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Resonator insensitive investigation to thermal lens for end-pumped lasers

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Optical pumping of solid-state lasers induces a thermal lens effect in the laser crystal; this thermal effect determines the beam properties. In this paper, the focal length of the thermal lens is calculated, and resonator parameter (g_1 g_2) is obtained, which is insensitive to thermal distortions when the product of the resonator parameters (g_1, g_2) equals 1/2.

By using our calculated results, we get the fundamental transverse mode laser output with optic–optic conversion of 48.6%, and slope efficiency of 56.3%.

Keywords: all solid-state laser, thermal lens, design of resonator, resonator insensitive to thermal distortions

PACC: 4230, 4260D

1. Introduction

All solid-state lasers pumped by laser diode[1,2] have received a great deal of attention, because of high coupling efficiency being obtained since there is a good spectral match between the emission spectrum of the pump beam and the absorption spectrum of the laser medium. Diode end-pumping schemes offer many advantages: high pumping efficiency and good mode selection for single mode operation of solid-state lasers. This scheme of pumping allows the pump energy to be deposited into the central region of the rod, thus exciting the lowest optical transverse mode (in contrast to side pumping, which illuminates the entire rod and tends to excite multiple modes). As a consequence of this pumping transverse nonuniformity, the heating distribution, which results from quantum inefficiencies is also nonuniform. Uniform heating distributions tend to parabolic temperature and phase error profile;[3] such phase errors are easily corrected by standard optical components. With end-pumped lasers, however, the temperature and optical phase profile, while still parabolic within the pumped region (assuming uniform pumping within the pumped region), are more complex (typically logarithmic) in the guard band surrounding the pumped region.

In the last few years, a number of neodymium-doped vanadium crystals have been identified as promising for diode-pumping lasers. These crystals consist of yttrium orthovanadate (YVO_4), gadolinium orthovanadate ($GaVO_4$), and strontium fluorovanadate ($Sr_5(VO_4)_3F$ or SVAP).[4,5] Several advantages with respect to Nd:YAG crystals include higher cross section, lower threshold, wider absorption bandwidth and polarized output. For example, the pumping absorption bandwidth of Nd: $GaVO_4$ crystals is 3.2nm, twice as much as that of Nd:YAG crystals. The wider absorption bandwidth means pump wavelength due to temperature or aging effect.

In Refs.[6–8], the authors analysed the thermal distributions, but did not discuss design of resonator in detail. Pfistner[10] analysed thermal lens effect; however he did not solve the algebra equation of resonator. Vittorio Magni[10] analysed the resonator in detail, whereas he did not analyse the thermal distribution. Moreover, he only discussed flash-lamp pumping rather than diode pumping.

In this paper, the focal length of the thermal lens is calculated and the thermal distribution and refraction induced by temperature are discussed in Sec.2. The focal length of the thermal lens is measured, and is in accord with the theory value. In Sec.3, we solve

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the problem of resonator at end pumping lasers, in which the resonator is insensitive to thermal lens.

2. Thermal lens at end-pumping solid-state lasers

Considering end-pumped solid-state lasers, the steady state temperature distributions in an axis symmetry is described as[11]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}+\frac{Q(r,z)}{K}=0, \tag{1}$$

where K is the thermal conductivity and $Q(r,z)$ is the heat density in the laser rod.

For a fibre-coupled pump beam, $Q(r,z)$ can be approximately described as a Gaussian distribution (or flat top distribution):

$$\begin{aligned}Q(r,z)=&\frac{2\alpha_{\rm a}P_{\rm h}}{\pi\omega_{\rm p}(z)[1-\exp(-\alpha_{\rm a}l)]}\\&\times\exp\left(\frac{-2r^2}{\omega_{\rm p}(z)}-\alpha_{\rm a}z\right),\end{aligned} \tag{2}$$

where $P_{\rm h}$ is total heat load, $\omega_{\rm p}$ is radius of the pump beam, $\alpha_{\rm a}$ is absorption coefficient and l is length of the rod.

