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Basic properties of an rf SQUID involving two Josephson junctions connected in series*

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We have studied the basic characteristics of a radio frequency superconducting quantum interference device (rf SQUID) involving two Josephson junctions connected in series, the case for the widely used grain boundary junction (GBJ) rf SQUID. It is found that the SQUID properties are determined mainly by the weaker junction when the critical current of the weaker junction is much lower than that of the other junction. Otherwise, the effect of the other junction is not negligible. We also find that only when the hysteresis parameter β is less than $1-\alpha$, where α is the critical current ratio of the two junctions, will the SQUID operate in the nonhysteretic mode.

Keywords: rf SQUID, grain boundary junction, hysteretic and nonhysteretic modes
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1. Introduction

Normally a radio frequency superconducting quantum interference device (rf SQUID) involves one Josephson junction integrated into a superconducting loop which is coupled to an LC-resonant circuit. Its basic properties have been studied in detail.[1−3] In the case of high temperature superconductor (HTS) rf SQUIDs fabricated with grain boundary junctions (GBJs), however, not one junction but two junctions connected in series are usually involved in the superconducting loop. Nowadays, mainly two types of high-T_c Josephson junctions are used in HTS SQUIDs: the step edge GBJ and the bicrystal GBJ.[4]

The step edge GBJ has been more often used in HTS rf SQUIDs because of its layout design flexibility. It was reported early on that actually two GBJs, the upper GBJ and the lower GBJ in series, are formed on the step edges.[5] For a large step angle ($> 70°$), the upper junction has a higher critical current I_c than the lower junction, typically by a factor of 3–5 or more.[6] Therefore, it is estimated that the step edge junction properties are solely defined by the lower junction. Although much experimental work has been done to investigate the microstructure of the step and the characteristics of the step edge GBJ rf SQUIDs,[7−9] the basic properties of such two-junction rf SQUIDs remain unclear due to the lack of corresponding theory. It is necessary to study how one more junction will influence the whole behaviour of a step edge GBJ rf SQUID.

The bicrystal GBJ is less widely used in the fabrication of HTS rf SQUIDs due to the extension of the grain boundary across the bicrystal substrate. In this case, a normal junction and a long junction connected in series are formed in the loop. The long junction will result in unwanted flux motion and low-frequency $1/f$ noise.[10] Recently, Fardmanesh *et al*[11] directly reduced the width of the long junction and made the bicrystal GBJ rf SQUID magnetometer using the layout design with about 4/1 asymmetric junction width ratio. Theoretic work is also needed here to clarify the basics of the bicrystal GBJ rf SQUID for its future application.

In this paper, we study the behaviour of an rf SQUID involving two Josephson junctions connected in series. Important results are discussed. Suggestions for further work are also made.

2. Basics of the two-junction rf SQUID

The schematic of the two-junction rf SQUID loop,

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namely a superconducting loop closed with two junctions in series, is shown in Fig.1. The loop inductance is L. The critical currents of the two junctions are I_1 and I_2 respectively. We assume$I_1 \leq I_2$, and define the critical current ratio $\alpha = I_1/I_2$, $0< \alpha \leq 1$.

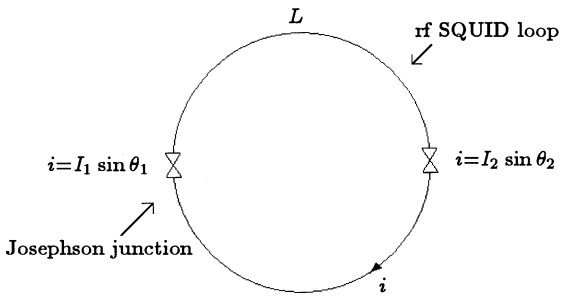


Fig.1. A schematic drawing of an rf SQUID loop closed with two Josephson junctions in series.

Assuming that each of the junctions is an ideal Josephson junction,[12] there is

$$i = I_1 \sin\theta_1 = I_2 \sin\theta_2, \tag{1}$$

where i is the circulating superconducting current flowing along the loop; θ_1, θ_2 are the phase differences of the two junctions.

Fluxoid quantization implies

$$\theta_1 + \theta_2 + \frac{2\pi\phi}{\phi_0} = 2k\pi, \tag{2}$$

where ϕ is the total magnetic flux through the loop, ϕ_0 is the flux quantum $h/2e \approx 2.07 \times 10^{-15}$Wb; k is an integer.

When the external flux applied through the loop is ϕ_x, we have

$$\phi = \phi_x + Li. \tag{3}$$

The free energy of this system can be written as

$$E = -\frac{\phi_0}{2\pi}(I_1 \cos\theta_1 + I_2 \cos\theta_2) + \frac{1}{2}Li^2, \tag{4}$$

where the first part is the total phase-coupling energy and the second term is the magnetic energy.[13]

From Eq.(1), considering that the stable system should have lower energy, we obtain

$$\theta_2 = \arcsin(\alpha \sin\theta_1) + 2k'\pi, \tag{5}$$

where k' is an integer.

