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Finite-amplitude vibration of a bubble and sonoluminescence*

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Numerical solutions of the differential equation for a bubble performing finite-amplitude vibration are given in detail for a variety of situations. The results demonstrate that in lower acoustic pressure (maximum Mach number very low) its vibration has bounce. When acoustic pressure is in excess of 1.18atm and the instantaneous radius of the bubble approaches its equivalent Van der Waals radius, the maximum velocity and acceleration on the surface of a bubble have a huge increase in a very short period, which seems to favour the sonoluminescence. In vacuum environment (0.1atm), an intensive sonoluminescence could be generated.

Keywords: bubble, finite-amplitude vibration, sonoluminescence
PACC: 4325

A bubble performing an infinitesimal-amplitude vibration can be regarded as a simple harmonic vibrator, of which the change of the volume from its equilibrium value satisfies a linear differential equation.[1] In actual cases, however, its amplitude is always finite and the Mach numbers are large enough so that the linear differential equation cannot be applied. Several decades ago, many papers dealing with this problem were published.[2–6] Typically, the Rayleigh–Plesset (RP) equation has been extensively used to investigate the bubble's sonoluminescence. As is well known, the RP equation can only be used in situations where the Mach numbers are small enough.[7,8] Unfortunately, when a bubble is compressed to the neighbourhood of the minimum volume, its maximum Mach number will be always near to or even far greater than 1. Another fact to be noticed is that the volume viscosity was not taken into account in the RP equation. From Refs.[8–10], however, we realized that this is not true in many liquids: for example, in water the volume viscosity is many times larger than the shear viscosity.[9] In consideration of these facts, a nonlinear differential equation for a bubble performing finite-amplitude vibration has been given in Ref.[8], i.e. (an error has been corrected in the formula (5) of Ref.[8], where the term $\frac{3}{2}(c+\dot{R})^2\dot{R}^2$ was typeset as $\frac{3}{2}(c+\dot{R})\left(c+\frac{2}{3}\dot{R}\right)^2\dot{R}^2$ incorrectly)

$$\begin{aligned}
&\left\{h+\left[\frac{1}{2}\dot{R}^2(\tau)+c\dot{R}(\tau)+c^2\right]-\frac{b}{\rho^2 c}\frac{\mathrm{d}p}{\mathrm{d}R}\right\}R(\tau)\ddot{R}(\tau)\\
&+\frac{3}{2}[c+\dot{R}(\tau)][c+\dot{R}(\tau)]\dot{R}^2\\
=&h[c+\dot{R}(\tau)][c+2\dot{R}(\tau)]+R(\tau)[c+\dot{R}(\tau)]\frac{\partial h}{\partial\tau}\\
&+\frac{b\dot{R}(\tau)}{\rho^2c^2}\frac{\mathrm{d}p}{\mathrm{d}r}\left\{[c+\dot{R}(\tau)][c+2\dot{R}(\tau)]-R(\tau)\frac{\partial c}{\partial\tau}\right\}\\
&+\frac{b}{\rho}\left\{\frac{\mathrm{d}p}{\mathrm{d}R}\frac{\partial}{\partial\tau}\left(\frac{1}{c^2\rho}\right)+\frac{\dot{R}(\tau)}{c^2\rho}\frac{\partial^2 p}{\partial^2 R}\right\}\\
&\times R(\tau)\dot{R}(\tau)[c+\dot{R}(\tau)]-\left[h+\frac{1}{2}\dot{R}^2(\tau)\right]R(\tau)\frac{\partial c}{\partial\tau},
\end{aligned}\tag{1}$$

where $R(\tau)$ is the instantaneous radius of the bubble, $\dot{R}(\tau)$ and $\ddot{R}(\tau)$ are the velocity and the acceleration on the surface of the bubble, respectively, c is the sound speed in the liquid surrounding the bubble, and [8]

$$b=\varsigma+\frac{4}{3}\eta,\tag{2}$$

$$h=\int_{p(\infty)}^{p(R)}\frac{\mathrm{d}p}{\rho},\tag{3}$$

$$p(R)=\left(p_0+\frac{2\sigma}{R_0}\right)\left(\frac{R_0^3-a^3}{R^3-a^3}\right)^{\gamma}-\frac{2\sigma}{R},\tag{4}$$

where p_0 is the environmental pressure, a is the equivalent Van der Waals radius, σ is the surface tension