For conventional edge cooling of the rod-end, the thermal boundary conditions can be approximated to adiabatic due to the fact that the cooling surface convection coefficient is a few orders larger than the natural convection coefficient at the end.[12] Under this approximation, the solution of (1) for an isotropic crystal is found to be

$$\begin{aligned}T(r,z)=&\frac{p_{\rm h}}{2\pi k}\frac{\alpha_{\rm a}\exp(-\alpha_{\rm a}z)}{1-\exp(-\alpha_{\rm a}l)}\\&\times\left[\left(1-\frac{r^2}{\omega_{\rm p}^2(z)}\right)\right.\\&\left.+\ln\left(\frac{r_0^2}{\omega_{\rm p}^2(z)}\right)+\ln\left(\frac{r_0^2}{r^2}\right)\right],\end{aligned} \tag{3}$$

where r_0 is the radius of the laser rod.

The temperature gradient causes thermal stresses. The thermally induced strains can be described by the equilibrium equation[13]

$$\frac{1-\mu}{1+\mu}\nabla(\nabla\cdot u)-\frac{1-2\mu}{1+\mu}\times\nabla\times\nabla\times u=\alpha_T\nabla(T), \tag{4}$$

where μ is Poisson's ratio, u is displacement vector and $\alpha_{\rm T}$ is thermal expansion coefficient.

The strains and displacement are related by

$$\varepsilon_{ik}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right). \tag{5}$$

Since a general analytical solution is useful for obtaining scaling relationships, we use the place stress approximation to solve the equilibrium equations. Based on this approximation, the strain distributions are determined to be

$$\begin{aligned}\varepsilon_{rr}=&(1+\mu)\alpha_{\rm T}\Delta T(r,z)+\frac{(1+\mu)}{r^2}\alpha_{\rm T}\\&\times\int_0^r\Delta T(r,z)r{\rm d}r+\frac{1+\mu}{r_0^2}\alpha_{\rm T}\\&\times\int_0^{r_{\rm o}}\Delta T(r,z)r{\rm d}r,\end{aligned} \tag{6}$$

$$\begin{aligned}\varepsilon_{\theta\theta}=&\frac{(1+\mu)}{r^2}\alpha_{\rm T}\int_0^r\Delta T(r,z)r{\rm d}r\\&-\frac{1+\mu}{r_0^2}\alpha_{\rm T}\int_0^{r_{\rm o}}\Delta T(r,z)r{\rm d}r,\end{aligned} \tag{7}$$

$$\varepsilon_{zz}=(1+\mu)\alpha_{\rm T}\Delta T(r,z). \tag{8}$$

The thermally induced strain caused changes of refractive index and length of rod by photo elastic effects. The detailed form of birefringence terms depend on the crystalline symmetries of the rod host material and the way in which the rod is cut from a crystal of the material. Born and Wolf[14] gave a general development for arbitrary material based on distortion of the refractive index ellipsoid. Here we indicate the form of this term for a typical material (YAG), which is a cubic crystal. Koechner and Rice[15] had derived the birefringence for the radially and azimuthally polarized component of the beam:

$$n(r)=n_0+T(r)\beta_{r,\theta}, \tag{9}$$

$$l(r)=l_0+T(r)\alpha_{\rm T}l, \tag{10}$$

$$\beta_{r,\theta}=\frac{\partial n}{\partial T}+\varepsilon_{r,\theta}=\frac{\partial n}{\partial T}+2n_0^3\alpha C_{r,\theta}, \tag{11}$$

$$\alpha_{\rm T}=\frac{1}{l}\frac{\partial l}{\partial T}, \tag{12}$$

where $n(r)$ is refractive index of rod, n_0 is refractive index of rod centre and $C_{r,\theta}$ is the thermal optic coefficient.

