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Stochastic resonance in a bistable system with coloured correlation between additive and multiplicative noise*

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The phenomenon of stochastic resonance (SR) in a bistable nonlinear system with coupling between additive and multiplicative noises is investigated when the correlation between two noise terms is coloured. It is found that the signal-to-noise ratio (SNR) of the system is affected not only by the coupling strength λ between two noise terms, but also by the noise correlation time τ. The SNR is changed from a single peak, to two peaks with a dip, and then to a monotonically decreasing function with noise strength. The dependence of the SR on the initial conditions is entirely caused by the coupling strength λ between two noise terms.

Keywords: coloured correlation, signal-to-noise ratio, stochastic resonance
PACC: 0540, 0250, 0545

1. Introduction

The phenomenon of stochastic resonance (SR) in a nonlinear system has attracted much attention both theoretically and experimentally in recent years due to its potential applications.[1−25] The SR phenomenon appears in many fields, such as periodical recurrence of ice ages,[1,2] electronic and magnetic systems,[3,4] a bistable ring laser,[5] neuroscience,[6] etc. Meanwhile, different kinds of SR phenomena have been investigated, such as coloured SR,[7] multiplicative SR,[8] etc. In most of the previous analysis, the SR in nonlinear systems with white noise was investigated intensively.[9−25] However, the coloured and multiplicative nature of the noise can also affect the SR phenomenon.[7,8] In certain situations, both additive and multiplicative noises may have a common origin and they may be coupled as well.[26−32]

In this paper, the SR phenomenon in a nonlinear system with coupling between additive and multiplicative noises is investigated when the coupling between different noise terms is coloured noise. The signal-to-noise ratio (SNR) is employed to study the SR phenomenon. In Section 2, the expression of SNR in a nonlinear bistable system with a weak periodic signal is derived. In Sec.3, the SNR is presented and the SR phenomenon is discussed when the noise correlation time τ and the coupling strength λ is changed. Section 4 investigates the return of the analytic expression of SNR to that for the bistable dynamics with additive noise only when the multiplicative noise goes to zero. The effect of initial condition on SR is explained. In Sec.5, the numerical simulation is presented to check the validity of the approximation method. A summary concludes the paper.

2. Theoretical analysis

2.1. Equation of motion

The nonlinear system with coupling between additive and multiplicative noises can be described by the following stochastic differential equation:[24,25]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = ax - x^3 + \varepsilon\cos\Omega t + p(t)x + q(t). \qquad (1)$$

Here all the variables and parameters are real, a is the bifurcation parameter, $\varepsilon\cos\Omega t$ is the periodic signal, and $p(t)$ and $q(t)$ are multiplicative and additive noises, respectively. Their mean and variance are given by

$$\begin{aligned} &\langle p(t)\rangle = \langle q(t)\rangle = 0, \\ &\langle p(t)p(t')\rangle = 2P'\delta(t-t'), \\ &\langle q(t)q(t')\rangle = 2P\delta(t-t'), \end{aligned} \qquad (2)$$

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$$\begin{aligned}\langle p(t)q(t')\rangle =&\langle q(t)p(t')\rangle\\ =&\frac{\lambda\sqrt{PP'}}{\tau}\exp\left[-\frac{|t-t'|}{\tau}\right].\end{aligned}\tag{3}$$

Here, P' and P are the strengths of multiplicative and additive noises, respectively, λ is the strength of the coupling between additive and multiplicative noise terms, τ is the noise correlation time.

According to the stochastic Liouville equation,[33] Eq.(1) satisfies

$$\begin{aligned}\frac{\partial Q(x,t)}{\partial t} =&-\frac{\partial}{\partial x}h(x)Q(x,t)\\ &-\frac{\partial}{\partial x}g_1(x)\langle p(t)\delta(x(t)-x)\rangle\\ &-\frac{\partial}{\partial x}g_2(x)\langle q(t)\delta(x(t)-x)\rangle,\end{aligned}\tag{4}$$

where

$$\begin{aligned}h(x) =&ax-x^3+\varepsilon\cos\Omega t,\\ g_1(x) =&x \text{ and } g_2(x)=1.\end{aligned}\tag{5}$$

The probability distribution function can be expressed as

$$Q(x,t)=\langle\delta(x(t)-x)\rangle.\tag{6}$$

Through the combination of functional analysis, Novikov's theorem[34] and Fox's approach,[35] the average $\langle p(t)\delta(x(t)-x)\rangle$ and $\langle q(t)\delta(x(t)-x)\rangle$ can be obtained.

