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Isospin and F-spin symmetry structure in low-lying levels of ^{48,50}Cr isotopes*

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The low-energy level structure and electromagnetic transitions of ^{48,50}Cr nuclei have been studied using the interacting boson model with isospin (IBM-3). A sequence of isospin excitation bands with isospin $T = T_z$, $T_z + 1$ and $T_z + 2$ has been assigned, and compared with available data. According to this study, the 2_3^+ and 2_2^+ states are the lowest mixed symmetry states in ^{48}Cr and ^{50}Cr, respectively. In particular, the present calculations suggest that a combination of isospin and F-spin excitation can explain the structure in these nuclei. The transition probabilities between the levels are analysed in terms of isoscalar and isovector decompositions which reveal the detailed nature of the energy levels. The results obtained are in good agreement with recent experimental data.

Keywords: IBM-3, isospin, mixed symmetry states, ^{48,50}Cr isotopes
PACC: 2160F, 2740, 2100H

1. Introduction

The interacting boson model (IBM)[1−3] is capable of giving a simple yet realistic description of nuclear collective motions. In its original version (IBM-1), only one kind of boson is considered, corresponding to fully proton neutron symmetric states. The neutron–proton extension of the model (IBM-2), predicts a new class of states[4] having mixed symmetry in the proton and neutron degrees of freedom, and it has been successfully observed in various experiments.[5−7] The interacting boson model has been successful in describing various properties of medium and heavy nuclei.[8−11]

In lighter nuclei, the valence protons and neutrons fill the same major shell and isospin should be taken into account. In order to include the isospin, IBM has been extended to the interacting boson model with isospin (IBM-3).[12] In IBM-3 three types of bosons are included: proton–proton (π), neutron–neutron (ν) and proton–neutron (δ), which forms the isospin $T = 1$ multiplet. The ν, δ and π bosons have the isospin projections $M_T = -1$, 0 and 1, respectively. The wavefunctions can be classified by the $U_c(3) \supset SU(2)_T$ group,[12] where $SU(2)_T$ is the usual isospin group. Dynamical symmetries of the IBM3 have been studied in Refs.[13–18]. The $U_{sd}(6)$ of IBM-1 goes to the $U(18)$ group for IBM-3 as its dynamical symmetry group. The natural chains below $U(18)$ start with $U_{sd}(6) \times U_c(3)$, and they must contain $O(3)$ and $SU_T(2)$ as subgroups because the angular momentum and the isospin are good quantum numbers. The $U_c(3)$ charge symmetry was described in detail in Ref.[19]. The chains beginning with $U_{sd}(6) \times U_c(3)$ and satisfying the above requirement are the following:[13]

$$\begin{aligned} &U(18) \supset (U_c(3) \supset SU_T(2)) \times (U_{sd}(6) \supset U_d(5) \supset O_d(5) \supset O_d(3)), \\ &U(18) \supset (U_c(3) \supset SU_T(2)) \times (U_{sd}(6) \supset O_{sd}(6) \supset O_d(5) \supset O_d(3)), \\ &U(18) \supset (U_c(3) \supset SU_T(2)) \times (U_{sd}(6) \supset SU_{sd}(3) \supset O_d(3)). \end{aligned} \tag{1}$$

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The subgroups $U_d(5)$, $O_{sd}(6)$ and $SU_{sd}(3)$ describe vibrational, γ- unstable and rotational nuclei, respectively.[20] The existence of the isospin excitations in light nuclei has aroused interest in the last few years.[21−25] Such renewed interest was sparked by discovery of states with isospin $T > T_z(T_z =\mid Z-N \mid /2)$. The Cr isotopes have been the subject of many theoretical and experimental investigations. On the experimental side, large γ- ray detector arrays are now available, and they have increased the detection sensitivity by orders of magnitude. Meanwhile, shell model calculations in the full fp shell can now be performed. Detailed description of the yrast band structure in the full fp space has been reported in ^{48}Cr[26] and ^{50}Cr.[27] Experimentally, the positive parity yrast band was extended up to the band termination and turned out to be in good agreement with the shell model prediction.[28,29] A detailed analysis of backbending mechanism of ^{48}Cr has been performed using the projected shell model.[30] In ^{50}Cr the first backbending at $J^\pi = 10^+$ and second backbending at $J^\pi = 18^+$ have been observed experimentally.[31] The high spin states and electromagnetic transitions have been investigated and compared with the shell model results.[32] Recently, microscopic three-cluster model calculations were also performed in an attempt to explain the band structure properties.[33] A systematic analysis of ^{48}Cr in the framework of the IBM-3 has been performed in Ref.[19]. However, a comprehensive analysis of the low-lying levels in these nuclei is still missing. It is the purpose of this paper to study the low-lying levels in the IBM-3, in particular to investigate the isospin and F-spin contents of the levels. This paper is arranged as follows. In Sec.2, we briefly discuss the interacting boson model with isospin. In Sec.3, we present the results of our calculation for the energy levels and compared with available data, and analyse the isospin and F-spin structure of the results. A discussion of electromagnetic transitions follows in Sec.4. Finally, in Sec.5 we summarize our results.