Radial variety of length causes curve of the endface. For the first order approximation, the curvature radio of the endface is obtained by optical geometry:

$$R_{\rm E}=\frac{2r_0}{\alpha_{\rm a}l}\cdot\frac{1}{\left.\dfrac{\partial^2T(r)}{\partial r^2}\right|_{r=0}}, \tag{14}$$

$$D_{\rm E}=-\frac{\alpha_{\rm a}l(n_0-1)}{r_0^2}\left.\frac{\partial^2T(r)}{\partial r^2}\right|_{r=0}. \tag{15}$$

If the effect of end-face curve is considered, the matrix of the thermal lens is derived:

$$M = \begin{pmatrix} \cos b - \dfrac{D_{\mathrm{E}}}{a}\sin b & \dfrac{1}{a}\sin b \\ -2D_{\mathrm{E}}\cos b + D_{\mathrm{E}}^2\dfrac{\sin b}{a} - a\sin b & -\dfrac{D_{\mathrm{E}}}{a}\sin b + \cos b \end{pmatrix}, \tag{16}$$

$$a^2 = -\beta_{r,\theta} n_0 \left.\frac{\partial^2 T(r)}{\partial r^2}\right|_{r=0}, \tag{17}$$

$$b = \frac{l\alpha}{n_0}. \tag{18}$$

Total dioptric power is caused by the thermal effect

$$\begin{aligned} D =& 2D_{\mathrm{E}}\cos b + a\sin b - D_{\mathrm{E}}^2\frac{\sin b}{a} \\ \approx& 2D_{\mathrm{E}} + ab - \frac{D_{\mathrm{E}}^2 l}{n_0}, \\ & b \ll 1, \end{aligned} \tag{19}$$

$$D_{\mathrm{T}} = \frac{P_{\mathrm{h}}}{2k\pi\omega_{\mathrm{p}}^2}\left[\frac{\partial n}{\partial T} + (1+\mu)(n_0-1)\alpha_{\mathrm{T}} + \varepsilon_{r,t}\right], \tag{20}$$

where $D_{\mathrm{E}} l \ll 1$.

If thw=e effect of the endface curve is not considered, total dioptric power is expressed as

$$D_{\mathrm{T}} \approx \frac{P_{\mathrm{h}}}{2k\pi\omega_{\mathrm{p}}^2}\left[\frac{\partial n}{\partial T} + \varepsilon_{r,t}\right]. \tag{21}$$

Based on the above equation, the thermal focal length for an end pumped solid-state laser can be derived as

$$\begin{aligned} f =& \frac{2\pi k\omega_{\mathrm{p}}^2}{P_{\mathrm{h}}}\left[\frac{\partial n}{\partial t} + (1+\mu)\right. \\ & \left.\times (n-1)\alpha_{\mathrm{T}} + n_0^3\alpha_{\mathrm{T}}C_{r,z}\right]^{-1}, \end{aligned} \tag{22}$$

$$f_{\mathrm{T}} = \frac{2k\pi\omega_{\mathrm{p}}^2}{P_{\mathrm{h}}}\left[\frac{\partial n}{\partial T} + n_0^3\alpha_{\mathrm{T}}C_{r.z}\right]^{-1}. \tag{23}$$

In Eq.(22), the first term results from the thermal dispersion $\partial n/\partial T$. The second term is caused by the axial strain distribution and the third term represents the strain-induced birefringence with the strain tensor. The thermal lens will affect the stability of a resonator and laser beam size. So, the laser cavity changes dynamically with pump power.

Experiments have been performed to observe changes of focal length with pumping power (Fig.1) by way of Ref.[16]. The laser crystal is a $GdVO_4$ rod with 5mm long and 3mm in diameter, with a Nd doping concentration of 3.5%. The 55mm long laser cavity consists of one endface of the laser rod and flat mirror with transmission of 90%. The pumping source is a fibre-coupled diode laser. The radius (ω_{p}) of the pump beam side in the laser rod is approximately 290μm. Fig.1 shows the focal length of thermal lens versus pump power. The parameters used are $\mathrm{d}n/\mathrm{d}T$=7.3×10^{-6}/K, α_{T}=4.7×10^{-6}/K, K=11.7W·cm/K, and μ=333, and 24% of pumping power is assumed to be generated as heat. From Fig.1, it can be seen that the experimental value is in accord with the theory value.