Then the approximate Fokker–Planck equation corresponding to Eq.(4) can be written as

$$\begin{aligned}\frac{\partial Q(x,t)}{\partial t} =&-\frac{\partial}{\partial x}h(x)Q(x,t)\\ &+P'\frac{\partial}{\partial x}g_1(x)\frac{\partial}{\partial x}g_1(x)Q(x,t)\\ &+\frac{\lambda\sqrt{PP'}}{1-\tau\left[h'(x_s)-\dfrac{g_2'(x_s)}{g_2(x_s)}h(x_s)\right]}\\ &\times\frac{\partial}{\partial x}g_1(x)\frac{\partial}{\partial x}g_2(x)Q(x,t)\\ &+P\frac{\partial}{\partial x}g_2(x)\frac{\partial}{\partial x}g_2(x)Q(x,t)\\ &+\frac{\lambda\sqrt{PP'}}{1-\tau\left[h'(x_s)-\dfrac{g_1'(x_s)}{g_1(x_s)}h(x_s)\right]}\\ &\times\frac{\partial}{\partial x}g_2(x)\frac{\partial}{\partial x}g_1(x)Q(x,t).\end{aligned}\tag{7}$$

According to Eq.(5), the steady state of the variable x is given by $x_s=\pm\sqrt{a}$. Thus one has $h'(x_s)-\dfrac{g_1'(x_s)}{g_1(x_s)}h(x_s)=h'(x_s)-\dfrac{g_2'(x_s)}{g_2(x_s)}h(x_s)=-2a$. Then the corresponding Fokker–Planck equation for the time-dependent probability density function $Q(x,t)$ of the variable x can be written as[36–39]

$$\frac{\partial Q(x,t)}{\partial t}=-\frac{\partial}{\partial x}F(x)Q(x,t)+\frac{\partial^2}{\partial x^2}D(x)Q(x,t)\tag{8}$$

with

$$\begin{aligned}F(x)=&ax-x^3+\varepsilon\cos(\Omega t)+P'x+\frac{\lambda\sqrt{PP'}}{1+2a\tau},\\ D(x)=&P'x^2+\frac{2\lambda\sqrt{PP'}}{1+2a\tau}x+P.\end{aligned}\tag{9}$$

The quasi-steady state probability distribution $Q_s(x,t)$ can be derived as [40]

$$\begin{aligned}Q_s(x,t) =&N\left|P'x^2+\frac{2\lambda\sqrt{PP'}}{1+2a\tau}x+P\right|^{-1/2}\\ &\times\exp\left[-\frac{\Phi(x,t)}{P'}\right]\end{aligned}\tag{10}$$

with

$$\begin{aligned}\Phi(x,t) =&\frac{1}{2}x^2-2\sqrt{\frac{P}{P'}}\frac{\lambda}{1+2a\tau}x\\ &-E_1\ln\left[P'x^2+\frac{2\lambda\sqrt{PP'}}{1+2a\tau}x+P\right]\\ &-\frac{E_2(1+2a\tau)}{\sqrt{(1+2a\tau)^2-\lambda^2}}\\ &\times\arctan\left[\frac{\lambda+\sqrt{P'/P}(1+2a\tau)x}{\sqrt{(1+2a\tau)^2-\lambda^2}}\right],\end{aligned}\tag{11}$$

where

$$\begin{aligned}E_1=&-\frac{a}{2}-\frac{P}{2P'}+\frac{2P}{P'}\left(\frac{\lambda}{1+2a\tau}\right)^2,\\ E_2=&a\frac{\lambda}{1+2a\tau}+3\frac{P}{P'}\frac{\lambda}{1+2a\tau}-4\frac{P}{P'}\left(\frac{\lambda}{1+2a\tau}\right)^3\\ &-\sqrt{\frac{P'}{P}}\varepsilon\cos\Omega t.\end{aligned}\tag{12}$$

From Eqs.(10)–(12), it is found that the quasi-steady state probability distribution $Q_s(x,t)$ is not a symmetric function about variable x. It is obvious that the symmetry of the system is broken by the coupling between the two noise sources.[41] When the coupling strength $\lambda=0$, $Q_s(x,t)$ returns to the symmetric case.

2.2. Signal-to-noise ratio (SNR)

The expressions of the SNR in the nonlinear system can be derived by the two-state approach.[9–11,24,25] In the absence of a weak periodic signal ($\varepsilon=0$), the deterministic potential $V(x)=-\dfrac{a}{2}x^2+\dfrac{1}{4}x^4$ of the nonlinear system has two stable

states $x_\pm = \pm\sqrt{a}$ and an unstable state $x_u = 0$. According to the two-state approach, there are $W_\pm(t)$ that represent non-stationary transition rates from the state x_+ to x_- and from x_- to x_+, respectively.

If the signal modulation is far smaller than the barrier height $\Delta V = V(0) - V(x_\pm)$, the transition rates $W_\pm$ can be expanded in terms of the small signal $\varepsilon\cos\Omega t$,

$$W_\pm = \frac{1}{2}[W_0 \pm W_1\eta\cos\Omega t + O(\eta^2\cos^2\Omega t)], \quad (13)$$

where

$$\frac{1}{2}W_0 = f(\mu), \text{ and } \frac{1}{2}W_1 = -\frac{\mathrm{d}f}{\mathrm{d}(\eta\cos\Omega t)}\bigg|_{\eta\cos\Omega t=0}. \quad (14)$$

Thus, the SNR in the nonlinear system reads

$$\mathrm{SNR} = \frac{\pi W_0\varepsilon^2 E_3^2}{4PP'[1-(\lambda/(1+2a\tau))^2]} \times\left[1-\frac{W_0^2\varepsilon^2E_3^2}{2PP'(W_0^2+\Omega^2)[1-(\lambda/(1+2a\tau))^2]}\right]^{-1}. \quad (15)$$