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coefficient, and γ is the ratio of specific heats of the gas in the bubble. Obviously, Eq.(1) is an intractable nonlinear differential equation, so any attempt to obtain an analytic solution seems to be impossible. In Ref.[8] only a crude numerical computation for a bubble in water was given. In this paper, the numerical solutions will be investigated thoroughly by means of the symbolic computation program at the frequency f=26kHz, the radius of the bubble R_0=4.5μm.

The curve in Fig.1 shows the relationship between $R(t)$ with p_0=p_a=1atm, which is a single period vibration with a serious bounce, where its maximum Mach number is 1.13×10^{-3}. When p_0=1atm and $1.18\geqq p_a \geqq 1.1$atm, the curves of the solutions are no longer single periodic. Figure 2 shows the relationship between $R(t)$ and t for p_a=1.18atm and the maximum Mach number approaching 0.2, where two groups of peaks occur alternately (double periodic), each of which is accompanied by serious bounce. This numerical computation was carried out until t=450 μs, but only the results up to t=160 μs are given in Fig.2.

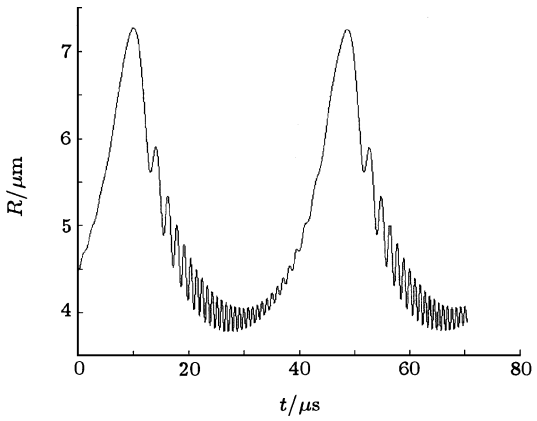


Fig.1. Radius of an air bubble in water vs time. p_0=1atm, p_a=1atm, f=26kHz, R_0=4.5μm.

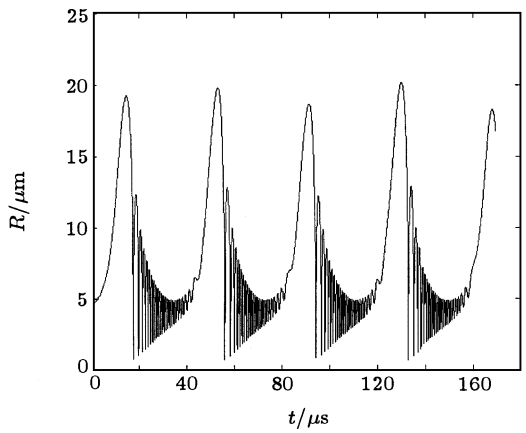


Fig.2. R(t) vs t. p_0=1atm, p_a=1.18atm, R_0=4.5 μm and f=26kHz.

As was pointed out by Ref.[11], the vibration of a bubble is very sensitive to the acoustical pressure,

$$\dot{R}(t) \propto (p_a)^n \tag{5}$$

and

$$n = \left[\frac{f_{\rm r}(R_0)}{f}\right], \tag{6}$$

where $f_{\rm r}(R_0)$ is the Minnaert resonant frequency of a bubble whose radius is R_0, f is the frequency of the acoustical pressure, and the notation $[x]$ means that only the integer part of x will be taken. For R_0=4.5μm and f=26kHz, $n \approx$28. Figure 3 shows the comparison of the numerical results from Eq.(1) with the results from (5) and (6), where the ordinate is normalized by the surface velocity at p_a=1atm.