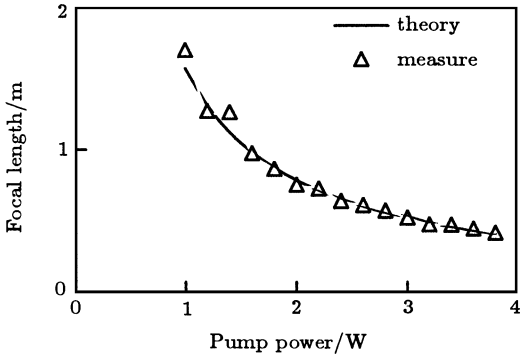


Fig.1. Thermal focal length versus input power of pumping.

3. Design of resonator

When a resonator of cw or high repetition rate pulse solid-state lasers is designed, some factors must be considered: (1) the mode volume inside the rod should be as large as possible to obtain high output power and efficiency; (2) misalignment sensitivity should be low to make the laser reliable and easy to use;(3) the mode volume inside the rod should be insensitive to the fluctuation of the focal length of thermally induced lens to avoid output power instability.

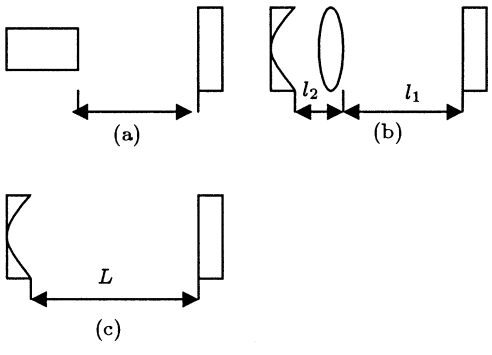


Fig.2. Equivalent resonator systems: (a) resonator with internal thermal focusing lens, (b) resonator with internal thin lens, (c) equivalent resonator with different mirror separation and curvatures.

On the basis of the above analysis, we assume that a rod of a cw laser, in which the heat generation is uniform, acts as a lenslike medium due to both thermal and stress-induced effects; the refractive index varies quadratically with the radial coordinate.[12] The focal length f of the rod is given by Eq.(22). From Eq.(20) the dioptric power $1/f$ is proportional to input power. In this case the lenslike medium is externally equivalent to a optical mode of a thin lens with a focal length given by Eq.(22).

The solid-state active medium is equivalent to f (focal length of thermal lens) at end-pumping (Fig.2). We regard the laser crystal as having curvature radius $\rho_1 = R_{\rm E}$, since the face of the crystal is plated and the length of the crystal is short: in general, several millimetres. The crystal is considered as a thin lens.

The beam radius at the mirror ω_1, ω_2 can be investigated with the ray-transfer matrix method using self-consistency for the return trip of the beam in the resonator. We refer mirror M_1 to the face:

$$\begin{aligned}\begin{pmatrix} a & b \\ c & d \end{pmatrix} &= \begin{pmatrix} 1 & l_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f_{\rm T}} & 1 \end{pmatrix} \begin{pmatrix} 1 & l_1 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1-\dfrac{l_2}{f_{\rm T}} & l_1+l_2-\dfrac{l_1 l_2}{f_{\rm T}} \\ -\dfrac{1}{f_{\rm T}} & 1-\dfrac{l_1}{f_{\rm T}} \end{pmatrix} \\ &= \begin{pmatrix} 1 & l_1 \\ -\dfrac{1}{f_{\rm T}} & 1-\dfrac{l_1}{f_{\rm T}} \end{pmatrix}, \end{aligned} \tag{24}$$

$$l_2 \ll l_1, f_{\rm T}. \tag{25}$$

The matrix is written as expression (26) when the ray is transferred on the return trip in the resonator:

$$\begin{aligned} M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} &= \begin{pmatrix} d & b \\ c & a \end{pmatrix} \\ &\times \begin{pmatrix} 1 & 0 \\ -\dfrac{2}{R_1} & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\dfrac{2}{R_{\rm E}} & 1 \end{pmatrix}, \end{aligned} \tag{26}$$

$$A = b\left(c - \frac{2d}{R_1}\right) + \left(a - \frac{2b}{R_1}\right)\left(d - \frac{2b}{R_2}\right), \tag{27}$$

$$B = bd + b\left(d - \frac{2b}{R_2}\right), \tag{28}$$

$$C = -a\left(c - \frac{2d}{R_1}\right) + \left(a - \frac{2b}{R_1}\right)\left(c - \frac{2a}{R_{\rm E}}\right), \tag{29}$$