Here

$$W_0 = \frac{\sqrt{2}a}{\pi}\exp\left[-\frac{1}{P'}\left\{-\frac{a}{2}\pm 2\sqrt{\frac{aP'}{P}}\frac{\lambda}{1+2a\tau} - E_1\ln(E_3) - E_4\frac{\lambda}{\sqrt{(1+2a\tau)^2-\lambda^2}}\times E_5\right\}\right], \quad (16)$$

where

$$E_3 = \frac{aP'}{P}\pm 2\sqrt{\frac{aP'}{P}}\frac{\lambda}{1+2a\tau}+1,$$
$$E_4 = a+\frac{3P}{P'}-\frac{4P}{P'}\left(\frac{\lambda}{1+2a\tau}\right)^2,$$
$$E_5 = \arctan\frac{\lambda\pm\sqrt{aP'/P}(1+2a\tau)}{\sqrt{(1+2a\tau)^2-\lambda^2}} - \arctan\frac{\lambda}{\sqrt{(1+2a\tau)^2-\lambda^2}}. \quad (17)$$

From Eqs.(15)–(17), it can be easily seen that the signal-to-noise ratio depends on both the coupling strength and correlation time between additive and multiplicative noise.

3. Effects of noise correlation time τ and coupling strength λ

From Eqs.(15)–(17), the SNR in the nonlinear system can be easily calculated when the parameters are varied.

In Fig.1, the SNR is plotted as function of the additive noise intensity P and the noise correlation time τ when the multiplicative noise intensity P' and the noise coupling strength λ are fixed. In Fig.1(a), it is seen that the SNR decreases monotonically as the additive noise strength P is increased and it also decreases slightly as the noise correlation time τ is increased. It is clear that there is no SR phenomenon for $\lambda > 0$ with the initial condition $x(0) = x_+$. In Fig.1(b), there is a maximum value in SNR and the height of the maximum increases when τ increases. It is seen that the SR phenomenon appears in the case of $\lambda > 0$ with initial condition $x(0) = x_-$.

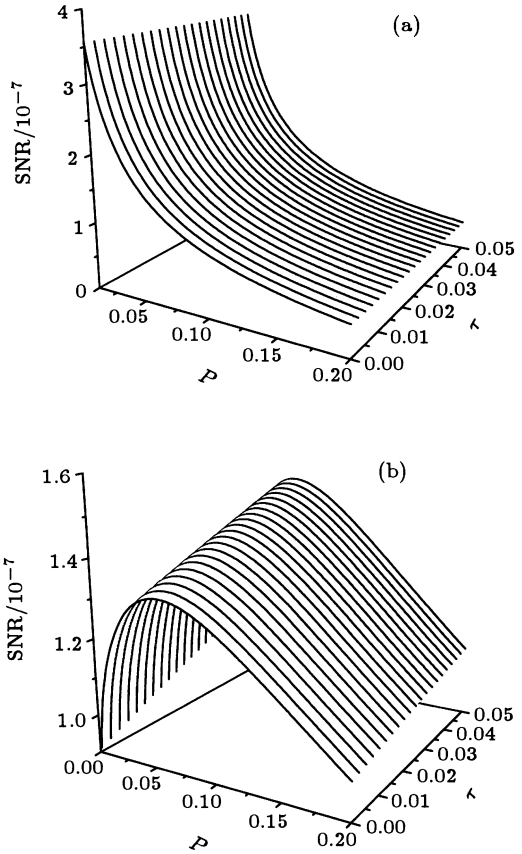


Fig.1. The SNR of the system as function of the additive noise intensity P and the noise correlation time τ. The parameters are chosen as λ=0.5, P' = 0.5, $\varepsilon = 0.0002$, $\Omega = 0.0001$. (a) The initial condition $x(0) = x_+$; (b) The initial condition $x(0) = x_-$.

The SNR as function of the multiplicative noise intensity P' and the noise correlation time τ is plotted in Fig.2 when the additive noise intensity P and the coupling strength λ are fixed. From Fig.2(a), it is seen that the SNR always has a maximum value when the multiplicative noise intensity P' increases. However, the maximum decreases with increasing value of the noise correlation time τ. It seems that the SR phe-

nomenon appears in the case of $\lambda > 0$ and $x(0) = x_+$. The height of the maximum in SNR decreases when the value of τ increases. In Fig.2(b), it is shown that the SNR is changed from one peak to one peak and one minimum, and finally to monotonically decreasing curve when the noise correlation time τ increases from zero to 0.1. It seems that the noise colour can suppress the phenomenon of SR.