2. The IBM-3 Hamiltonian

The most general IBM-3 Hamiltonian can be written as

$$H = \epsilon_s \hat{n}_s + \epsilon_d \hat{n}_d + H_2, \tag{2}$$

where

$$\begin{aligned} H_2 =& \frac{1}{2}\sum_{L_2T_2} C_{L_2T_2}((d^\dagger d^\dagger)^{L_2T_2}.(\tilde{d}\tilde{d})^{L_2T_2}) \\ &+ \frac{1}{2}\sum_{T_2} B_{0T_2}((s^\dagger s^\dagger)^{0T_2}.(\tilde{s}\tilde{s})^{0T_2}) \\ &+ \sum_{T_2} A_{2T_2}((s^\dagger d^\dagger)^{2T_2}.(\tilde{d}\tilde{s})^{2T_2}) \\ &+ \frac{1}{\sqrt{2}}\sum_{T_2} D_{2T_2}((s^\dagger d^\dagger)^{2T_2}.(\tilde{d}\tilde{d})^{2T_2}) \\ &+ \frac{1}{2}\sum_{T_2} G_{0T_2}((s^\dagger s^\dagger)^{0T_2}.(\tilde{d}\tilde{d})^{0T_2}), \end{aligned} \tag{3}$$

and

$$\begin{aligned} &(b_1^\dagger b_2^\dagger)^{L_2T_2}.(\tilde{b_3}\tilde{b_4})^{L_2T_2} \\ =&(-1)^{(L_2+T_2)}\sqrt{(2L_2+1)(2T_2+1)} \\ &\times[(b_1^\dagger b_2^\dagger)^{L_2T_2}\times(\tilde{b_3}\tilde{b_4})^{L_2T_2}]^{00} \end{aligned} \tag{4}$$

is the dot product in both angular momentum and isospin. The tilted quantity is defined as

$$\tilde{b}_{lm,m_z} = (-1)^{(l+m+1+m_z)} b_{l-m-m_z}. \tag{5}$$

The symbols T_2 and L_2 represent the two-boson system isospin and angular momentum. The parameters A, B, C, D and G are related to the two-body matrix elements by $A_{T_2} = \langle sd20 \mid H_2 \mid sd20\rangle$, with $T_2 = 0,1,2$, $B_{T_2} = \langle s^2 0T_2 \mid H_2 \mid s^2 0T_2\rangle$, $G_{T_2} = \langle s^2 0T_2 \mid H_2 \mid d^2 0T_2\rangle$, $D_{T_2} = \langle sd2T_2 \mid H_2 \mid d^2 2T_2\rangle$ and $C_{L_2T_2} = \langle d^2 L_2T_2 \mid H_2 \mid d^2 L_2T_2\rangle$, with $T_2 = 0,2$ and $L_2 = 0,2,4$ and by $C_{L_21} = \langle d^2 L_21 \mid H_2 \mid d^2 L_21\rangle$ with $L_2 = 1,3$. The parameters A_1, C_{11} and C_{31} are similar to the Majorana interactions in the IBM-2 which and will be referred to also as Majorana interactions. These interactions are important to shift the states with mixed symmetry with respect to the total symmetric ones. Since only a little experimental information is known about such states in the nuclei under study, we attempt to vary the parameters appearing in these terms to fit the energy of available experimental data on 1^+ and other states which are sensitive to these parameters.

Microscopic studies on the IBM-3 parameters[34,35] shows that the IBM-3 Hamiltonian not only depends on the boson number, but also depends on isospin value. The dependence on isospin is more dramatic than on the boson number. The best fit to the whole spectrum can be found. The normal product form of the Hamiltonian 1 and the Casimir operators of IBM-3 limits are related. They can be given, after

some tedious algebra. We have rewritten the Hamiltonian in terms of linear combination of Casimir operators which is convenient for analysis of the dynamical symmetry. The expressions of the Casimir operators can be found in Ref.[13]. In Casimir operator form, the Hamiltonians are

$$\begin{aligned} H_{48} =& \lambda C_{2U_{sd}(6)} + 1.460T(T+1) + 0.030C_{1U_d(5)} \\ & - 0.091C_{2SU_{sd}(3)} + 0.025C_{2U_d(5)} \\ & + 0.173C_{2O_d(5)} + 0.01C_{O_d(3)}, \end{aligned} \tag{6}$$

$$\begin{aligned} H_{50} =& \lambda C_{2U_{sd}(6)} + 1.322T(T+1) + 0.780C_{1U_d(5)} \\ & - 0.119C_{2SU_{sd}(3)} + 0.030C_{2U_d(5)} \\ & + 0.091C_{2O_d(5)} + 0.002C_{O_d(3)}, \end{aligned} \tag{7}$$

for ^{48,50}Cr isotopes, respectively. The corresponding parameters in the form of Eq.(3) are also given in Table 1. From comparison of this table with the figures of Ref.[35], with consideration of the contribution of the $-\chi a_0 \hat{N}_s \hat{N}_d$ term with $\chi = (N-T)(N+T+1)/3N(N-1)$ interactions in the Hamiltonian, we found that our Hamiltonian is consistent with this microscopic study. The dependence on N and T is considered in the Hamiltonian parameters.

Table 1. Parameters of the IBM-3 hamiltonian used for the description of the Cr-isotopes.