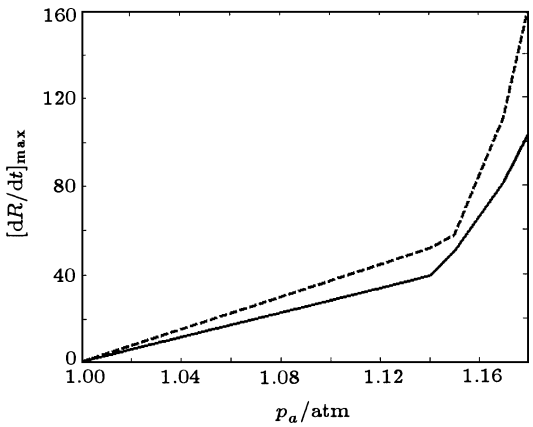


Fig.3. Maximum velocity on the surface of bubble vs sound pressure p_a. p_0=1atm, R_0=4.5μm, and f=26kHz. Solid line: from the numerical solutions of Eq.(1). Dashed line: from Eqs.(5) and (6).

When $p_a \geqq$1.19atm, all the computation will overflow near the neighbourhood of the Van der Waals radius, where the maximum Mach number will be larger than 16 at $R(t)$=0.63μm. Obviously, the $R(t) \sim t$ curves bounce no more. Figures 4(a), (b), and (c) show the comparison of $R(t)$, $\dot{R}(t)$, $\ddot{R}(t)$ in p_a=1.18atm (bounce) with p_a=1.19atm (no bounce), respectively, from which we can see that in the neighbourhood of the Van der Waals radius the velocity and acceleration on the surface of the bubble have a huge increase in a very short period (less than one picosecond). It seems that there is a 'pressure barrier' between 1.18 and 1.19atm of the sound pressure, across which the vibration of the bubble no longer bounces and its instantaneous radius can approach the Van der Waals' radius more easily, so that the maximum kinetic energy of the bubble will have a huge increase and will be beneficial to sonoluminescence. When the

sound pressure is lower than 1.18atm, the air bubble will have bounce, which means that most parts of its energy cannot enter into the interior of the gas so that the light emission will be very weak or even vanish. From these facts we can understand why in the low sound pressure region only a few measurements showed very weak light emission.[12,13] Figures 5(a)–(c) display the results for p_a=1.35atm, where some errors in the scale of coordinates in the Fig.3 of Ref.[8] have been corrected.