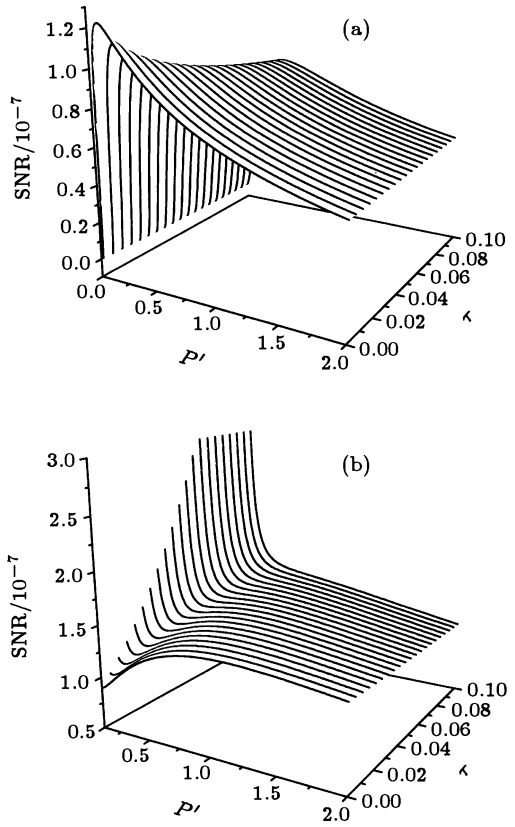


Fig.2. The SNR of the system as function of the multiplicative noise intensity P' and the noise correlation time τ. The parameters are chosen as $\lambda = 0.5$, $P = 0.1$, $\varepsilon = 0.0002$, $\Omega = 0.0001$. (a) The initial condition $x(0) = x_+$; (b) The initial condition $x(0) = x_-$.

The SNR as function of the additive noise intensity P and the noise coupling strength λ is plotted in Fig.3 when the multiplicative noise intensity P' and the noise correlation time τ are fixed. For the initial condition $x(0) = x_+$ in Fig.3(a), the shape of the picture looks like a saddle. When the value of λ is increased from -1.0 to $+1.0$, the height of the peak in SNR is increased and the position of the peak is shifted to smaller values of P. Then the curve of the SNR decreases monotonically as a function of P and the SR phenomenon disappears for $\lambda \geq 0$. For initial condition $x(0) = x_-$, the SNR is shown in Fig.3(b) as function of P and λ. It is very interesting to note that the shape of the SNR is similar to that in Fig.3(a) but with almost opposite tendency and behaviour. The SR phenomenon with maximum height in SNR appears at λ=1.0 while the SR disappears for $\lambda \leq 0$.

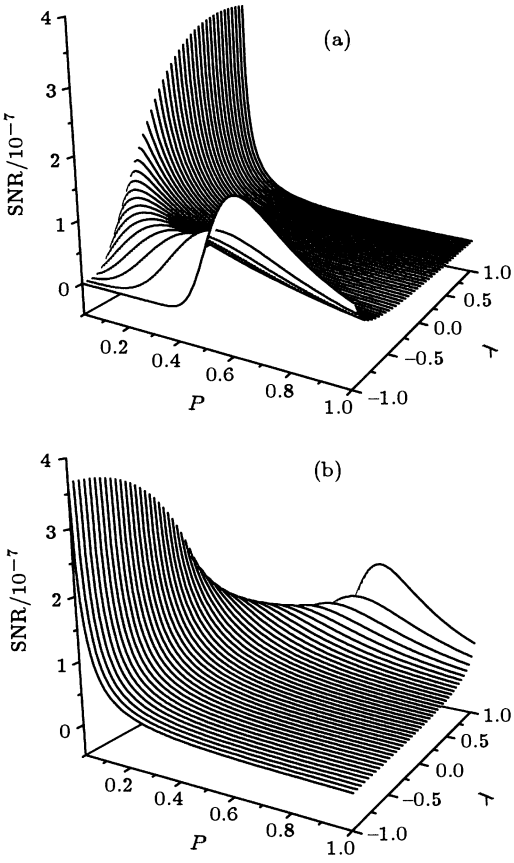


Fig.3. The SNR of the system as function of the additive noise intensity P and the noise coupling strength λ. The parameters are chosen as $P' = 0.5$, $\tau = 0.005, \varepsilon = 0.0002$, $\Omega = 0.0001$. (a) The initial condition $x(0) = x_+$; (b) The initial condition $x(0) = x_-$.

The SNR is plotted in Fig.4 as function of the multiplicative noise intensity P' and the noise coupling strength λ when the values of additive noise intensity P and noise correlation time τ are fixed. For initial condition $x(0) = x_+$ in Fig.4(a), it is seen that the curve of the SNR decreases monotonically as P' is increased when $\lambda < 0$. That is, no SR phenomenon occurs for $\lambda < 0$. When λ is increased from negative to positive, the peak appears in SNR at relatively small value of P'. There is also a peak in the SNR when the noise coupling strength λ is increased. For the initial condition $x(0) = x_-$ in Fig.4(b), it is seen that there is a peak in the curve of SNR when the multiplicative noise intensity P' is increased for $\lambda \leq 0$. The peak of SNR is shifted to smaller values of P' and disappears for $P' = 0$ when λ is increased from negative to positive value. If the value of P' is greater than zero, there is also a peak in the SNR when SNR is plotted as a

function of λ. It is seen that the SR-like phenomenon appears as the noise coupling strength λ is varied.