$$D = ad + b\left(c - \frac{2a}{R_{\rm E}}\right). \tag{30}$$

The resonator parameters are introduced:

$$g_1 = a - \frac{b}{R_1}, \quad g_2 = d - \frac{b}{R_{\rm E}}. \tag{31}$$

The resonator stability condition is given:

$$|A + D| \le 2 \quad \text{namely,} \quad 0 \le |g_1 g_2| \le 1. \tag{32}$$

In the resonator, the spot size of beam waist is written to be the following expression:

$$\omega_0^2 = \frac{\lambda b}{\pi} \frac{\sqrt{g_1 g_2 (1 - g_1 g_2)}}{g_1 + a^2 g_2 - 2a g_1 g_2}, \tag{33}$$

$$l_0 = \frac{b g_2 (a - g_1)}{g_1 + a^2 g_2 - 2a g_1 g_2}, \tag{34}$$

$$q_0^{-1} = -\,{\rm i}\frac{\lambda}{\pi \omega_0^2}, \tag{35}$$

where l_0 is the distance from beam waist to mirror M_1 and q_0 is the parameter of the beam waist.

From Eq.(24), we know that

$$b = l_1. \tag{36}$$

The q parameter of any position in the resonator is expressed as follows:

$$\frac{1}{q_i} = \frac{1}{R_i(z)} - {\rm i}\frac{\lambda}{\pi \omega_i^2(z)}, \tag{37}$$

$$R = \frac{2B}{D - A}, \tag{38}$$

$$\omega = \left(\frac{\lambda}{\pi n}\right)^{1/2} \frac{|B|^{1/2}}{\left[1 - \left(\dfrac{D + A}{2}\right)^2\right]^{1/4}}, \tag{39}$$

where R is the radius of curvature and ω is the radius of optic beam in the referred face.

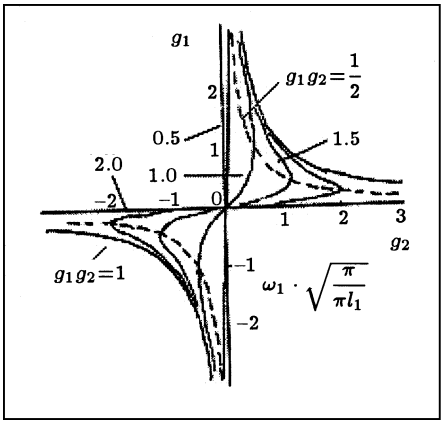


Fig.3. Curves of constant spot size ω_1 in the (g_1 g_2) diagram calculated by Eq.(41). Parameter values ω_1 $(\Pi/\lambda l_1)^{1/2}$=0.5, 1, 1.5, 2.

From Eqs.(37)–(39), the beam radius ω_1 is obtained in the mirror M_1:

$$\omega_1^2 = \frac{\lambda b}{\pi}\left[\frac{g_2}{g_1(1-g_1g_2)}\right]^{1/2}. \tag{40}$$

Figure 3 shows the curves of constant value of the spot size ω_1. Large spot ω_1 appears for resonators with large g_2 values. ω_1 increases with increasing mirror separation l_1 (Eq.(36)). Figure 3 demonstrates additionally that the spot size is insensitive to variations of g_2 and g_1 for resonators by products (g_1, g_2) on the hyperbola, Eq.(46), indicated by the dotted line. On the dotted line, ω_1 changes the least if g_1 varies at constant g_2; if g_2 varies at constant g_1, the changes of ω_1 are the smallest also on this line. The influence of thermal focal length f on g_2 and g_1 depends on the geometrical dimension given by Eq.(31).

A resonator is called insensitive to thermal lens if beam divergence and mode volume are insensitive to variations of the average thermal focal length f of the laser rod. Mode volume V and divergence determine the output mirror size ω_1.[17] From Refs.[17, 18], we know that the output mirror size ω_1 is insensitive to thermal focusing length f.