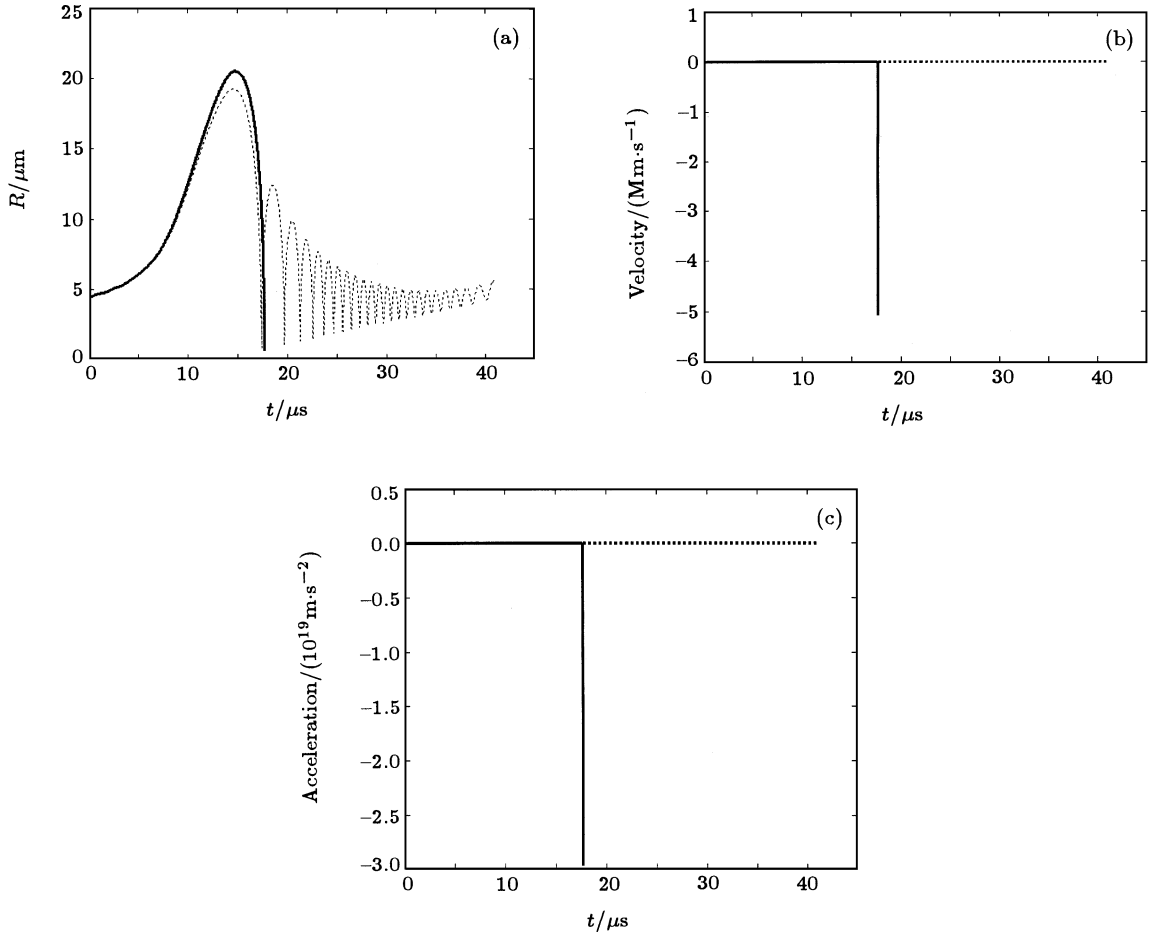


Fig.4. (a) Radius versus time. Dotted line: p_a=1.18atm; solid line: p_a=1.19atm. (b) Velocity versus time. Dotted line: p_a=1.18atm; solid line: p_a=1.19atm. (c) Acceleration versus time. Dotted line: p_a=1.18atm;. solid line: p_a=1.19atm

An intense implosive collapse of gas or vapour in a bubble can lead to ultrahigh compressions and temperatures and to the generation of sonoluminescence. Many papers, for example Refs.[12–17], reported experimental results in this area. Many years ago, many theoretical works, for example [18–26], were published and various mechanisms of sonoluminescence were considered. In this paper, we start only from total energy of the bubble to investigate this problem.

When a bubble is compressed in the neighbourhood of the Van der Waals radius, its kinetic energy has a huge increase in a short time, which leads easily to the generation of sonoluminescence. At the same time, the bubble releases its energy and finally returns to its equilibrium state. Under continuous drive of the acoustic pressure, the process will be repeated. Thus, there is reason to say that a bubble that has no bounce will generate far more intensive sonoluminescence than a bouncing bubble, or that a bubble with serious bounce can hardly emit light.