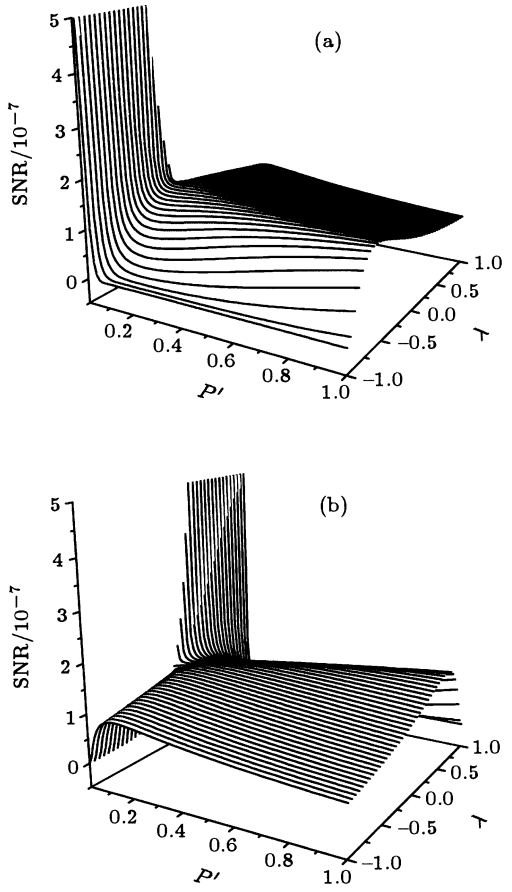


Fig.4. The SNR of the system as function of the multiplicative noise intensity P' and the noise coupling strength λ. The parameters are chosen as $P = 0.1$, $\tau = 0.005, \varepsilon = 0.0002$, $\Omega = 0.0001$. (a) The initial condition $x(0) = x_+$; (b) The initial condition $x(0) = x_-$.

4. Disappearance of multiplicative noise and effect of initial conditions

From Eqs.(10)–(12), it can be seen that there is a singular point when the multiplicative noise intensity P' goes to zero. The results from Eqs.(10)–(12) are only valid for $P' > 0$. However, when the multiplicative noise intensity P' goes to zero, the nonlinear dynamical system with a weak periodic signal of Eqs.(1)–(3) is reduced to[11]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = ax - x^3 + q(t) + \varepsilon\cos(\Omega t). \tag{18}$$

Here $q(t)$ is additive noise, its mean and variance are given by

$$\langle q(t)\rangle = 0,\ \langle q(t)q(t')\rangle = 2P\delta(t-t'). \tag{19}$$

The quasi-steady state probability distribution $Q_s(x,t)$ can be obtained as

$$Q_s(x,t) = N\frac{1}{P}\exp\left[-\frac{U(x,t)}{P}\right], \tag{20}$$

where N is a normalization constant, $U(x,t) = -\dfrac{a}{2}x^2 + \dfrac{x^4}{4} - \varepsilon\cos(\Omega t)x$ is the potential of the system.

The signal-to-noise ratio can be written as

$$\mathrm{SNR} = \frac{\sqrt{2}a^2\varepsilon^2\exp[-a^2/4P]}{4P^2} \times \left[1 - \frac{\dfrac{2a^2\varepsilon^2}{\pi^2P^2}\exp[-a^2/2P]}{2\left(\dfrac{2a^2}{\pi^2}\exp[-a^2/2P] + \Omega^2\right)}\right]^{-1}. \tag{21}$$

This is exactly the result obtained in the pioneering work of Ref.[11].

The numerical results from Eq.(15) with $P' = 0.001$ is indistinguishable with that from Eq.(21) with $P' = 0$. The deviation is less than 0.13%. Even for P'=0.03, the deviation between Eqs.(15) and (21) of $P' = 0$ is still less than 3.7%. The deviation is within the thickness of the curves shown in Figs.1–4. This means that for very small multiplicative noise intensity P', one can approximately return to the results of standard SR with additive noise only.[11] The singularity of $P' = 0$ appeared in Eq.(15) is mainly due to the approximation used in deriving the Fokker–Planck equation.

The standard SR, as it is classically defined with a weak periodic modulation, is essentially a steady-state property in the response of a nonlinear system. The steady-state conditions exclude any dependence of the SR effect on the initial condition $x(0)$. This is the philosophy of standard SR and indeed what is observed in practice. However, the SNR derived from Eqs.(1)–(3) depends on the initial condition $x(0)$. The SNR as a function of additive noise strength P are plotted for different initial conditions $x(0)$ in Figs.5(a)–5(d) when the coupling strength λ and noise correlation time τ are varied. The other parameters Ω, ε, and P' are fixed. From Figs.5(a) and 5(b) it can be seen that there is no dependence of the SNR on $x(0)$ for $\lambda = 0$ and the curve of the SNR is a monotonically decreasing function of P. From Figs.5(c) and 5(d) it is clear that the curve of the SNR depends on the initial condition $x(0)$ when the coupling strength between two noise terms is $\lambda = 0.5$. The curve of the SNR descends monotonically when the initial condition is $x(0) = x_+$. There appears a peak when the initial condition is $x(0) = x_-$.

