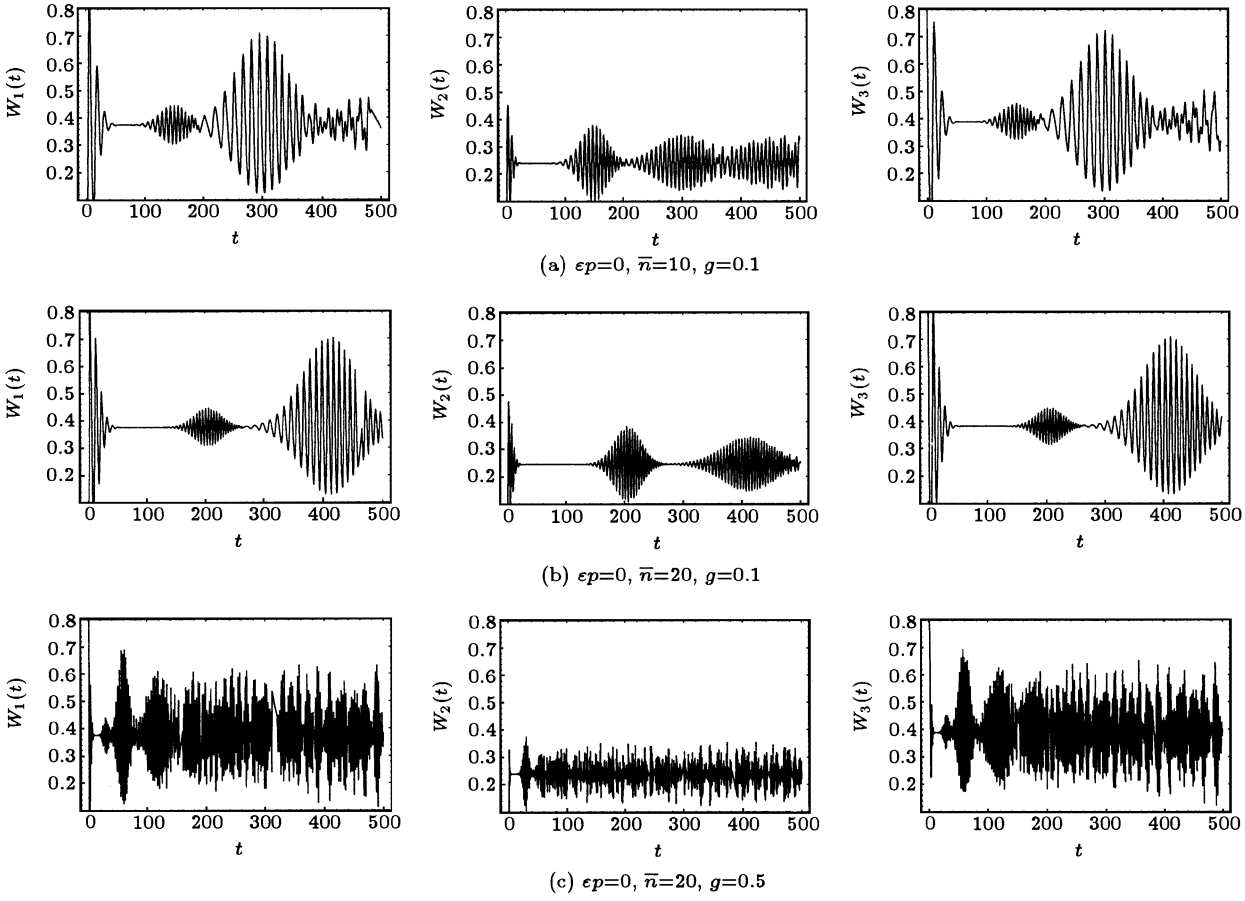


Fig.2. Evolution of the atomic population when the atomic motion is neglected.

It can be seen from Fig.2(a) that, in the condition of no atomic motion, the atomic population shows the collapse–revivals repeatedly, and the amplitude of Rabi oscillation in each revival period is not the same. Increasing intensity of photon field (Fig.2(b)) induces the collapse–revivals more clearly, while increasing of the coupling constant g (Fig.2(c)) induces the population to oscillate so drastically that the phenomenon of collapse–revival is not so clear.