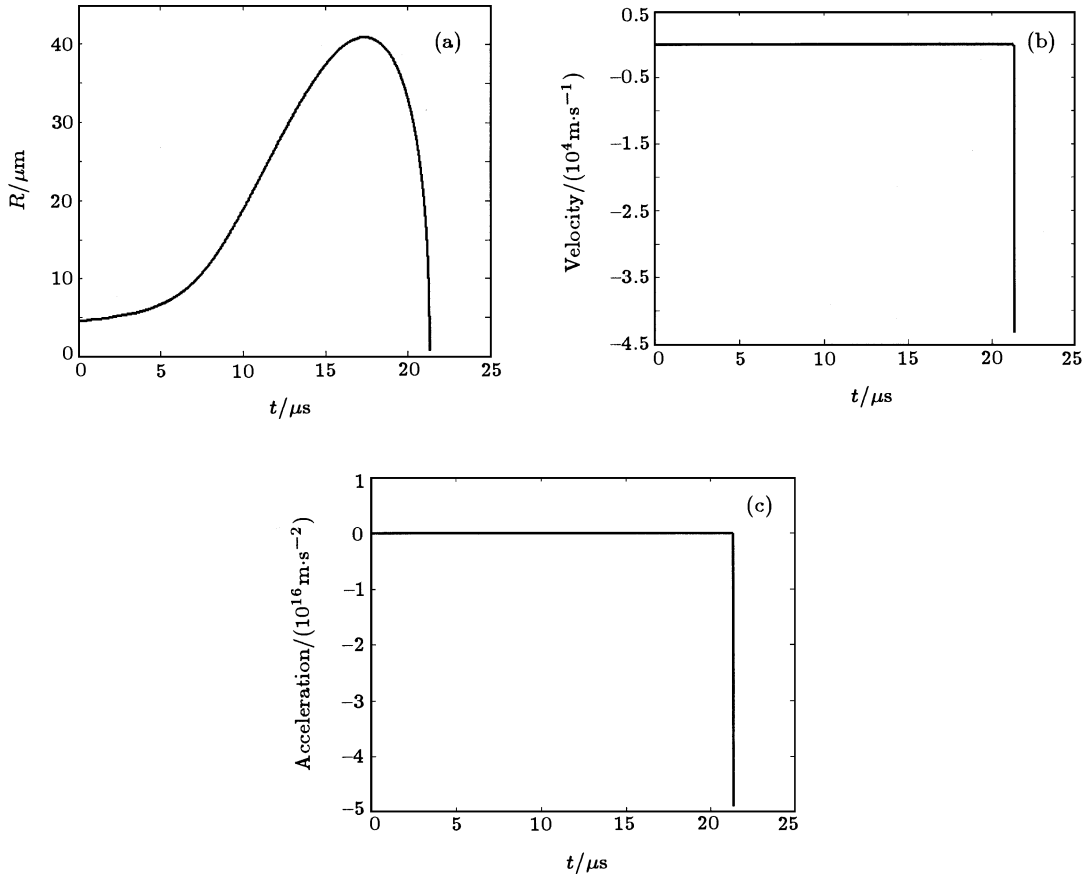


Fig.5. (a) $R(t)$ versus t. p_0=1atm, p_a=1.35atm, R_0=4.5μm, and f=26kHz. (b) $\dot{R}(t)$ versus t. p_0=1atm, p_a=1.35atm, R_0=4.5μm, and f=26kHz. (c) $\ddot{R}(t)$ versus t. p_0=1atm, p_a=1.35atm, R_0=4.5μm, and f=26kHz.

Another important quantity is the FWHM, which depends upon the driving sound pressure.[13,17] Furthermore, the shape of the light pulse approaches the Gaussian. If the emission light is proportional to the kinetic energy density of the bubble, one has

$$\alpha(p_a,\beta)\dot{R}^2 \propto \mathrm{e}^{-\left(\frac{t}{\tau(p_a,\beta)}\right)^2}, \qquad (7)$$

where α and β depend upon the mechanism of sonoluminescence, and τ is proportional to the FWHM. From Eqs.(5)–(7), the relationship between the FWHM and sound pressure can be obtained and the numerical results are listed in Table 1, where the dependence of α on sound pressure is not taken into account.

