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Synchronization of chaotic erbium-doped fibre lasers and its application in secure communication

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We have investigated the synchronization of chaotic dual-ring erbium-doped fibre lasers using the delay feedback–injection scheme. Numerical simulation shows that two chaotic dual-ring erbium-doped fibre lasers can be well synchronized if the strengths of delay feedback and injection are suitable. Even though the effects of the noises and the difference between the two lasers are considered, the twins can still reach good synchronization. Secure communication could be realized using the scheme of message-masking based on the synchronization of the chaotic dual-ring erbium-doped fibre lasers.

Keywords: chaos, synchronization, secure communication, dual-ring erbium-doped laser
PACC: 0545, 4255

1. Introduction

Erbium-doped fibre ring lasers are attracting considerable interest for their applications in optical fibre telecommunication system as potential comparable laser source[1,2] with high output power and narrow line-width. Synchronization between the coupled nonlinear oscillators has recently attracted the attention of many researchers. A rich palette of behaviours has been observed in a wide variety of systems, including, among others, populations dynamics, coupled neurons, and lasers.[3−13] The interest in the synchronization of chaotic erbium-doped lasers has been motivated by its potential applications: for example, in secure communication.[14−16] We have theoretically investigated the synchronization of dual-ring erbium-doped fibre lasers by different methods,[17,18] which is instructive for secure communications using chaotic erbium-doped fibre lasers. However, the delayed effect was not considered in previous works.

In this paper, we investigate the synchronization of chaotic dual-ring erbium-doped fibre lasers using delay feedback-injection method, and find that two dual-ring erbium-doped fibre lasers can be well synchronized with each other even though they have different parameters. On the other hand, when the effect of noise is taken into consideration, the two lasers can also reach synchronization only if the noises are not greater than 0.1%. Based on the synchronization of chaotic dual-ring erbium-doped fibre lasers, we find that the secure communications can be realized by the message-masking scheme, and the delay effect leads to a retarded synchronization which is beneficial to secure telecommunications.

2. A scheme of synchronization

The basic dual-ring erbium-doped fibre laser system is shown in Fig.1. Both rings a and b contain a wave division multiplex (WDM) coupler for each erbium-doped fibre, and the rings a and b are coupled to each other by coupler C_0. $I_{\mathrm{p}a}$ and $I_{\mathrm{p}b}$ are the pumping intensity of the rings a and b, respectively. As is known, a dual-ring erbium-doped fibre laser can show periodic, chaotic and developed chaotic states under different conditions.[11] In fact, a dual-ring erbium-doped fibre laser has two erbium-doped fibre lasers, which are coupled to each other by coupler C_0 to produce chaotic lasing output.

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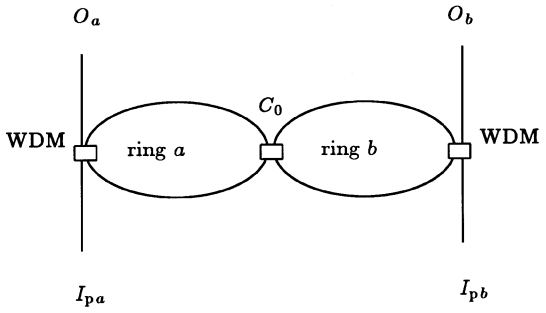


Fig.1. Dual-ring erbium-doped laser system, where C_0 is coupler, WDM is wavelength division multiplexer, I_{pa} and I_{pb} are the pumping intensities, O_a and O_b are outputs.

In our scheme of synchronization, a dual-ring erbium-doped fibre laser (driven system called S) is driven by a delay signal from the other dual-ring erbium-doped fibre laser (driving system called M) through a unidirectional coupler. M is not affected by S, but is affected by the delay feedback signal from itself. The dynamic equations of the whole system keep their original type[11] and are shown as follows:

The system M:

$$\frac{\mathrm{d}E_{aM}(\tau)}{\mathrm{d}\tau} = -k_{aM}(E_{aM}(\tau) + \eta_{0M}E_{bM}(\tau) - \varepsilon_1 E_{aM}(\tau-\tau_{\mathrm{d}1})) + g_{aM}E_{aM}(\tau)D_{aM}, \quad (1)$$

$$\frac{\mathrm{d}E_{bM}(\tau)}{\mathrm{d}\tau} = -k_{bM}(E_{bM}(\tau) - \eta_{0M}E_{aM}(\tau)) + g_{bM}E_{bM}(\tau)D_{bM}, \quad (2)$$

$$\frac{\mathrm{d}D_{aM}}{\mathrm{d}\tau} = -(1 + I_{paM} + E_{aM}^2(\tau))D_{aM} + I_{paM} - 1, \quad (3)$$

$$\frac{\mathrm{d}D_{bM}}{\mathrm{d}\tau} = -(1 + I_{pbM} + E_{bM}^2(\tau))D_{bM} + I_{pbM} - 1; \quad (4)$$

The system S:

$$\frac{\mathrm{d}E_{aS}(\tau)}{\mathrm{d}\tau} = -k_{aS}(E_{aS}(\tau) + \eta_{0S}E_{bS}(\tau) - \varepsilon_2 E_{aM}(\tau-\tau_{\mathrm{d}2})) + g_{aS}E_{aS}(\tau)D_{aS}, \quad (5)$$

$$\frac{\mathrm{d}E_{bS}(\tau)}{\mathrm{d}\tau} = -k_{bS}(E_{bS}(\tau) - \eta_{0S}E_{aS}(\tau)) + g_{bS}E_{bS}(\tau)D_{bS}, \quad (6)$$

$$\frac{\mathrm{d}D_{aS}}{\mathrm{d}\tau} = -(1 + I_{paS} + E_{aS}^2)D_{aS} + I_{paS} - 1, \quad (7)$$

$$\frac{\mathrm{d}D_{bS}}{\mathrm{d}\tau} = -(1 + I_{pbS} + E_{bS}^2)D_{bS} + I_{pbS} - 1, \quad (8)$$

where E_a and E_b are the lasing field, while D_a and D_b are the population inversion in rings a and b respectively. k_a and k_b are the products of τ_2 and decay rate, g_a and g_b are the products of τ_2 and gain coefficient of lasing field in rings a and b, respectively. Here, $\tau = \gamma t$ with $\gamma = 1/\tau_2$, τ_2 is the decay time of the lasing upper level and is about 10ms for erbium-doped laser; in this paper, we take τ_2=10ms. η_0 is the coupling coefficient of the coupler C_0, ε_1 and ε_2 are the unidirectional feedback and injection strength respectively. $\tau_{\mathrm{d}1}$ and $\tau_{\mathrm{d}2}$ are the delay time of the feedback and injection signal for M and S, respectively and are all normalized to τ_2. The subscripts M and S indicate the systems M and S.

Luo *et al*[11] investigated the periodic and chaotic behaviour of the dual-ring erbium-doped fibre laser without delay feed-back signal. In this paper the equations of systems M and S are the same as those in Ref.[11] except that the delay feedback term and delay injection term are added in M and S, respectively. We choose appropriate system parameters to assure that M and S are in chaotic states. The parameters are as follows: $k_{aM} = k_{bM} = 1000$; $g_{aM} = 10500$, $\eta_{0M} = 0.2$; $I_{paM} = I_{pbM} = 4$, the parameters in S are the same as those in M, $\varepsilon_1 = \varepsilon_2 = 0.09$, $\tau_{\mathrm{d}1} = \tau_{\mathrm{d}2} = 5.0 \times 10^{-5}$s. Figure 2(a) shows the strange attractor projection on the plane E_{aM}-E_{bM}, and Fig.2(b) shows the power spectrum of E_{aM}. The corresponding phase graph and power spectrum for system S are the same as those for M and we need not provide them. From the strange attractor and power spectrum, one can find that the systems M and S are chaotic, which is consistent with Luo's results.[11]