In Fig.3(a), as the parameter εp, which denotes the atomic motion and structure of field mode, equals 0.1, the population shows a phenomenon of regular collapse–revival in which the atom occupies the upper-level $|3\rangle$ and lower-level $|1\rangle$ periodically, and the evolution of the population presents two small amplitudes and one big amplitude alternately. With the increasing of εp (Fig.3(b)), the segments of small amplitude disappear gradually and the period of revivals is shortened. We can see from Fig.3(b) that the amplitude of W_1, W_2 and W_3 hold steady and are equal to 0 periodically: namely, the atomic motion makes the transitions between three levels more frequent than the situation without atomic motion.

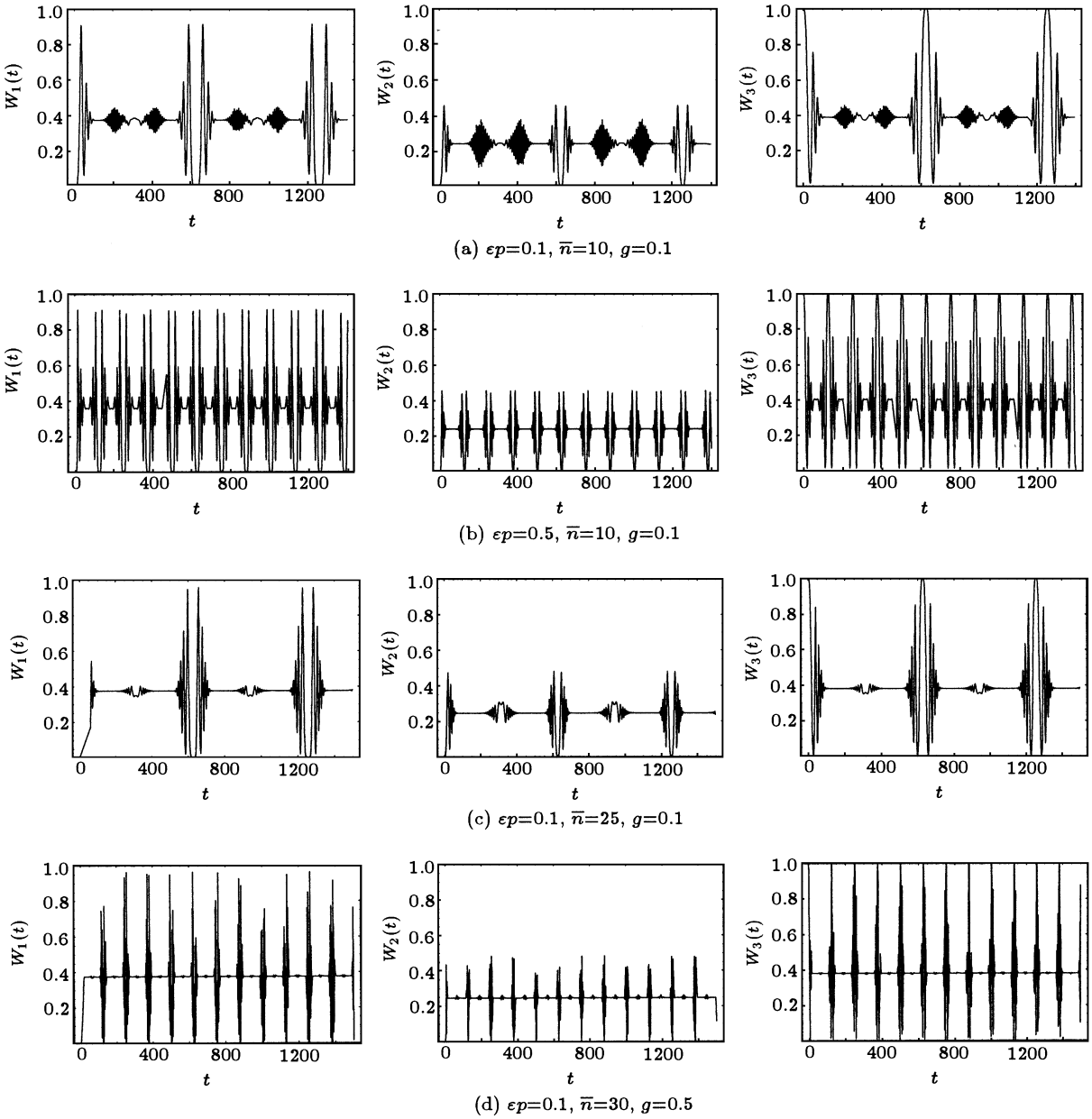


Fig.3. Evolution of the atomic population when the atomic motion is considered.