Table 1.

p_a/atm	1.20	1.25	1.30	1.35	1.40	1.50
$\tau(p_a)/\tau(1.2)$	1.00	1.51	2.12	2.57	2.94	3.53

In previous paragraphs, the vibration of a bubble was investigated only at the normal environmental pressure (one atmosphere). In the following the vibration in vacuum environment (for example, 0.1atm) will be investigated. In general, at lower environmental pressure the maximum values of $\dot{R}(t)$, $\ddot{R}(t)$ as well as $R(t)$ have a huge increase. Figures 6(a), (b) and (c) display the relationship of $R(t)$, $\dot{R}(t)$, and $\ddot{R}(t)$ vs t, respectively, where p_a=1.15atm; the solid line refers to p_0=0.1atm and the dashed line to p_0=1atm. These results demonstrate that in the vacuum environment the bounce disappears and the maxima of $R(t)$, $\dot{R}(t)$ and $\ddot{R}(t)$ have a huge increase that seems to favour the sonoluminescence. In Figs.7(a), (b) and (c), the curves describe the radius, velocity and acceleration of a bubble (the acoustical pressure p_a=15atm), where the solid line and the dashed line refer to the environmental pressure p_0=1atm and p_0=0.1atm, respectively, where some errors (in the scale of coordinates) in Fig.4 of Ref.[8] have been corrected. In a vacuum environment, obviously, the resonant peak of a bubble splits into three and its maximum value increases a little. However, the maximum velocity and acceleration have a huge increase.

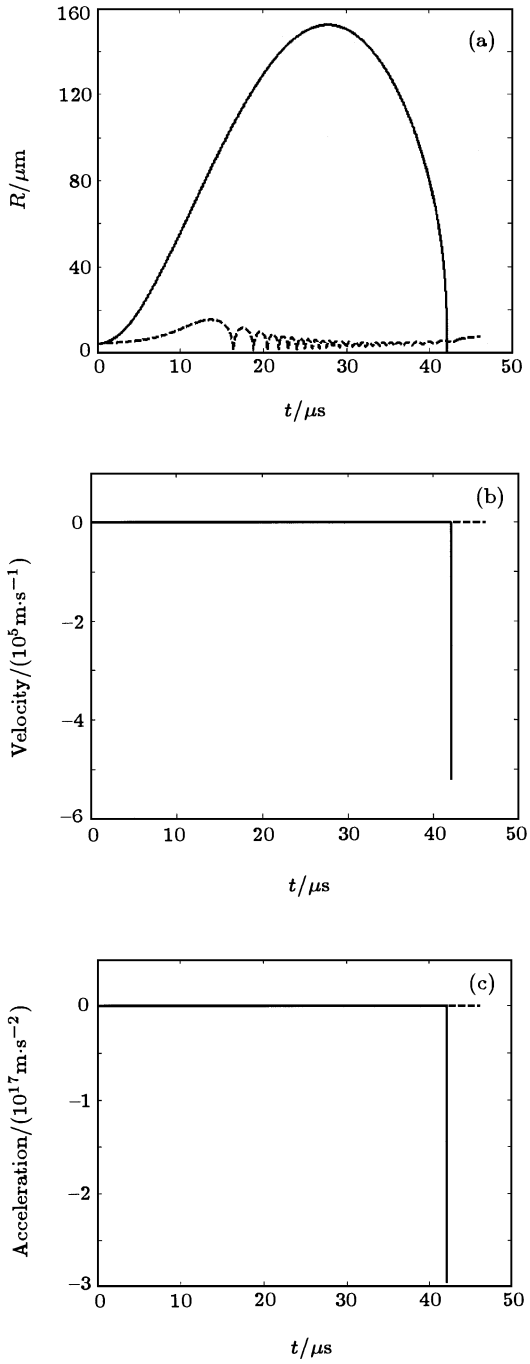


Fig.6. (a) $R(t)$ versus t. p_a=1.15atm. Solid line: p_0=0.1atm; dashed line: p_0=1atm. (b) Velocity versus time. p_a=1.15atm. Solid line: p_0=0.1atm, dashed line: p_0=1atm. (c) Acceleration versus time. p_a=1.15atm. Solid line: p_0=0.1atm, dashed line: p_0=1atm.