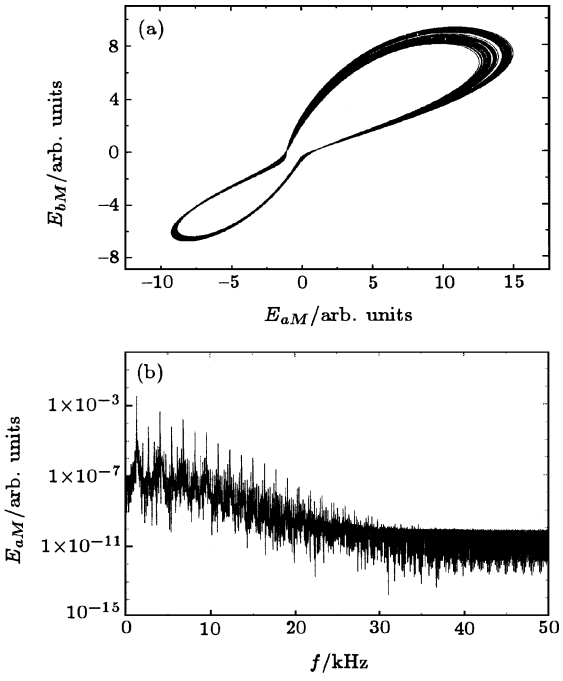


Fig.2. The attractor projection on the plane E_{aM}-E_{bM} and the power spectra of E_{aM}. The system parameters are as follows: k_{aM}, k_{bM}, k_{aS}, k_{bS}:1000; g_{aM}, g_{aS}:10500; g_{bM}, g_{bS}: 4700; η_0:0.2; I_{paM}, I_{pbM}, I_{paS}, I_{pbS}: 4; $\tau_{\mathrm{d}1}$, $\tau_{\mathrm{d}2}$: 5.0×10^{-5}; ε_1, ε_2: 0.09.

Figure 3 shows the synchronization between chaotic systems S and M. The system parameters are the same as those in Fig.2. The corresponding parameters in systems M and S are the same and the noise effects are not considered in Figs.3 (a_1), (a_2) and (a_3). Figures 3 (a_1) and (a_2) show the time series of lasing fields E_{aM} and E_{aS}, respectively, which indicate that S and M are both in chaotic states. Figure 3(a_3) shows the time series of $\delta E = E_{aM} - E_{aS}$; one can find that system S can well be synchronized with M after the transient process has died out. In order to test the robustness of the synchronization, as an example, Gauss noise terms $\xi_{1(\tau)}$ and $\xi_{2(\tau)}$ ($0 < \xi_{1(\tau)}, \xi_{2(\tau)} < 0.1\%$) are added in Eqs.(1) and (5), respectively. Figs.3 (b_1), (b_2) and (b_3) are similar to Figs.3 (a_1), (a_2) and (a_3) respectively except that the Gauss noise is considered, and the systems S and M are still chaotic and synchronize well with each other ($\delta E < 10^{-4}$) after the transient process has died out. If the noise terms are added in other equations, similar results would be obtained. However, the system S is not identical with M, for the system parameters of S are different from those of M. As an example, k_{bS} and g_{bS} differ from the corresponding k_{bM} and g_{bM} as follows:

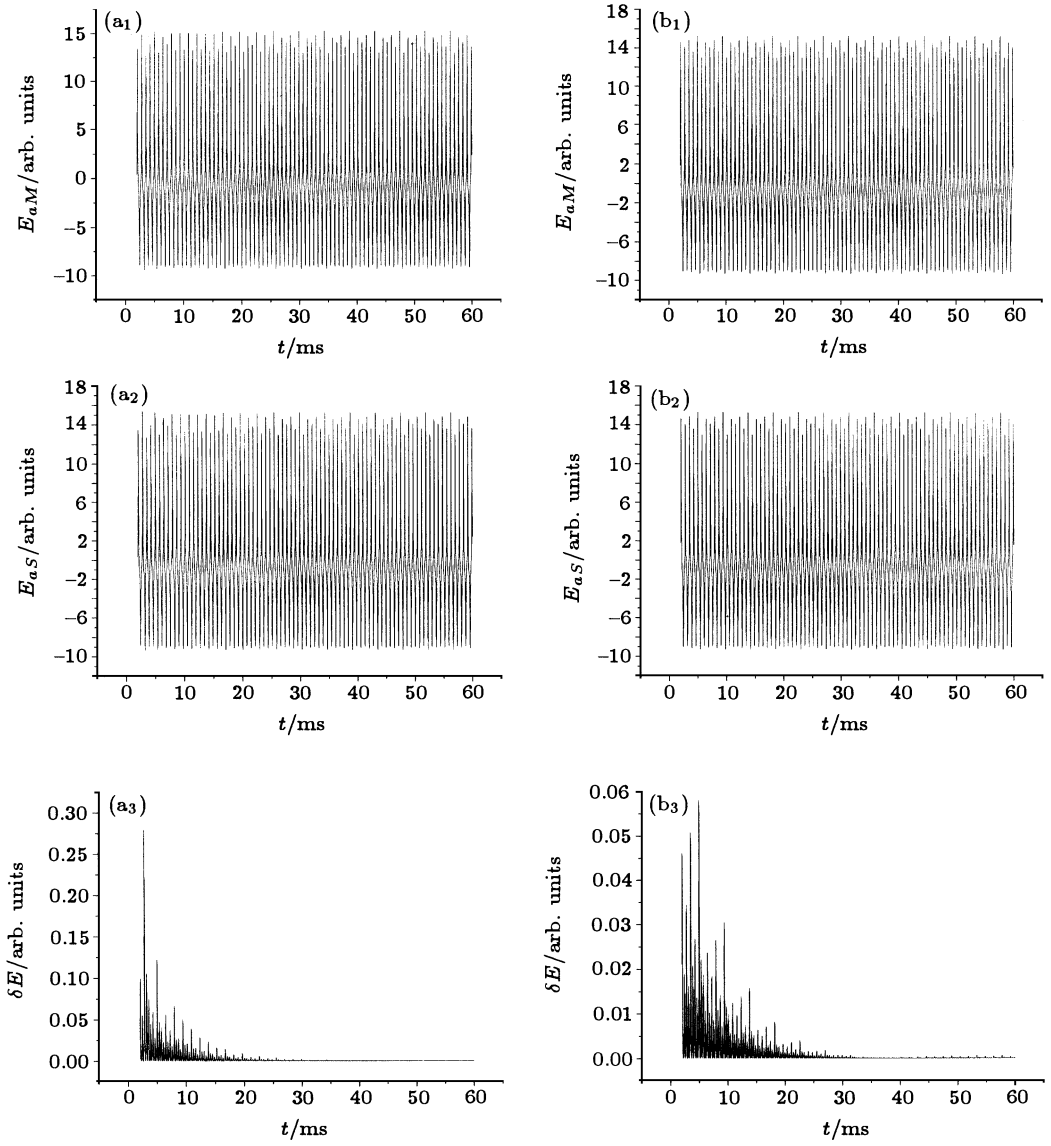


Fig.3. Synchronization of S and M after transient process. $\delta E = |E_{aM} - E_{aS}|$, the parameters are the same as those in Fig.2. The Gauss noises are not considered in (a_1), (a_2), and (a_3), but considered in (b_1), (b_2) and (b_3).

$$k_{bS} = k_{bM}(1 + \delta k), \tag{9}$$

$$g_{bS} = g_{bM}(1 + \delta g); \tag{10}$$

other parameters are the same as those in Fig.2. We calculate the maximum difference $\delta E = |E_{aM} - E_{bM}|$ over a long time after the transient process died out

with $-0.1 \leq \delta k \leq 0.1$ and $-0.1 \leq \delta g \leq 0.1$. δE in decibel can be taken as the loss scale of synchronization. Figure 4 shows the relationship among δE, δk and δg. One can find that there exists a minimum value for δE if $\delta g = 0$ and $\delta k = 0$; in other words, if all corresponding parameters of the systems S and M are identical, S and M can be best synchronized with the feedback and the injection strength: $\varepsilon_1 = \varepsilon_2 = 0.09$ (of course they can still reach a synchronization for other values of ε_1 and ε_2). Otherwise, the loss scale of synchronization becomes larger and larger as $|\delta g|$ and $|\delta k|$ become larger and larger. On the other hand, there are 'valleys' along the line from $(-0.036, -0.1)$ to $(0.036, 0.1)$, i.e. if the corresponding parameters in system S can match those in M, S can well synchronize with M. Of course, the difference of the corresponding parameters should not be too big, which can be found from the depth of the 'valley'. For other parameters, we can obtain a similar result using the same method.