Hence from Eqs.(2), (3) and (5)

$$\sin\theta_1 = \frac{-\sin\left(2\pi\frac{\phi}{\phi_0}\right)}{\left[1+\alpha^2+2\alpha\cos\left(2\pi\frac{\phi}{\phi_0}\right)\right]^{1/2}}, \tag{6}$$

$$\phi_x = \phi + \frac{LI_1 \sin\left(2\pi\frac{\phi}{\phi_0}\right)}{\left[1+\alpha^2+2\alpha\cos\left(2\pi\frac{\phi}{\phi_0}\right)\right]^{1/2}}. \tag{7}$$

Equation (7) describes the behaviour of the total flux as a function of the applied flux. Define the hysteresis parameter $\beta=2\pi LI_1/\phi_0$. For $\beta=2$ and the critical current ratio α having different values, a plot of Eq.(7) is illustrated in Fig.2. It is noticed immediately when α is increased with β remaining constant that the curve demonstrates more hysteretic properties. When $\alpha \to 0$, Eq.(7) turns into $\phi_x = \phi + LI_1 \sin(2\pi\phi/\phi_0)$, i.e. the equation in the case of single-junction rf SQUIDs,[1] and the effect of the junction having higher critical current is trivial.

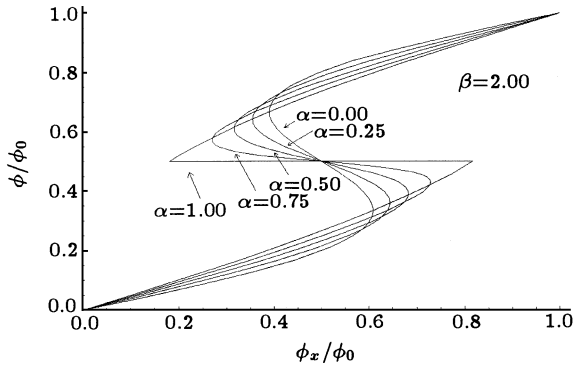


Fig.2. Total magnetic flux ϕ of a two-junction rf SQUID as a function of the external flux ϕ_x with constant hysteresis parameter β=2 and different critical current ratios: α=0, 0.25, 0.5, 0.75 and 1.

It is found[1)] that when $\beta < 1-\alpha$, $\mathrm{d}\phi/\mathrm{d}\phi_x > 0$ everywhere, ϕ is a single-valued function of ϕ_x and when $\beta > 1-\alpha$, ϕ is a multi-valued function of ϕ_x. In other words, when $\beta < 1-\alpha$, the system operates in the nonhysteretic mode and when $\beta > 1-\alpha$, the system operates in the hysteretic mode.

1) $\dfrac{\mathrm{d}^2\phi_x}{\mathrm{d}\phi^2} = -\dfrac{2\pi\beta}{\phi_0}\sin\left(\dfrac{2\pi\phi}{\phi_0}\right)\dfrac{\left[\alpha\cos\left(\dfrac{2\pi\phi}{\phi_0}\right)+\dfrac{1+\alpha^2}{2}\right]^2+\dfrac{3}{4}(1-\alpha^2)^2}{\left[1+\alpha^2+2\alpha\cos\left(\dfrac{2\pi\phi}{\phi_0}\right)\right]^{5/2}} \Rightarrow \left(\dfrac{\mathrm{d}\phi_x}{\mathrm{d}\phi}\right)_{\min} = \left.\dfrac{\mathrm{d}\phi_x}{\mathrm{d}\phi}\right|_{\phi=\phi_x=\phi_0/2} = 1-\dfrac{\beta}{1-\alpha},$

therefore when $\beta < 1-\alpha$, $\dfrac{\mathrm{d}\phi_x}{\mathrm{d}\phi} > 0$ i.e. $\dfrac{\mathrm{d}\phi}{\mathrm{d}\phi_x} > 0$ everywhere; otherwise, $\dfrac{\mathrm{d}\phi}{\mathrm{d}\phi_x} < 0$ in some region.