Besides the above features, the influence of the intensity of photon field ($\bar{n}$) is also distinct from that without atomic motion. With $\bar{n}$ increasing, the segments of two small amplitudes collapse gradually (Fig.3(c)). When $\bar{n} = 30$ these two segments collapse completely into a straight line. After that the increasing of $\bar{n}$ can only change the frequency of Rabi oscillation, while the period of collapse–revival does not change with $\bar{n}$. In addition, the invariance of g changes the frequency of Rabi oscillation, but the period of collapse–revival holds steady (Fig.3(d)).

4. Dipole squeezing effect of the atom

In order to study the dipole squeezing effect of the atom, the total creation and annihilation operators of a three-level atom can be defined as follows:[18]

$$D_+ = C_2^\dagger C_1 + C_3^\dagger C_2, \tag{18}$$

$$D_- = C_1^\dagger C_2 + C_2^\dagger C_3. \tag{19}$$

In terms of D_+ and D_-, a pair of conjugate dipole operators of the atom is given by

$$D_x = D_+ + D_-, \tag{20}$$

$$D_y = -\mathrm{i}(D_+ - D_-), \tag{21}$$

with the commutation relation

$$[D_x, D_y] = 2\mathrm{i}D_3, \tag{22}$$

where $D_3 = D_+D_- - D_-D_+$. Hence, if the operators D_x and D_y satisfy the relations, respectively,

$$\langle(\Delta D_x)^2\rangle < |\langle D_3\rangle|, \tag{23}$$

$$\langle(\Delta D_y)^2\rangle < |\langle D_3\rangle|, \tag{24}$$

we can say that the x-component or y-component of the dipole moment is squeezed. So we investigate the dipole squeezing effect by introducing the following functions:

$$\begin{aligned} f_1 =&\langle(\Delta D_x)^2\rangle - |\langle D_3\rangle| \\ =&\langle D_x^2\rangle - \langle D_x\rangle^2 - |\langle D_3\rangle|, \end{aligned} \tag{25}$$

$$\begin{aligned} f_2 =&\langle(\Delta D_y)^2\rangle - |\langle D_3\rangle| \\ =&\langle D_y^2\rangle - \langle D_y\rangle^2 - |\langle D_3\rangle|. \end{aligned} \tag{26}$$

In terms of Eqs.(9) and (11)–(13), the expected values of the dipole operators are, respectively,

$$\begin{aligned} \langle D_x\rangle =& \sum_n [F_{n+1}^* F_n (a_{n+1}^* b_{n+1} + b_{n+2}^* c_{n+2}) \\ &+ F_{n-1}^* F_n (c_{n+1}^* b_{n+1} + b_n^* a_n)], \end{aligned} \tag{27}$$

$$\begin{aligned} \langle D_x^2\rangle =& \sum_n [|F_n|^2 (2|b_{n+1}|^2 + |a_n|^2 + |c_{n+2}|^2) \\ &+ F_{n+2}^* F_n a_{n+2}^* c_{n+2} + F_{n-2}^* F_n c_n^* a_n], \end{aligned} \tag{28}$$

$$\begin{aligned} \langle D_y\rangle =& -\mathrm{i} \sum_n [F_{n+1}^* F_n (a_{n+1}^* b_{n+1} + b_{n+2}^* c_{n+2}) \\ &- F_{n-1}^* F_n (c_{n+1}^* b_{n+1} + b_n^* a_n)], \end{aligned} \tag{29}$$

$$\begin{aligned} \langle D_y^2\rangle =& \sum_n [|F_n|^2 (2|b_{n+1}|^2 + |a_n|^2 + |c_{n+2}|^2) \\ &- F_{n+2}^* F_n a_{n+2}^* c_{n+2} + F_{n-2}^* F_n c_n^* a_n], \end{aligned} \tag{30}$$