Nucleus	^{48}Cr	^{50}Cr
$\epsilon_{s\nu} = \epsilon_{s\pi}$	1.830	1.274
$\epsilon_{d\nu} = \epsilon_{d\pi}$	2.737	2.58
$A_i (i = 0, 1, 2)$	−6.264,−2.496,2.496	−5.824,−2.108,2.108
$C_{i0} (i = 0, 2, 4)$	−7.172,−5.018,−6.152	−5.802,−4.404,−6.042
$C_{i2} (i = 0, 2, 4)$	1.588,3.742,2.608	2.130,3.528,1.890
$C_{i1} (i = 1, 3)$	−2.993,−2.892	−2.370,−2.350
$B_i (i = 0, 2)$	−5.900,2.860	−5.348,2.584
$D_i (i = 0, 2)$	0.681,0.681	0.890,0.890
$G_i (i = 0, 2)$	−0.814,−0.814	−1.064,−1.064

The λ determines the position of the mixed symmetry states. In ^{50}Cr, there is one 1^+ state at 3.629MeV, and this requires $\lambda = -0.03$MeV. We then use the same value for the ^{48}Cr isotope because no experimental information is available for this isotope. It can be seen from the Casimir operator form that the Hamiltonian of ^{48}Cr is more rotational than that of ^{50}Cr because the coefficient of $C_{U_d(5)}$ in ^{48}Cr is much less than that in ^{50}Cr. In ^{50}Cr the coefficient of $C_{1U_d(5)}$ is very large, and this indicates that it is closer to the $U(5)$ limit and is in transition from $U(5)$ to $SU(3)$.

3. Isospin and F-spin symmetry structure

We assume ^{56}Ni as the closed core and the bosons in the current study corresponding to pairs of hole fermions. The energy levels are shown in Figs.1 and 2. Good agreement exists in the ground state band between the calculated and observed spectrum. In other bands a one-to-one correspondence between calculation and the experiment is established for those levels that have been experimentally observed. The energy levels and wavefunctions are obtained using a computer program written by Van Isacker.[36] It is very interesting to see that the sequence of bands with $T = T_z$, $T_z + 1$ and $T_z + 2$ have been produced, and they correspond to available experimental data in ^{48}Cr, while in ^{50}Cr the experimental data are more densely distributed than the calculation. The β- band with $T = T_z$ is the first and second excited band in ^{48}Cr and ^{50}Cr, respectively. In Fig.1 we also draw the ground state band of ^{48}Ti where the lowest isospin is $T = 2$ which should be close to the isospin analogue state in ^{48}Cr. It is worth pointing out that the ^{48}Ti spectrum is calculated with the same Hamiltonian as that of ^{48}Cr. Here we have assumed that the ground state energy of ^{48} Ti is equal to that of the IBM-3 calculated $0^+_{T=2}$ state in ^{48}Cr. This is supported by the following estimate. We estimate the energy of the isospin analogue state in ^{48}Cr by considering the binding energy difference of ^{48}Cr and ^{48}Ti and then subtracting the Coulomb energy difference. This estimation is crude because Coulomb energy depends on the shape of the nucleus sensitively. By using the tables in Ref.[37] and the following Coulomb energy formula:

$$E_{\text{Coulmob}} = 0.70\frac{Z^2}{A^{1/3}}(1 - 0.76Z^{-2/3}), \tag{8}$$

we obtained the energy of the $T = 2$ isospin analogue state in ^{48}Cr to be 8.350MeV which is close to the energy of the $0^+_{T=2} = 8.760$MeV in our IBM-3 calculation.

From the comparison between the IBM-3 results and experiment data, the following points merit special attention. In ^{48}Cr, the $3^+_1(T = 0)$ state appears at 6.188MeV in our calculation, and it is not yet seen in experiment. In ^{50}Cr the IBM-3 predicts the first 3^+ at 3.823MeV and the second 3^+ at 5.921MeV. These states are both from the $[N-1, 1]$ $U(6)$ irreducible

representation. Here we have the $U(6)$ labelling as it is a good quantum number approximately. The first and second $J = 4^+$ in ^{48,50}Cr are in good agreement with experimental data, and 4_3^+ state in ^{48}Cr and 4_2^+ state in ^{50}Cr are mixed symmetry states. In both nuclei the first $J = 5^+$ state is a solely mixed symmetry state.

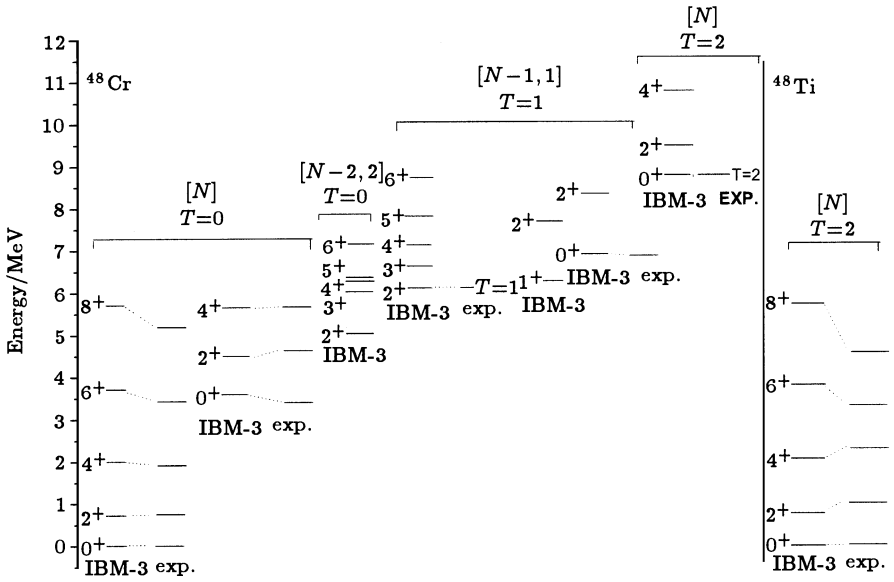


Fig.1. Comparison between lowest excitation energy bands ($T = T_z, T_z + 1$ and $T_z + 2$) of the IBM-3 calculation and experimental excitation energies of ^{48}Cr. The experimental data are taken from Ref.[45].