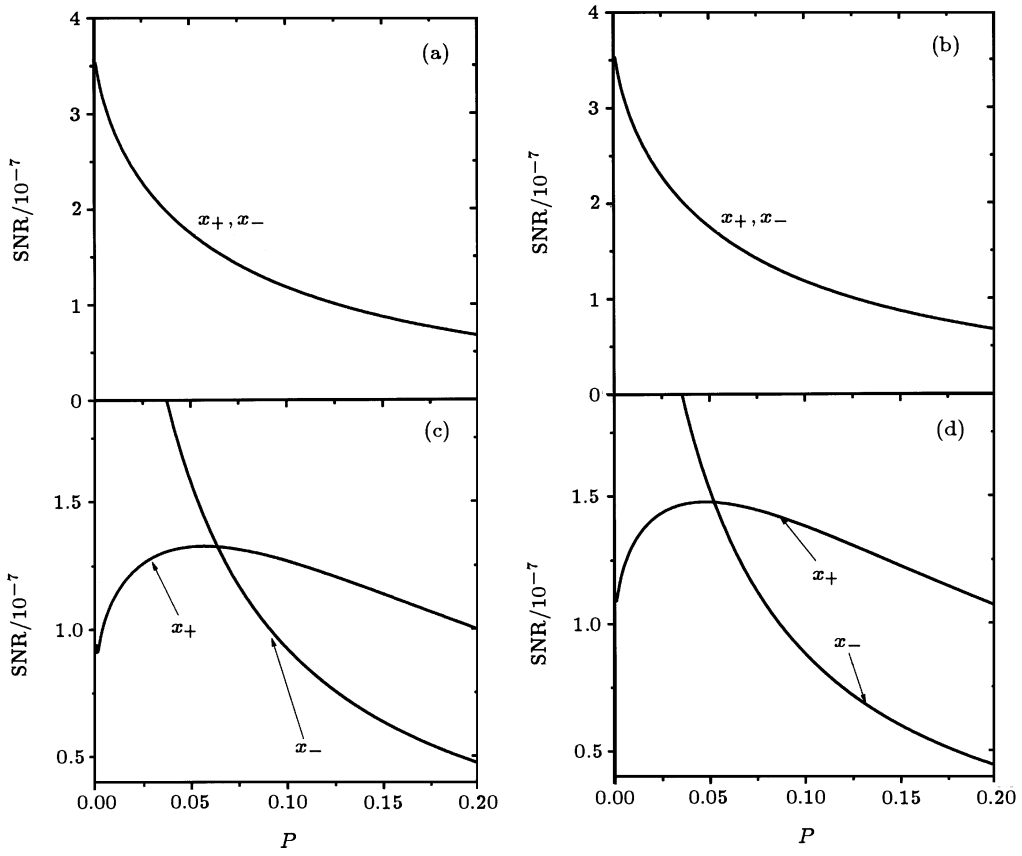


Fig.5. The SNR of the system as a function of the additive noise intensity P when initial conditions are changed with different noise coupling constant λ. The parameters are chosen as P'=0.08, $\varepsilon = 0.0002$, $\Omega =$ 0.0001. (a) λ=0, τ=0; (b) λ=0, τ=0.04; (c) λ=0.5, τ=0; (d) λ=0.5, τ=0.04.

Figures 6(a)–(d) are plots of the SNR as a function of P' for different initial conditions $x(0)$ when the coupling strength λ and noise correlation time τ are varied. The other parameters Ω, ε, and P are fixed. From Figs.6(a) and (b) it can be seen that there is no dependence of SNR on $x(0)$ when the noise coupling strength is $\lambda = 0$. From Figs.6(c) and (d) it is clear that the curve of the SNR depends on the initial condition $x(0)$ when the noise coupling strength is $\lambda = 0.5$. There is a peak in SNR at nonzero value of P' when the initial condition is $x(0) = x_+$, while there are a valley and a peak in SNR when the initial condition is $x(0) = x_-$. When the noise correlation time is increased from $\tau = 0$ to $\tau = 0.04$ with initial condition $x(0) = x_+$, the height of the peak in SNR decreased and the position of the peak is shifted to larger values of P'. For the initial condition $x(0) = x_-$, the height of the dip in SNR increases while the height of the peak only increases slightly. The tendency of the SR phenomenon is to disappear.

From Figs.5 and 6 it can be seen that the dependence of SR on the initial condition $x(0)$ is entirely due to the coupling between additive and multiplicative noise terms.

The SNR, which comes from the power spectrum, is a quantity that characterizes the process over an infinitely long timescale. After a few transitions and/or periods of the external signal, such a stochastic system should forget their initial state. The source of the dependence of SNR on the initial condition can be traced back to Eqs.(13) and (16). Because the correlation breaks the symmetry of the system, the original calculation cannot be repeated with a single W_0, which is replaced by either W_{0+} or W_{0-} at the very end. From the beginning of the calculation starting with Eq.(13), W_{0+} and W_{0-} have to be supposed separately. This can also be seen clearly from Eq.(16). If $\lambda \neq 0$, $W_0 = W_{0\pm}$, the SNR will depend on the initial condition $x(0)$. The initial conditions $x(0) = x_\pm = \pm\sqrt{a}$ are multiplied by λ and appear in the second, third, and fourth terms in the exponential in the right-hand side of Eq.(16). However, if $\lambda = 0$, all terms associated with the initial condition will disappear. Equation (16) is reduced to $W_0 = \frac{\sqrt{2}a}{\pi}\exp\left[-\frac{1}{P'}\left\{-\frac{a}{2}+\left(\frac{P}{2aP'}+\frac{a}{2}\right)\ln\left(\frac{aP'}{P}+1\right)\right\}\right]$.

Obviously, the SNR does not depend on the initial condition $x(0)$ when the coupling between two noise terms is $\lambda = 0$.