$$\langle D_3\rangle = \sum_n |F_n|^2 [|a_n|^2 - |c_n|^2]. \tag{31}$$

According to Eqs.(25)–(31), the evolution of f_1 and f_2 can be obtained by the technique of numerical analysis. Firstly, by comparing Fig.5(a) and 5(e) with Fig.4(a) and 4(b), we can see that the atomic motion reduces the probability that D_x and D_y are squeezed, but the duration of squeezing is increased. When the parameter ϵp is increased from 0.1 to 1 (Fig.5(b) and 5(f)), the dipole moment of the atom presents the periodical squeezing effect. The squeezed degree and number of D_y is greater than D_x and the uncertainty relation of these two components D_x and D_y periodically satisfy the minimum uncertainty relation: that is to say, $f_1 = 0$ and $f_2 = 0$. If εp is increased continually (Figs.5(c), 5(g)), the dipole moment of the atom will present a steady Rabi oscillation. D_y will remain in the squeezed state but its squeezed degree become smaller. D_x can only present a very feeble squeezing. So we confirm that a dipole squeezed atom can be obtained by choosing an approximate parameter εp which denotes the atomic motion and the structure of field mode.

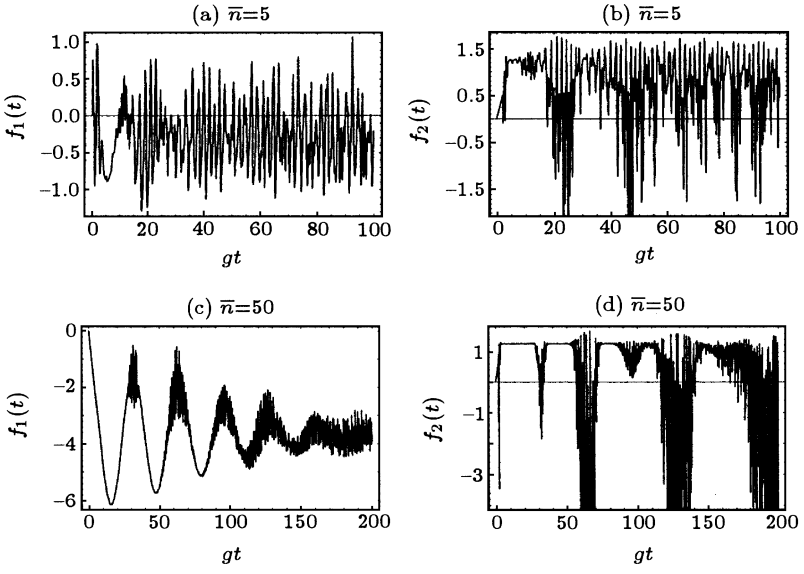


Fig.4. Evolution of $f_1(t)$ and $f_2(t)$ when the atomic motion is neglected.

Secondly, when the intensity of initial photon field become stronger, we can find a squeezing effect completely distinct from that of the weak field (Figs.5(d), 5(h)). When $\bar{n}$ increases to 50, f_1 and f_2 represent a phenomenon of collapse–revival, and this collapse–revival of f_1 is modulated by a carrier wave. The

period of the carrier wave increases with the initial photon number. When the atomic motion is neglected (Fig.4(c)), the amplitude of the carrier wave of f_1 attenuates gradually. When the atomic motion is considered (Fig.5(d)), the amplitude of the carrier wave holds steady. As $\varepsilon p = 0.1$, D_x is squeezed in most time, at the same time D_y is squeezed only in a short time (Figs.5(d), 5(h)). The increasing of εp makes the squeezing degree of D_x decreased and that of D_y increased; meanwhile the frequency of f_1 and f_2 is also increased.