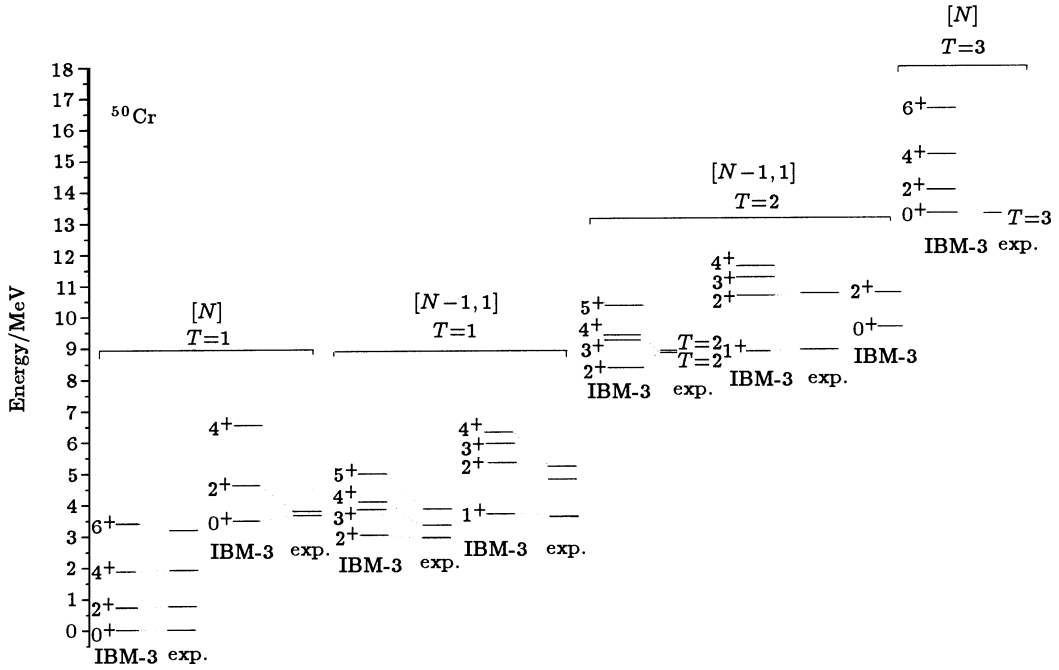


Fig.2. Comparison between lowest excitation energy bands ($T = T_z, T_z + 1$ and $T_z + 2$) of the IBM-3 calculation and experimental excitation energies of ^{50}Cr. The experimental data are taken from Ref.[45].

To identify the mixed symmetry states, one can make use of their electromagnetic transition properties: weakly $E2$ and strongly $M1$ decay to ground state and first 2^+ state respectively.[38] The mixed symmetry $J = 2^+$ state in light nuclei has been identified in ^{54,56}Cr, ^{56,58}Fe,[39] ^{56}Fe[40,41] and $^{64-68}$Ge,

$^{60-66}$Zn.[42] The mixed symmetry structure of wavefunctions can be seen by calculating the $\langle J \mid C_{2U(6)} \mid J\rangle$ value. The calculated 2_3^+ and 2_2^+ in ^{48}Cr and ^{50}Cr exhibit $[N-1,1]$ $U(6)$ partition, indicating that those states are the lowest mixed symmetry states in ^{48,50}Cr, respectively. For ^{48}Cr($T_z=0$), the lowest mixed symmetry state comes from $[N-2,2]$ because $[N-1,1]$ does not contain $T=0$, while for ^{50}Cr it comes from $[N-1,1]$. From Figs.1 and 2 one can see this fact and therefore the lowest mixed symmetry state in ^{48}Cr has high energy.

The IBM-3 analysis gives a 1_1^+ level at 6.218MeV with partition $[N-1,1]$, and it is higher than the lowest mixed symmetry 2^+ state. No experimental evidence for this level is available. The energy of the first 1^+ in ^{50}Cr is equal to 3.613MeV, and it is quite close to the experimental one. A possible candidate for 1_2^+ mixed symmetry has also been identified at 7.876MeV; it is close to the observed state $J=(1,2)$ at 7.646MeV. The energy of these observed states are well reproduced by the calculation. We have varied each of the Majorana parameters around the best-value while keeping all other parameters at their best-fit values; the variations of the energy of these states with the parameters are shown in Figs.3–6. In ^{48}Cr the mixed symmetry component of the lowest mixed symmetry band belongs predominantly to the partition $[N-2,2]$ with $T=T_z$. The second mixed symmetry state 2^+ has the partition $[N-1,1]$ with $T=T_z+1=1$ with energy 6.110MeV; it is very close to experimental level at 6.100MeV ($T=1$).[45] Starting with this mixed state, a whole band of mixed states is predicted by this IBM-3 calculation. At the same time the lowest mixed symmetry bands in ^{50}Cr have the partition $[N-1,1]$ with $T=T_z,T_z+1$ as shown in Fig.2. Because IBM-3 has three charge states, it is possible to construct $U(6)$ partitions into three rows, namely the $[N_1,N_2,N_3]$ states which are characteristic of IBM-3. We found that such state comes high in energy, upwards at about 8.5MeV, the lowest example being a scissor mode state in ^{48}Cr at 8.599MeV which is predominantly the $[2,1,1]$ partition with $T=1$. The present calculation predicts that it decays to 2_3^+ by strong $M1$ transition with $B(M1)=0.24\mu_N^2$ and weak $E2$ transition with $B(E2)=0.0013e^2b^2$. It would be very significant if these properties are observed in experiment.