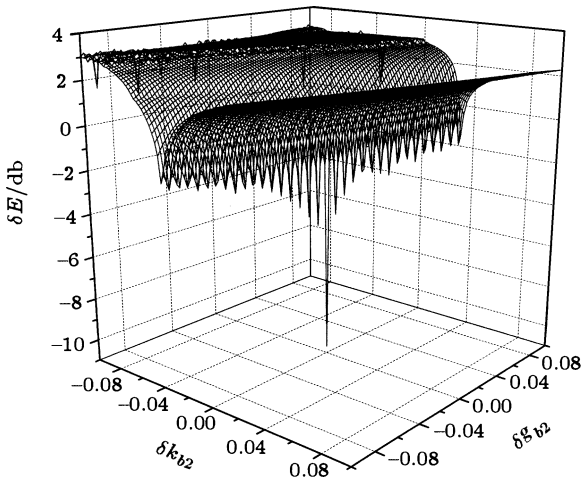


Fig.4. The loss scale of synchronization (δE in decibel) versus variation of k_{bS} and g_{bS}: δk, δg. Other parameters are the same as those in Fig.2.

Hence, by means of the delayed feedback–injection method, two dual-ring erbium-doped fibre lasers can well synchronize with each other even though there is some difference between them; on the other hand, the twins reach good synchronization even though the noises are taken into consideration. In a word, the synchronization is robust in our scheme, which is very useful in practice, for example, in secure communications.

In the process of numerical simulation, we find that, if $\tau_{\mathrm{d}1} \neq \tau_{\mathrm{d}2}$, $E_{aS}(\tau - \tau_{\mathrm{d}1})$ would synchronize with $E_{aM}(\tau - \tau_{\mathrm{d}2})$. The reason is that the states of the two lasers is determined by the delay feedback signal $\varepsilon_1 E_{aM}(\tau - \tau_{\mathrm{d}1})$ and the delay injection signal $\varepsilon_2 E_{aM}(\tau - \tau_{\mathrm{d}1})$ respectively, i.e. $E_{aS}(\tau - \tau_{\mathrm{d}1})$ and $E_{aM}(\tau - \tau_{\mathrm{d}2})$ are both determined by the lasing field $E_{aM}(\tau - \tau_{\mathrm{d}1} - \tau_{\mathrm{d}2})$, hence the delay feedback and injection signal play an important role, and $E_{aS}(\tau - \tau_{\mathrm{d}1})$ synchronize with $E_{aM}(\tau - \tau_{\mathrm{d}2})$. This is consistent with the generalized synchronization of S and M.[19] In practice, $\tau_{\mathrm{d}1} < \tau_{\mathrm{d}2}$ is always true for S is in a remote place, hence the output of the system S would synchronize with that of the system M in an earlier time: that is to say, the delay effect leads to retarded synchronization, which is fit for the secure telecommunications.

3. A scheme for masking signal

Figure 5 shows the system for the message-masking scheme, which contains a transmitter and a receiver. The main part of the transmitter (left part of the dashed line) or receiver (right part of the dashed line) is a dual-ring erbium-doped fibre laser, shown in Fig.1. We still call the laser in transmitter and receiver as M and S, respectively. $C_1 - C_5$ are couplers and D is the delayed line. The signal S_{i} is coupled into the output optic field through C_1, then the mixed signal $S_i + E_{aM}$ is divided into two parts: one is fed back into ring a of M with a certain delay time which is determined by D. The other is transmitted to the receiver and is divided into two parts by coupler C_4. One is injected into the ring a of S as driving signal, the other with the output of S is used to decode the message signal from the mixed chaotic signal and message signal through the decoder. S_0 is the extracted message signal that one needs.

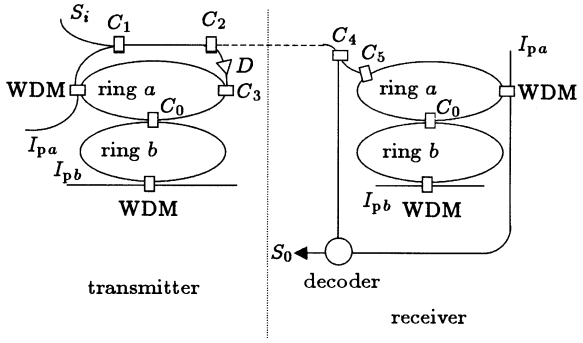


Fig.5. Message-masking scheme. $C_1, \ldots, C_5$: coupler; D: delayed line. S_i, message signal; S_0, extracted message signal.