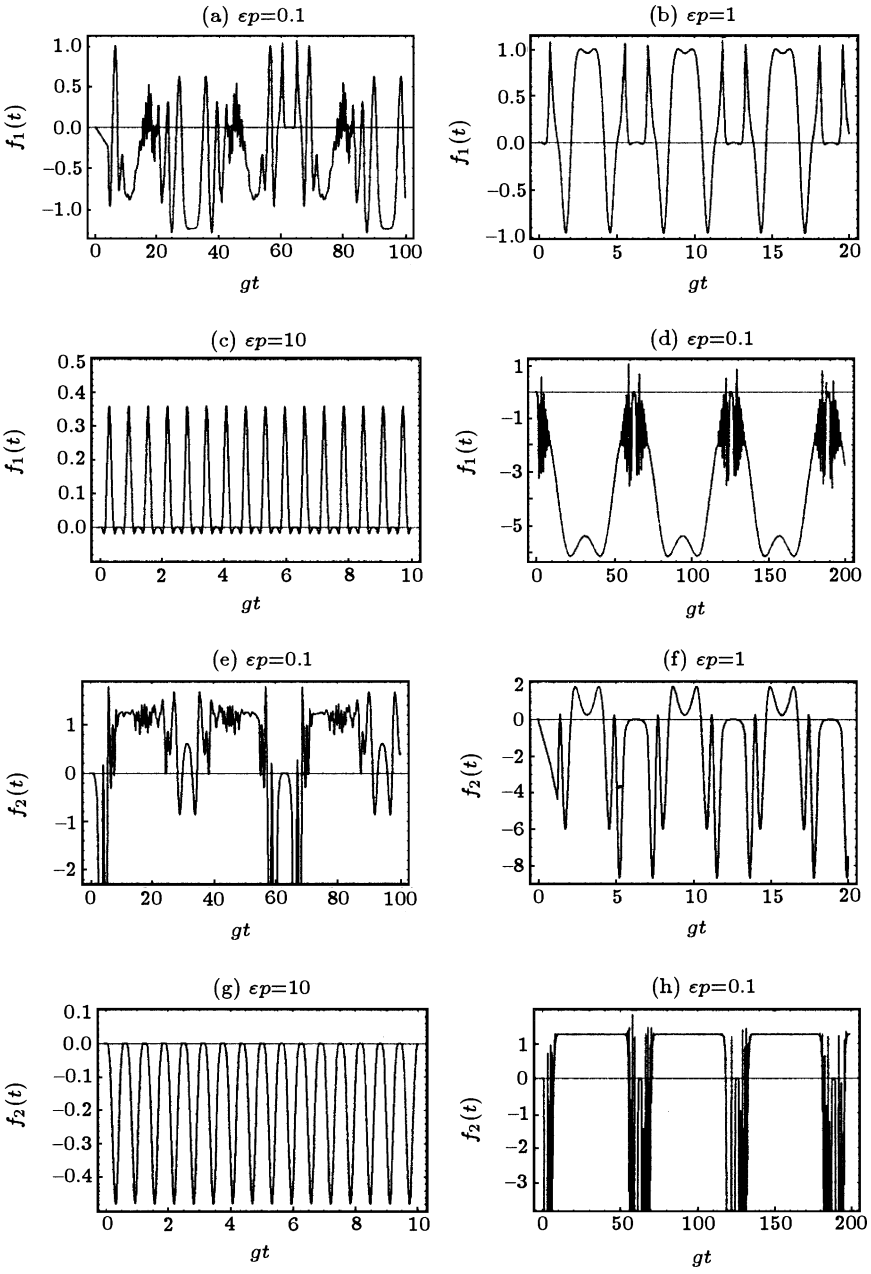


Fig.5. Evolution of $f_1(t)$ and $f_2(t)$ when the atomic motion is considered, where $\bar{n} = 50$ for $(d)(h)$ and $\bar{n} = 5$ for the others.

5. Conclusions

On the basis of the above discussion, we conclude as follows: (a) The atomic population is influenced by the initial photon number, coupling constant and the parameter of atomic motion and structure of field mode collectively. The atomic motion causes the amplitude of atomic population to be steady and the atom to be in the ground state $|1\rangle$ and the upper state $|3\rangle$ periodically. Increasing εp can shorten the

period of collapse–revival. When $\bar{n} \geq 30$, the period of collapse–revival is not influenced by $\bar{n}$, and the increasing of $\bar{n}$ and g can increase the frequency of Rabi oscillation. (b) The influence of atomic motion on dipole squeezing effect of the atom is different according to the intensity of the initial field. In the condition of a very weak field, choosing an appropriate parameter εp, we can obtain an atom which is kept in the squeezed state. With the increasing of $\bar{n}$, the squeezing effect presents periodical collapse–revival modulated by a carrier wave. The amplitude of the carrier wave does not attenuate in the case of a moving atom, and the period of it increases with the increasing of $\bar{n}$.

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