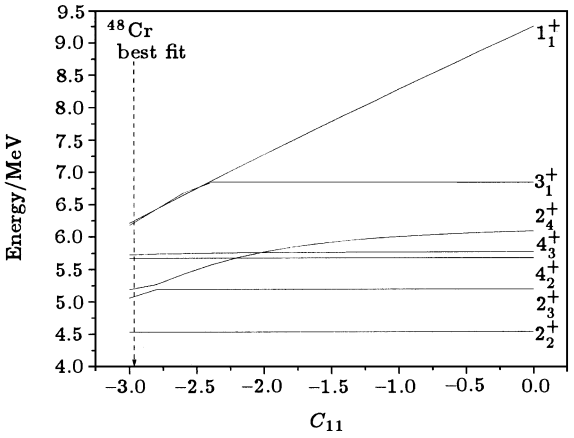


Fig.3. Variation in level energy of ^{48}Cr as a function of C_{11}; all other parameters were kept at their best-fit values.

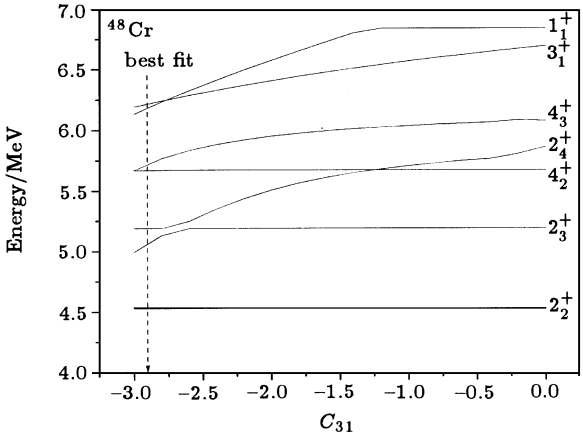


Fig.4. Variation in level energy of ^{48}Cr as a function of C_{31}; all the other parameters were kept at their best-fit values.

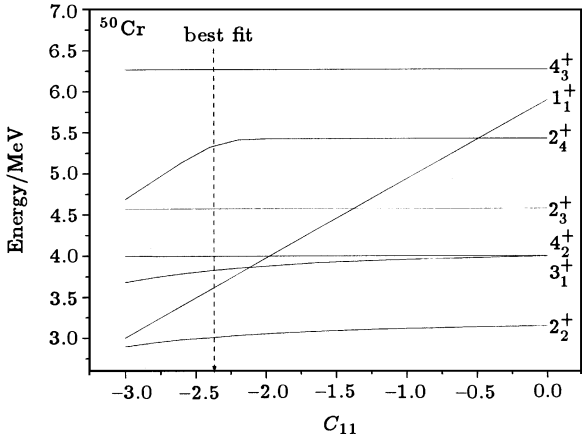


Fig.5. Variation in level energy of ^{50}Cr as a function of C_{11}, all other parameters being kept at their best-fit values.

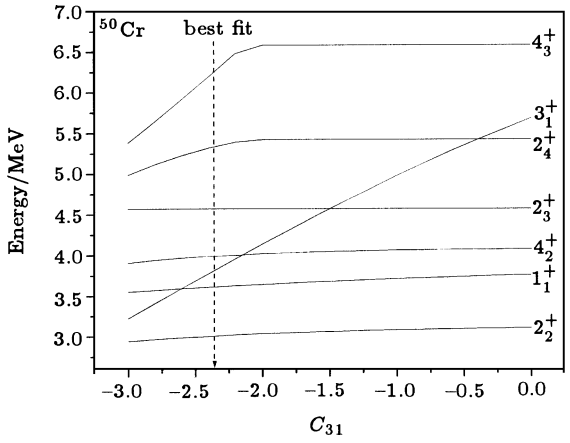


Fig.6. Variation in level energy of ^{50}Cr has a function of C_{31}, all the other parameters being kept at their best-fit values.

We now compare the present results with those by Garcia-Ramos and Isacker,[19] which have been one of the most successful IBM-3 calculations in this region. Low-lying levels in ^{48}Cr nucleus are determined by using a simplified Hamiltonian. Up to E_x=4MeV the two results are similar except for the 0_2^+ state, where our result is close to the experiment one at 3.42MeV. The Garcia-Ramos and Isacker result did not produce the 2^+ (T=1) state, which has been observed at 6.1MeV (see Fig.4 of Ref.[19]). At the same time the energies of 1^+ states are in good agreement with the result of Abdelaziz *et al*[43] (see Table 5 of Ref.[43]). It is demonstrated that the $T = T_z, T_z + 1, T_z + 2$ states are well reproduced.