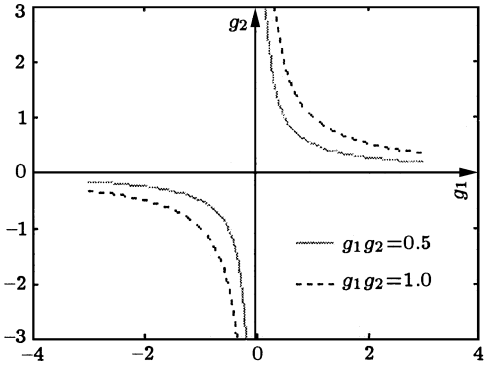


Fig.4. A resonator is insensitive to thermal lens; resonator parameter is content with Eq.(46).

$$\frac{\mathrm{d}\omega_1}{\mathrm{d}f_\mathrm{T}} = 0, \tag{41}$$

$$\frac{\mathrm{d}\omega_1}{\mathrm{d}f_\mathrm{T}} = \frac{\partial\omega_1}{\partial g_1}\cdot\frac{\partial g_1}{\partial f_\mathrm{T}} + \frac{\partial\omega_1}{\partial g_2}\cdot\frac{\partial g_2}{\partial f_\mathrm{T}} + \frac{\partial\omega_1}{\partial L}\cdot\frac{\partial L}{\partial f_\mathrm{T}}, \tag{42}$$

$$\frac{\mathrm{d}\omega_1}{\mathrm{d}f}\cdot\frac{f_\mathrm{T}}{\omega_1} = \frac{L}{4f_\mathrm{T}} \times \frac{2g_2 - \frac{1}{g_1} + \frac{1}{g_2}\left(\frac{l_2}{l_1}\right)^2 + 2\left(\frac{l_2}{l_1}\right)}{1-g_1g_2}. \tag{43}$$

Substituting Eq.(36) into Eq.(42), we obtain the relation of g_1 and g_2:

$$\frac{1}{g_1} = 2g_2 + \frac{1}{g_2}\left(\frac{l_2}{l_1}\right)^2 + 2\frac{l_2}{l_1}. \tag{44}$$

For end-pumped lasers, from Eqs.(24), (25), and (44), a new formula is obtained:

$$2g_2 - \frac{1}{g_1} = 0,$$

namely

$$g_1g_2 = \frac{1}{2}. \tag{45}$$

For practical use, it is important that the derivative of ω_1 versus f not only vanishes for a fixed average value f, but is insensitive to variations of the mean volume V, and of the mirror separation l_1. The derivative was therefore studied in the vicinity of the zero line. For end-pumped lasers, l_2=0, Eq.(44) then becomes

$$\frac{\mathrm{d}\omega_1}{\mathrm{d}f}\cdot\frac{f_\mathrm{T}}{\omega_1} = \frac{1}{4}\frac{l_1}{f_\mathrm{T}}\frac{2g_1g_2}{g_1(1-g_1g_2)}. \tag{46}$$

The derivative increases with increasing mirror separation l_1 and decreases with increasing mean volume V.

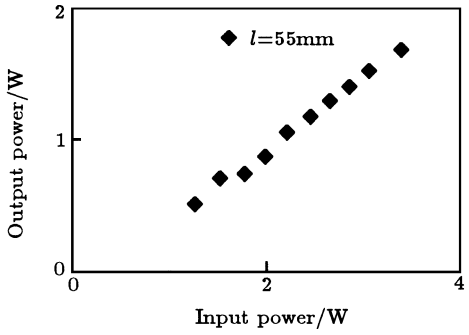


Fig.5. Output power versus input power.

Figure 5 shows the relation of output and input power. From Fig.5, output power increases with input power increasing. The output mirror is plated by film with reflection of 95%. TEM_{00} is obtained (M_x^2=1.08, M_y^2=1.13). The largest output power is 1.68W. Optic–optic conversion is 48.6%. Slope efficiency is 56.3%. Optimized resonator length is 55mm, which is calculated by Eqs.(32) and (47).

4. Conclusions

In this paper, the focal length of thermal lens is calculated. We design the resonator of end-pumped lasers, and obtain the resonator parameter equation which illuminates that the resonator is insensitive to thermal focal length and length of resonator. The largest output power (1.68W) is achieved, optic–optic conversion is 48.6% and slope efficiency is 56.3%.

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