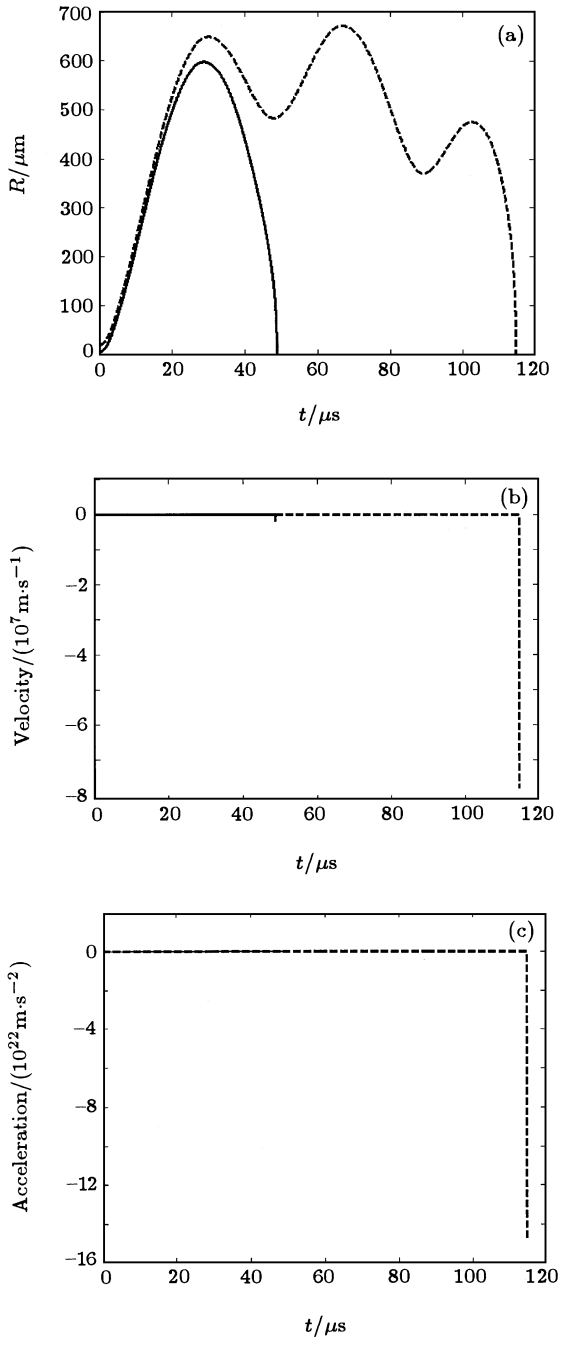


Fig.7. (a) Radius of an air bubble in water versus time. p_a=15atm, f=26kHz, R_0=4.5μm; dashed line: p_0=0.1atm; solid line: p_0=1atm. (b) Velocity versus time. p_a=15atm. Solid line: p_0=1atm; dashed line: p_0=0.1atm. (c) Acceleration versus time. p_a=15atm. Solid line: p_0=1atm (acceleration too small); dashed line: p_0=0.1atm.