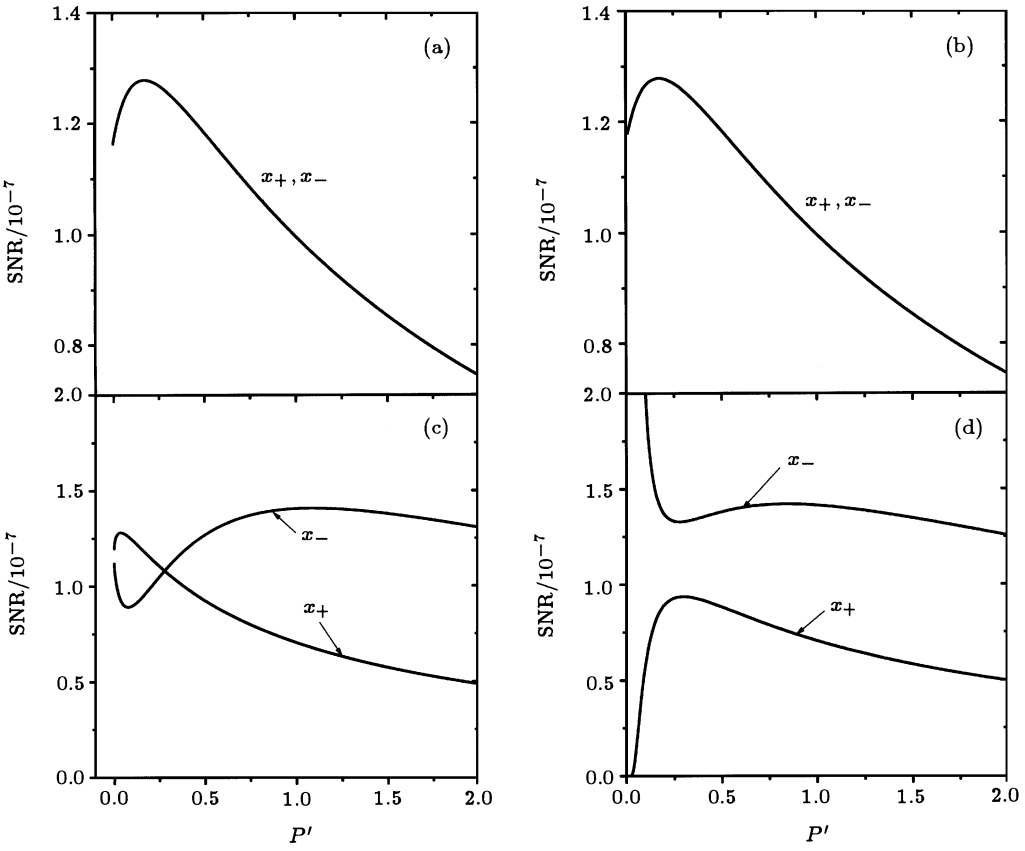


Fig.6. The SNR of the system as a function of the multiplicative noise intensity P' when initial conditions are changed with different noise coupling constant λ. The parameters are chosen as P=0.03, $\varepsilon = 0.0002$, $\Omega = 0.0001$. (a) λ=0, τ=0; (b) λ=0, τ=0.04; (c) λ=0.5, τ=0; (d) λ=0.5, τ=0.04.

5. Numerical simulation

In order to check the validity of the approximation method employed in the derivation, it is necessary to perform numerical simulation. The coloured noise term in Eq.(3) can be described by a differential equation of Ornstein–Uhlenbeck (OU) noise with constant intensity. Equations (1) and (3) can be stochastically equivalent to

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= ax - x^3 + \varepsilon\cos\Omega t + p(t)x \\ &\quad + \sqrt{2x}\varsigma(t) + q(t), \\ \frac{\mathrm{d}\varsigma(t)}{\mathrm{d}t} &= -\frac{\varsigma(t)}{\tau} + \frac{\rho(t)}{\tau}, \end{aligned} \tag{22}$$

where $\rho(t)$ is Gaussian white noise with zero mean and correlation

$$\langle \rho(t)\rho(t')\rangle = 2\lambda\sqrt{PP'}\delta(t-t'). \tag{23}$$

Here $2\lambda\sqrt{PP'}$ is the strength of the coloured noise.

The numerical simulation can be performed by integrating the dynamical equations of (22) and (23). Gaussian white noise is generated using the Box–Muller algorithm and a pseudo-random number generator.[42,43] The numerical data of time series are obtained using Euler procedure. Then the data are calculated using a fast Fourier transform. To reduce the variance of the result, typical 500 ensembles of power spectra, each containing 32 periods of the signal are averaged. The output signal-to-noise ratio is defined as the ratio of the strength of the single peak to the mean amplitude of the background noise at the input signal frequency.

The results of numerical calculations of the SNR are plotted in Fig.7. In Figs.7(a) and (b) the SNR are plotted as functions of additive noise intensity P when the noise correlation time is $\tau = 0.01$. It is seen that the curve of SNR decreases monotonically for the initial condition $x(0) = x_+$. While the initial condition

is $x(0) = x_-$, there is a peak in the curve of SNR. In Figs.7(c) and (d) the SNR are plotted as functions of multiplicative noise intensity P' when the noise correlation time is $\tau = 0.05$. It is shown that there is a peak in the curve of SNR for the initial condition $x(0) = x_+$. While the initial condition is $x(0) = x_-$, the curve of SNR decreases for small values of multiplicative noise intensity P', and then increases with a peak when P' is increased further. It is clear that the approximate analytic results of the SNR plotted in Figs.1 and 2 are consistent with the computer simulations.