4. Electromagnetic transition

The $E2$ transition is described by the following isoscalar and isovector one-boson $E2$ operators $Q = Q^0 + Q^1$:[42]

$$Q^0 = \alpha_0\sqrt{3}[(s^+\hat{d})^{20} + (d^+\hat{s})^{20}] + \beta_0\sqrt{3}[(d^+\hat{d})^{20}, \quad (9)$$

$$Q^1 = \alpha_1\sqrt{2}[(s^+\hat{d})^{21} + (d^+\hat{s})^{21}] + \beta_1\sqrt{2}[(d^+\hat{d})]^{21}. \quad (10)$$

The $M1$ transition is also a one-boson operator with an isoscalar part and an isovector part $M = M^0 + M^1$,[42]

$$M^0 = g_0\sqrt{3}(d^+\hat{d})^{10} = g_0 L/\sqrt{10}, \quad (11)$$

$$M^1 = g_1\sqrt{2}(d^+\hat{d})^{11}, \quad (12)$$

where g_1 and g_0 are the isovector and isoscalar g-factors, respectively, and L is the angular momentum operator.

Having obtained the wavefunction for states, we can calculate the electromagnetic transition rate between states using the subroutine (BEM3TR). In order to link it with energy code, we have made some modification. The parameters in the $E2$ operator are adjusted to fit the experimental data of $B(E2; 2_1^+ \to 0_1^+)$, where $\alpha_0 = \beta_0 = 0.1$ and 0.13 for ^{48}Cr and ^{50}Cr, respectively and $\alpha_1 = \beta_1 = 0.1$ for both isotopes. The results are summarized in Tables 2 and 3.

Table 2. Experimental[32,45] and calculated $B(E2)$ (e^2B^2), calculated $B(M1)$ (μ_N^2) and mixing ratio for ^{48}Cr isotope.

	$B(E2)$		$B(M1)$	δ
$J_i^+ \to J_f^+$	Exp.	Calc.	Calc.	Calc.
$2_1^+ \to 0_1^+$	0.0320(41)	0.0309		
$2_2^+ \to 0_1^+$		0.0067		
$2_2^+ \to 2_1^+$		0.0007	0.0000	
$2_2^+ \to 0_2^+$		0.0101		
$1_1^+ \to 0_1^+$			0.6437	
$1_1^+ \to 0_2^+$			0.2362	
$1_1^+ \to 2_1^+$		0.0077	0.3923	0.3141
$1_1^+ \to 2_2^+$		0.0091	0.2103	−0.1434
$1_1^+ \to 2_3^+$		0.0016	0.1365	−0.0517
$2_2^+ \to 4_1^+$		0.0166		
$4_1^+ \to 2_1^+$	0.0329(110)	0.0383		
$4_2^+ \to 2_1^+$		0.0012		
$6_1^+ \to 4_1^+$	0.0301(78)	0.0345		

Table 3. Experimental[32,44,45] and calculated $B(E2)$ (in unit e^2b^2), $B(M1)$ (in unit μ_N^2) and mixing ratio for ^{50}Cr isotope.

	$B(E2)$		$B(M1)$		δ	
$J_i^+ \to J_f^+$	Exp.	Calc.	Exp.	Calc.	Exp.	Calc.
$2_1^+ \to 0_1^+$	0.0217(25)	0.0247				
$2_2^+ \to 2_1^+$		0.0072	0.3938(716)	0.6319	$0.03^{+0.06}_{-0.04}$	0.1383
$2_2^+ \to 0_1^+$	0.0023(8)	0.0062				
$0_2^+ \to 2_2^+$		0.0385				
$2_3^+ \to 0_1^+$		0.0109				
$2_3^+ \to 2_1^+$		0.0209		0.0000		
$2_3^+ \to 2_2^+$		0.0013		0.0238		0.1291
$1_1^+ \to 0_1^+$			0.3300(400)	0.2778		
$1_1^+ \to 0_2^+$				0.378		
$1_1^+ \to 2_1^+$		0.0072		0.2225		0.2113
$1_1^+ \to 2_2^+$		0.0318		0.0013		−1.2259
$1_1^+ \to 2_3^+$		0.0086		0.6559		−0.0447
$1_2^+ \to 2_1^+$		0.0002		0.0026		− 0.2118
$1_2^+ \to 2_2^+$		0.0048		0.0012		−4.0168
$3_1^+ \to 2_1^+$		0.0177		0.1421		−0.4452
$3_1^+ \to 4_1^+$		0.0031		0.4741		−0.0636
$4_1^+ \to 2_1^+$	0.0204(25)	0.0256				
$4_2^+ \to 4_1^+$		0.0110	0.1325(375)	0.4885	$-0.02^{+0.16}_{-0.52}$	− 0.1281
$5_1^+ \to 3_1^+$		0.0031				
$5_1^+ \to 4_1^+$	0.0001	0.0108		0.0671	−0.47(16)	−0.5005
$6_1^+ \to 4_1^+$	0.0235(47)	0.0165				

The $B(E2)$ values between the yrast states are well reproduced. The calculated $B(E2; 4_1^+ \to 2_1^+)$ and $B(E2; 6_1^+ \to 4_1^+)$ are quite close to the experimental values, at the same time the calculated $B(E2; 2_3^+ \to 0_1^+)$ in ^{50}Cr is larger than the experimental by about $0.004e^2b^2$. From these tables one can see that the transitions between states with the same partition are larger than transition between states with different $U(6)$ labels.