Figure 3 is an example, where α is chosen to be 1/4, equal to the width ratio reported by Fardmanesh *et al.*[11] In Fig.3, we see that due to the effect of the junction having higher critical current, only when the hysteresis parameter β is smaller than 3/4, not 1 as expected in the case of single junction rf SQUIDs,[1] can the nonhysteretic mode be obtained. If Fardmanesh *et al*[11] wanted their bicrystal GBJ rf SQUID to operate in the nonhysteretic mode which is preferred for lower noise level compared with the hysteretic mode,[14] they should have to reduce the hysteresis parameter β to a reasonable value less than $1-\alpha$. This result also applies to the step edge GBJ rf SQUID. When the lower and upper junctions have the same critical superconducting current, i.e. α=1, the step edge GBJ rf SQUID must operate in the hysteretic mode. Although it is not the case for a large step angle rf SQUIDs, the effect of the upper junction cannot be trivial because α is typically 1/3–1/5.[6]

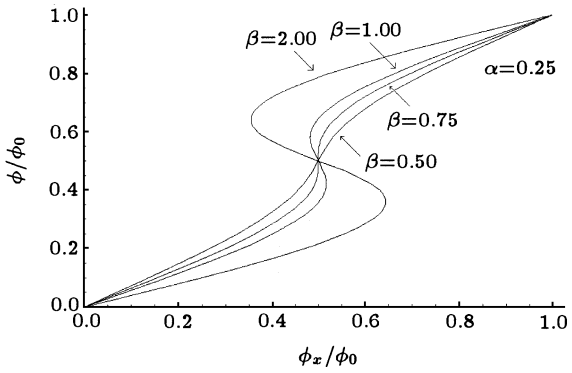


Fig.3. ϕ/ϕ_0 vs ϕ_x/ϕ_0 for a two-junction rf SQUID with constant critical current ratio α=0.25 and different hysteresis parameters: β=0.5, 0.75, 1, 2.

From Eqs.(1), (2), (4) and (5), the circulating superconducting current i and the free energy E can be written as functions of the total flux ϕ:

$$i = I_1 \sin\theta_1 = \frac{-I_1 \sin\left(2\pi\dfrac{\phi}{\phi_0}\right)}{\left[1+\alpha^2+2\alpha\cos\left(2\pi\dfrac{\phi}{\phi_0}\right)\right]^{1/2}}, \quad (8)$$

$$E = -\frac{\phi_0 I_1}{2\pi\alpha}\left[1+\alpha^2+2\alpha\cos\left(2\pi\frac{\phi}{\phi_0}\right)\right]^{1/2} + \frac{1}{2}LI_1^2\frac{\left[\sin\left(2\pi\dfrac{\phi}{\phi_0}\right)\right]^2}{1+\alpha^2+2\alpha\cos\left(2\pi\dfrac{\phi}{\phi_0}\right)}. \quad (9)$$

Although the analytic expressions of $i(\phi_x)$ and $E(\phi_x)$ cannot be given, the numerical results are shown in Fig.4. In the hysteretic mode, the quantum transition to lower energy state is indicated.[1]

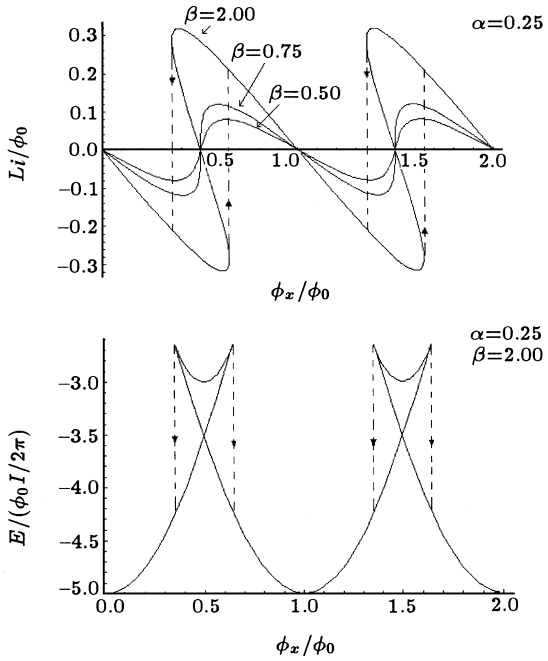


Fig.4. Circulating current i and free energy E of a two-junction rf SQUID as functions of the external flux ϕ_x with constant critical current ratio. The arrows indicate the quantum transitions in the hysteretic mode.

3. Conclusions

We have presented a comprehensive description of the basic characteristics of two-junction rf SQUIDs, and the case for GBJ rf SQUIDs. The effect of the junction having higher critical current is negligible only when the critical current ratio α is much less than 1. When the hysteresis parameter β is less than $1-\alpha$, the two-junction rf SQUID will operate in the nonhysteretic mode. Otherwise, it will operate in the hysteretic mode. To further the investigation in theory, we must use the resistively and capacitively shunted junction (RCSJ) model to study the time-dependent behaviour of the two junctions.[15] This work is necessary for the investigation of the total noise level. Chesca[16] gave the theory of single-junction rf SQUIDs operating in the presence of large thermal fluctuations. The corresponding theory for two-junction rf SQUIDs will be more practical and useful. Experimentally, we can design the bicrystal GBJ rf SQUID and study its operating mode and noise level with different critical current ratios by changing the junction width ratio. The work is in progress.

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