As an example, the analogue and digital message signal is extracted from the mixed message and chaotic signal by the synchronization of two lasers. The chaotic signal is used to mask the message signal in order to realize secure communication.

The analogue signal is written in the following expression:

$$S_i = m(1 + \sin(\omega t)), \tag{11}$$

where m is the amplitude of S_i, $\omega = 2\pi f$, f is the frequency of S_i, here f=50kHz. After adding the signal S_i, the feedback signal $E_{aM}(\tau - \tau_{d1})$ and injection signal $E_{aM}(\tau - \tau_{d2})$ in Eqs.(1) and (5) are changed to $E_{aM}(\tau - \tau_{d1}) + S_i(\tau - \tau_{d1})$ and $E_{aM}(\tau - \tau_{d2}) + S_i(\tau - \tau_{d2})$ respectively and other quantities are changeless. τ_{d2} is determined by the distance of transmission. In our numerical simulation, $\tau_{d2} = 5.0 \times 10^{-4}$, $\tau_{d1} = 5.0 \times 10^{-5}$, $m = 0.01$, f=5kHz. The digital signal is a series random 0,1 signal and is expressed as follows:

$$S_i = \begin{cases} 0.01, & \text{if digital signal is 1,} \\ 0, & \text{if digital signal is 0.} \end{cases} \tag{12}$$