The isovector part $M1$ of the transition from 1^+ and higher energy 2^+ mixed symmetry states to ground state and first 2^+ state respectively is analysed. The result is consistent with the identification of the 1^+ and the lowest 2_{ms}^+ in the these two nuclei. From the $M1$ operator, the isoscalar part is simply the angular momentum; $M1$ transition is determined by g_1 part only. For a good reproduction of calculated $M1$ transition we used the $g_1 = -2.8\mu_N$ for the ^{48}Cr and ^{50}Cr. They lead to pure isovecter $M1$ transitions. In ^{50}Cr isotope the calculated $B(M1; 1_1^+ \to 0_1^+)$ and $B(M1; 4_2^+ \to 4_1^+)$ values are in good agreement with the experimental data. We obtain $B(M1; 2_2^+ \to 2_1^+)$=0.6319 μ_N^2 which compares well with the 0.3938 μ_N^2 value observed experimentally. The $2_2^+ \to 2_1^+$ decay with its $E2/M1$ mixing ratio of δ=0.03 found experimentally is nearly pure $M1$ transition.

In ^{50}Cr the calculated 1^+ state at 8.901MeV decays to ground state with reduced $M1$ transition 0.100 μ_N^2 and is in good agreement with the experimental data of $0.0823(89)\mu_N^2$ reported in Ref.[45] The relative $B(E2)$ transitions between the states $J = 0^+, 1^+$ and 2^+ having different isospin values are calculated and summarized in Figs.7 and 8, respectively.

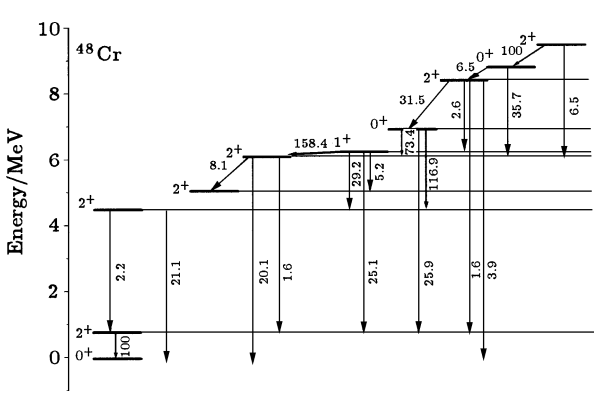


Fig.7. Energy levels and relative $B(E2)$ for ^{48}Cr isotope.

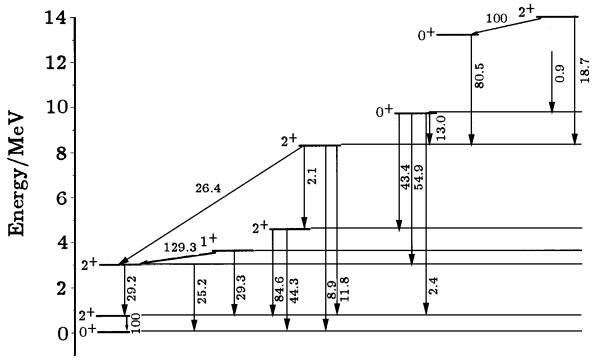


Fig.8. Energy levels and relative $B(E2)$ for ^{50}Cr isotope.

We have also calculated the magnetic dipole moments for the ground state band in ^{50}Cr. The parameters used in the calculation are g_0=1.0 μ_N and $g_1 = -2.8\mu_N$. To show the sensitivity of the magnetic dipole moments to the g_0 parameter, we fixed g_1 which is determined by the $M1$ transition and varied g_0. The result is shown in Fig.9 where the best fit appears at g_0=1.0μ_N.

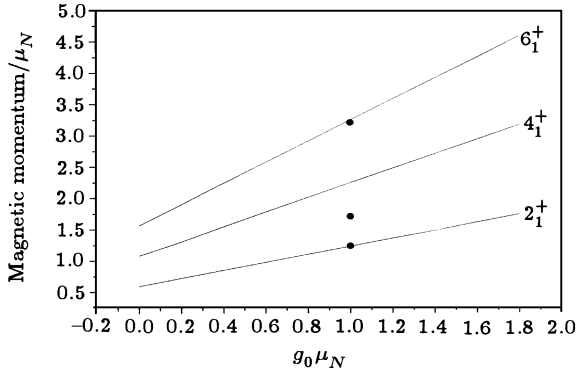


Fig.9. Magnetic momentum of ^{50}Cr as a function of g_0, g_1 being kept at best-fit values =−2.8. The experimental data (denoted by •) are taken from Ref.[45].

5. Conclusion

Summarizing our results we may conclude that the IBM-3 description of the low-lying levels in the ^{48}Cr and ^{50}Cr nuclei is satisfactory. We have shown, on the basis of energy levels and electromagnetic properties, the presence of mixed symmetry states near 5 and 3MeV in ^{48}Cr and ^{50}Cr, respectively. In ^{48}Cr the 2_3^+ state at 5.069MeV decays predominantly to the 2_1^+ state via a pure $M1$ transition, while in ^{50}Cr the 2_2^+ state decays strongly through $M1$ and weakly through $E2$ transition decay to 2_1^+. Theses states are the candidates of the lowest mixed symmetry states in ^{48,50}Cr, respectively. In our calculation the scissor mode 1^+ states in these nuclei lie higher than the lowest mixed symmetry 2^+ states, which were observed at 3.629MeV in ^{50}Cr. No mixed symmetry state labelled with $[N-1,1]$ and $T=0$ exists in ^{48}Cr, and this is a natural result of group reduction. As a result, the scissor state in ^{48}Cr nucleus is higher than that in ^{50}Cr.