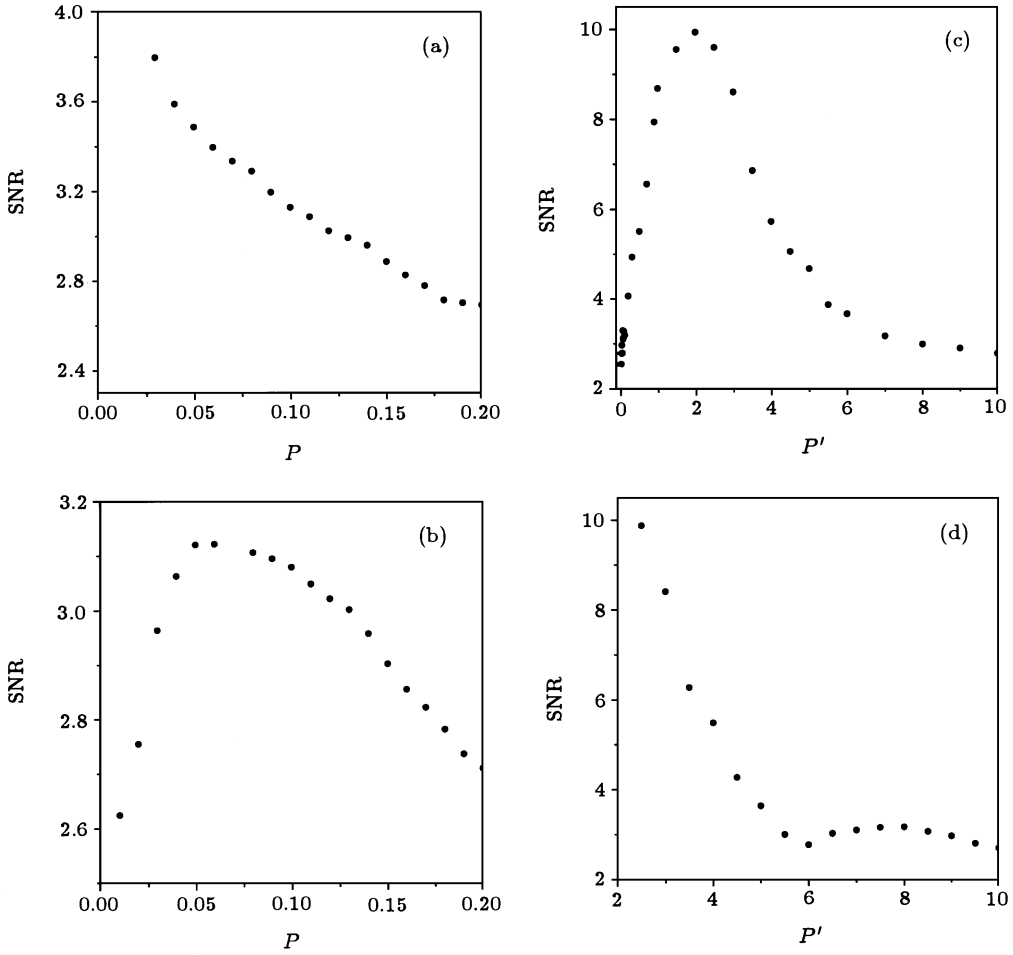


Fig.7. The numerical simulations of the SNR as function of the additive and multiplicative noise intensity P and P' respectively. The parameters are chosen as $\varepsilon = 0.0002, \Omega = 0.0001$. (a), (b) P'=0.5, τ=0.01; (c), (d) P=0.1, τ=0.05; (a), (c) The initial condition $x(0) = x_+$; (b) (d) The initial condition $x(0) = x_-$.

6. Summary

The stochastic resonance in a nonlinear bistable system driven by a weak periodic signal is investigated when the additive and multiplicative noise terms are coupled in the system.

It is shown that the SR phenomenon depends on the initial conditions of the system. For positive initial condition $x(0) = x_+$, either no SR appears for additive noise or there is no significant change for multiplicative noise as the noise correlation time τ is varied. Meanwhile, for negative initial condition $x(0) = x_-$, the SR is changed from one peak to two peaks with one dip, and then to a monotonically decreasing function of multiplicative noise when the noise correlation time τ is increased.

For additive noise, strong SR appears for the noise coupling strength λ=-1.0 and the SR disappears for $\lambda \geq 0.0$ with positive initial condition $x(0) = x_+$. The SNR shows opposite behaviour with strong SR at λ=1.0 for negative initial condition of $x(0) = x_-$.

For multiplicative noise, the peak in SNR appears at $\lambda \leq 0.0$ for positive initial condition $x(0) = x_+$ and at $\lambda \geq 0.0$ for negative initial condition $x(0) = x_-$.

It is clear that the dependence of the SR phe-

nomenon on the initial conditions $x(0)$ is entirely due to the coupling λ between two different noise terms. If the coupling $\lambda = 0$, it is reduced to standard SR and there is no dependence of the SR on $x(0)$.

The noise correlation time τ can change the phenomenon of SR dramatically. Increasing τ can not only reduce the height of the peak in SNR but also change the shape of SNR from one peak to two peaks with one dip, and finally monotonically decreasing. The phenomenon of SR will disappear when the noise correlation time τ is very large.

Finally, the results of numerical simulation verifies that the approximation method used in the derivation is valid.

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