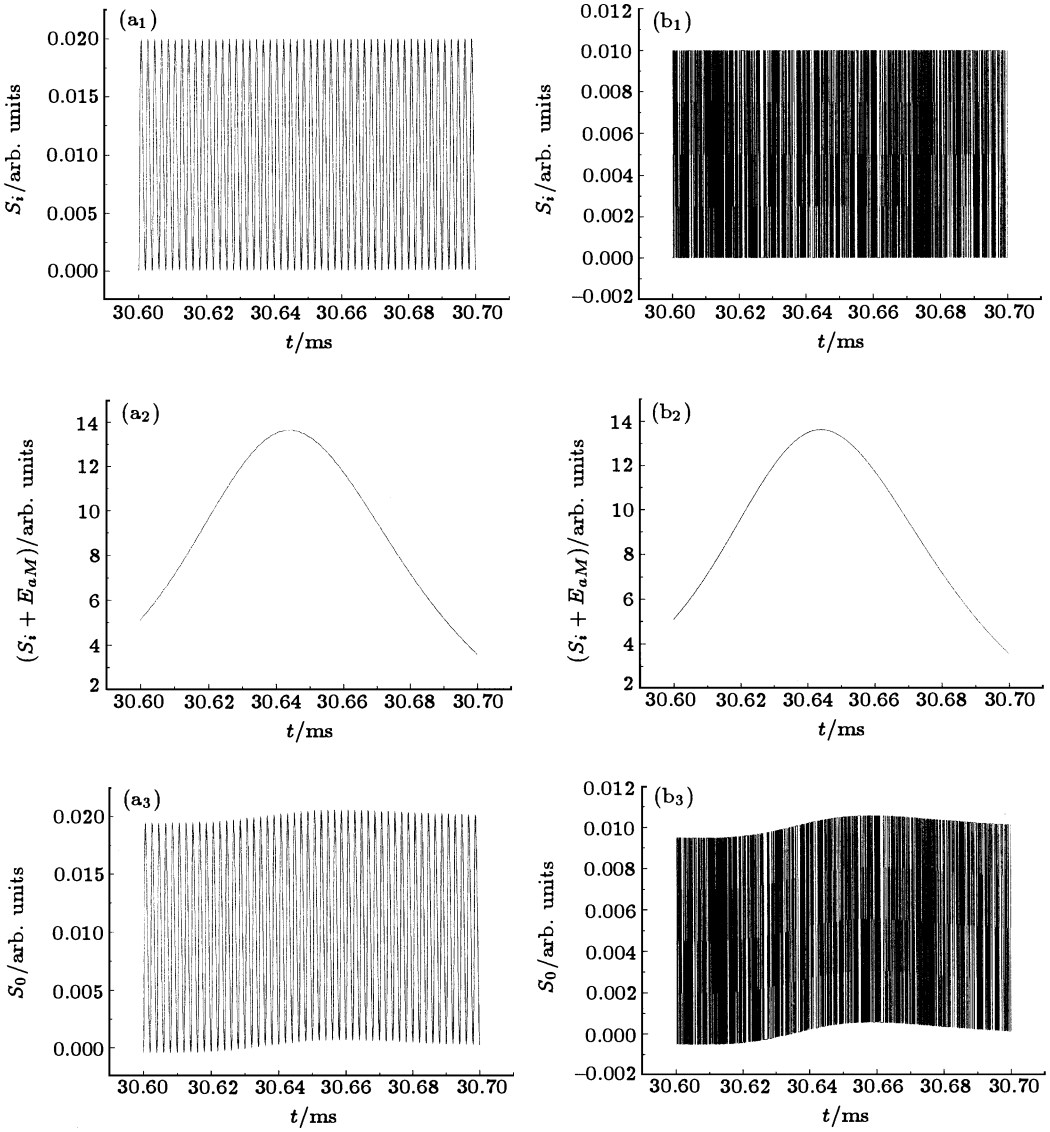


Fig.6. Numerical simulation results of the secure communications using message-masking scheme. The system parameters are the same as those in Fig.3. except that $\tau_{d1} = 5.0 \times 10^{-5}$, $\tau_{d2} = 5.0 \times 10^{-4}$. ($a_1$), ($a_2$), and ($a_3$) are for analogue signal, (b_1), (b_2) and (b_3) are for digital signal. $S_0(\tau) = E_{aM}(\tau - (\tau_{d1} - \tau_{d2})) + S_i(\tau - (\tau_{d1} - \tau_{d2})) - E_{aS}(\tau)$.

Figure 6 shows the result of the secure communications after the transient process has died out. We only present the behaviours of the signals for a very short time (0.1ms) in order to see clearly the analogue and digital signals. Figures 6 (a_1) and (b_1) show the analogue and digital signals S_i. Figures 6 (a_2) and (b_2) show the $E_{aM}(\tau - \tau_{d2}) - \tau_{d1})) + S_i(\tau - (\tau_{d2} - \tau_{d1}))$; in fact, the behaviours of the curves for long time are

similar to those in Fig.3 (a_1) which shows the chaotic behaviour of E_{aM}. One can find that S_i is completely masked by the chaotic signal E_{aM}; of course, S_i is a small signal compared with E_{aM}. Figures 6 (a_3) and (b_3) show the extracted analogue and digital signals $S_0(\tau) = E_{aM}(\tau - (\tau_{d2} - \tau_{d1})) + S_i(\tau - (\tau_{d2} - \tau_{d1})) - E_{aS}(\tau)$ respectively. Comparing Fig.6 (a_1) with (a_3) and Fig.6 (b_1) with (b_3), one can find that the extracted message signal S_0 is not identical with the imported message signal S_i, but the basic outlines of S_0 are the same as those of S_i. A receiver can easily recognize the message signal. Hence, the message signals are successfully decoded by using the synchronization of the two chaotic dual-ring erbium-doped fibre lasers. Of course, S_i and E_{aM} both affect the state of M and S; however, S_i is very small and could be neglected compared with E_{aM}, the driving signal, and the synchronization of the two lasers is almost not affected by S_i. If S_i is big, it would change the state of S and M, and affect the realization of synchronization. Hence, secure telecommunication can be realized by message-masking and decoding through synchronization of chaotic dual-ring erbium-doped fibre lasers for small message signals. It is pointed out that our method is also suitable for high or low frequency signal, for the chaotic signal has a very wide frequency range.

4. Conclusion

Two dual-ring erbium-doped fibre lasers can synchronize with each other by delayed feedback-injection method and the synchronization is robust in our scheme. The delayed time and strength of feedback and injection play an important role in realizing synchronization. For remote S, the output at time τ would synchronize with the output of M at the earlier time $(\tau - \tau_d)$, which is determined by the transmitting distance; this retarded synchronization is fit to secure telecommunication. Optical secure communications can be realized based on the retarded synchronization of chaotic dual-ring erbium-doped fibre lasers. The message-masking scheme is useful for a small message signal and is not successful for a large message signal. Our method is established successfully and has potential use in practice.

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