The calculated results are in good agreement with available experimental data, but more experiment data for those nuclei are needed to validate this predicted nuclear structure. In particular, the present calculations suggest a combined isospin and mixed symmetry excitation in the low-lying levels of these nuclei. The present calculation also predicts the existence of row-rowed partition state at about 9Mev, and this type of state is typical of IBM-3, not present in IBM-1 and IBM-2. It will be highly desirable to substantiate these predictions in future experiment.

References

[1] Arima A and Iachello F 1976 *Ann. Phys.* (N.Y.) **99** 253
[2] Arima A and Iachello F 1978 *Ann. Phys.* (N.Y.) **111** 201
[3] Arima A and Iachello F 1979 *Ann. Phys.* (N.Y.) **123** 468
[4] Iachello F and Arima A 1987 *The Interacting Boson Model* (Cambridge: Cambridge University Press)
[5] Hamilton W D, Irbäck A and Elliott J 1984 *Phys. Rev. Lett.* **53** 2469
[6] Fazekas B *et al* 1992 *Nucl. Phys.* A **548** 249
[7] Wiedenhover I *et al* 1997 *Phys. Rev.* C **56** R2354
[8] Liu F Y *et al* 2001 *Chin. Phys.* **10** 194
[9] Shi Z Y and Ji S Y 2003 *Acta Phys. Sin.* **52** 42 (in Chinese)
[10] Di Y M 1998 *J. Ningxia University (Natural Science Edition)* **19** 340
[11] Li J S, Liu Y X and Gao M 2002 *Sci. Chin.* A **32** 1074
[12] Elliott J P and White A P 1980 *Phys. Lett* B **97**169
[13] Long G L 1994 *Chin. J. Nucl. Phys.* **16** 331
[14] Ginocchio J N 1996 *Phys. Rev. Lett.* **77** 28
[15] Kota V K B 1997 *Phys. Lett.* B **399** 185
[16] Lac V S, Elliott J P and Evans J A 1997 *Phys. Lett.* B **394** 231
[17] Long G L, Gan C Y and Shen T Y 1997 *Commun. Theor. Phys.* **27** 317
[18] Kota V K B 1998 *Ann. Phys.* **265** 101
[19] Garcia-Ramos J E and Van Isacker P 1999 *Ann. Phys.* **274** 45
[20] Lipas P O 1984 *International Review of Nucl. Phys.* **Vol. 2** (Edited by T. England, J. Rekestad and J-S Vadgen)
[21] Langanke K, Vogel P and Zheng D C 1997 *Nucl. Phys.* A **626** 735
[22] Frauendorf F and Sheikh J 1999 *Nucl. Phys.* A **645** 509
[23] Devi Y D, Robinson S and Zamick L 2000 *Phys. Rev.* C **61** 037305
[24] Fischer S M *et al* 2002 *Phys. Rev. Lett.* **87** 132501

[25] Von Brentano P Frießner *et al* 2002 *Nucl. Phys.* A **704** 15c
[26] Caurier E *et al* 1995 *Phys. Rev. Lett.* **75** 2466
[27] Martinez-Pinedo *et al* 1996 *Phys. Rev.* C **54** R2150
[28] Cameron J A *et al* 1998 *Phys. Rev.* C **58** 808
[29] Langanke K 1998 *Phys. Lett.* B **438** 235
[30] Hara K, Sun Y and Mizusaki T 1999 *Phys. Rev. Lett.* **83** 1922
[31] Lenzi S M *et al* 1997 *Phys. Rev.* C **56** 1313
[32] Hasegawa M, Kaneko K and Tazaki S 2000 *Nucl. Phys.* A **674** 411
[33] Descouvemont P 2002 *Nucl. Phys.* A **709** 275
[34] Evans J A, Long G L and Elliott J P 1993 *Nucl. Phys.* A **561** 201
[35] Evans J A *et al* 1995 *Nucl. Phys.* A **593** 85
[36] Audi G and Wapstra A H 1993 *Nucl. Phys.* A **565** 1
[37] Van Isacker P *et al* 1986 *Ann. Phys.* (N. Y.) **171** 253
[38] Nakada H and Otsuka T 1997 *Phys. Rev.* C **55** 2418
[39] Eid S A A A, Hamilton W D and Elliott J P 1986 *Phys. Lett.* B **166** 267
[40] Nakada H, Otsuka T and Sebe T 1991 *Phys. Rev. Lett.* **67** 1086
[41] Elliott J P *et al* 1996 *Nucl. Phys.* A **609** 1
[42] Elliott J P, Evans J A and Lac V S 1996 *Nucl. Phys.* A **597** 341
[43] Abdelaziz M *et al* 1989 *Nucl. Phys.* A **503** 452
[44] Lang J *et al* 1982 *Rev. Mod. Phys.* **54** 119
[45] Firestone R B 1998 *Table of Isotopes* 8th Edition