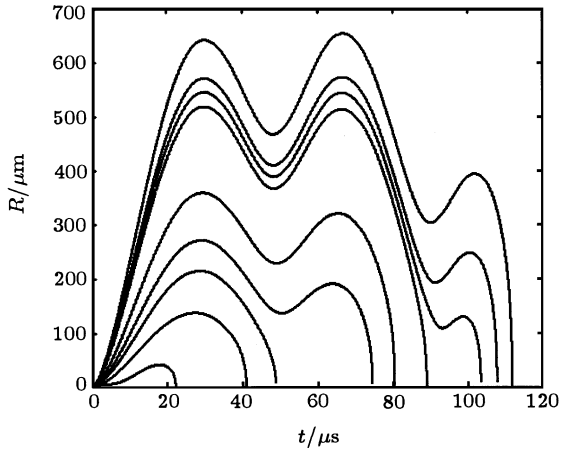


Fig.8. Radii of bubble in water versus time for various sound pressure (p_a=15, 12, 11, 10, 5, 3, 2, 1.35, and 1atm from top to bottom). In all cases, the environment pressure and radius are 0.1atm and 4.5μm, respectively.

The curves in Fig.8 show the relationship between the instantaneous radius of the bubble and the time, where the environmental pressure p_0=0.1atm. The Van der Waals' radius $a \approx R_0/18$, and the acoustical pressure are 15, 12, 11, 10, 5, 3, 2, 1.35, and 1.0atm from the top to bottom of the figure. These results demonstrate how the resonant peaks of the bubble come to split with the increase of the sound pressure, where each run of computation is continued until it overflows when the instantaneous radius of the compressed bubble approaches its Van der Waals value.

References

[1] Devin C Jr. 1959 *J. Acoust. Soc. Am.* **31** 1654
[2] Rayleigh L 1917 *Phil. Mag.* **34** 94
[3] Trilling L 1952 *J. Appl. Phys.* **23** 14
[4] Girmore P R 1952 *Calif. Inst. Tech. Report* No.26-4
[5] Flynn H 1957 *J. Acoust. Soc. Am.* **57** 1379
[6] Plesset M S and Chapman R B 1971 *J. Fluid Mech.* **47** 283
[7] Qian Z W 2000 *Tech. Acoust.* **19** 106 (in Chinese)
[8] Qian Z W and Xiao Ling, 2003 *Chin. Phys. Lett.* **20** 80
[9] Markham J J *et al* 1951 *Rev. Mod. Phys.* **23** 353
[10] Qian Z W 1998 *J. Sound Vib.* **211** 791
[11] Qian Z W 2001 *Chin. Phys.* **10** 636
[12] Gompf B *et al* 1997 *Phys. Rev. Lett.* **79** 1405
[13] Pecha B *et al* 1998 *Phys. Rev. Lett.* **81** 717
[14] Gaitan G F *et al* 1992 *J. Acoust. Soc. Am.* **91** 3166
[15] Barber B P *et al* 1997 *Phys. Rep.* **281** 65
[16] Moran M J *et al* 1995 *Nucl. Instrum. Methods Phys. Res.* B **96** 651
[17] Lu M J *et al* 2003 *Chin. Phys. Lett.* **20** 141
[18] Wu C C and Roberts P H 1993 *Phys. Rev. Lett.* **70** 3424
[19] Moss W C *et al* 1994 *Phys. Fluids* 6 2979
Moss W C *et al* 1997 *Science* **276** 1398
[20] Brenner M P *et al* 1996 *Phys. Rev. Lett.* **76** 1158
[21] Hilgenfeld S *et al* 1999 *Nature* **398** 402
[22] Xie Z X *et al* 1998 *Chin. Phys. Lett.* **15** 813
[23] Xu N *et al* 1998 *Phys. Rev.* E **57** 1615
Xu N *et al* 1999 *Phys. Rev. Lett.* **83** 2441
[24] Yuan L *et al* 1998 *Phys. Rev.* E **57** 4265
Cheng H Y *et al* 1998 *Phys. Rev.* E **58** R2705
[25] Xie C G *et al* 2003 *Acta Phys. Sin.* **52** 102 (in Chinese)
[26] Liu H J *et al* 2003 *Acta Phys. Sin.* **52** 620 (in Chinese)