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# Generation of motional Fock states of a trapped ion in the mediate-excitation regime*

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We propose a scheme to prepare Fock states for the vibrational motion of a trapped ion. Unlike previous schemes, the present scheme works in the mediate-excitation regime, in which the corresponding Rabi frequency is equal to the trap frequency. Thus, the required time is greatly shortened, which is important in view of decoherence.

**Keywords:** Fock state, trapped ion, mediate-excitation regime
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In recent years, there has been much interest in the problem of generating arbitrary quantum states of a harmonic oscillator. In the context of cavity QED, a number of novel schemes have been proposed for generating various nonclassical states of an electromagnetic field.[1−4] Of special interest are Fock states, which are the most basic quantum states. Recently, two-photon states of a cavity field have been observed.[5]

On the other hand, recent advances in laser cooling and ion trapping have opened new prospects in quantum state engineering. The extremely weak coupling between the vibrational motion of the trapped ion and the external environment provides the possibility of generating various nonclassical states of the ion motion with high efficiency. Recently, proposals have been made for generating various nonclassical vibrational states of a trapped ion, such as Fock states,[6] squeezed states [7] and Schrödinger cat states.[8−10] To date, motional Fock states, squeezed states, coherent states,[11] and Schrödinger cat states [12] have been observed.

The above-mentioned schemes [6,11] for generating motional Fock states operate in the low-excitation regime, in which the Rabi frequency $\Omega$ characterizing the laser–ion interaction is much smaller than the trap frequency $\nu$. In this paper, we show how we can generate motional Fock states in the mediate-excitation regime, in which the corresponding Rabi frequency is on the order of the trap frequency. From the viewpoint of decoherence, the present scheme may be of importance due to the great reduction of the operation time by using a laser of higher intensity.

We consider an ion trapped in a one-dimensional harmonic potential. We excite this ion with a laser beam tuned to the ion transition. In the rotating-wave approximation, the Hamiltonian for this system is given by (assuming $\hbar = 1$)

$$\hat{H} = \hat{H}_0 + \hat{H}_{\mathrm{I},\phi}, \tag{1}$$

where

$$\hat{H}_0 = \nu \hat{a}^+ \hat{a} + \omega_0 \hat{\sigma}_z, \tag{2}$$

$$\hat{H}_{\mathrm{I},\phi} = \frac{\Omega}{2}[\hat{\sigma}^+ \mathrm{e}^{-\mathrm{i}(\omega_0 t - k_0 \hat{x} + \phi)} + \mathrm{H.c.}], \tag{3}$$

$\hat{\sigma}_z = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$ and $\hat{\sigma}^+ = |e\rangle\langle g|$, with $|e\rangle$ and $|g\rangle$ being the excited and ground states of this ion; $\hat{a}^+$ and $\hat{a}$ are the creation and annihilation operators for the vibrational mode, $\nu$ is the vibrational frequency of the centre-of-mass mode, and $\omega_0$ is the transition frequency for the two-level ion, and $\Omega$, $\phi$, and $k_0$ are the Rabi frequency, phase and wavevector of the laser frequency, respectively. The position operator $\hat{x}$ can be expressed by $\hat{x} = \eta(\hat{a}^+ + \hat{a})/k_0$, with $\eta = k_0\sqrt{1/(2\nu M)}$ being the Lamb–Dicke parameter and $M$ being the

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mass of the trapped ion. Then the interaction Hamiltonian $\hat{H}_{\mathrm{I},\phi}$ of Eq.(3) can be rewritten as [13]

$$\hat{H}_{\mathrm{I},\phi} = \frac{\Omega}{2}\mathrm{e}^{-\eta^2/2}\Big[\hat{\sigma}^+\mathrm{e}^{-\mathrm{i}(\omega_0 t+\phi)} \times \sum_{m,n=1}^{\infty}\frac{(\mathrm{i}\eta)^{m+n}}{m!n!}\hat{a}^{+m}\hat{a}^n + \mathrm{H.c.}\Big]. \quad (4)$$

Consider the behaviour of the trapped ions in the Lamb–Dicke regime, where $\eta\sqrt{\bar{n}+1} \ll 1$, with $\bar{n}$ being the mean quantum number of the centre-of-mass mode. In this case we can retain the terms only to the first order in $\eta$ and $\mathrm{e}^{-\eta^2/2} \simeq 1$. Then we obtain

$$\hat{H}_{\mathrm{I},\phi} = \frac{\Omega}{2}\{\hat{\sigma}^+\mathrm{e}^{-\mathrm{i}(\omega_0 t+\phi)}[1+\mathrm{i}\eta(\hat{a}^+ + \hat{a})] + \mathrm{H.c.}\}. \quad (5)$$

In the interaction picture, the interaction Hamiltonian is given by

$$\hat{H}'_{\mathrm{I},\phi} = \mathrm{e}^{\mathrm{i}\hat{H}_0 t}\hat{H}_{\mathrm{I},\phi}\mathrm{e}^{-\mathrm{i}\hat{H}_0 t} = \frac{\Omega}{2}\{\hat{\sigma}^+\mathrm{e}^{-\mathrm{i}\phi}[1+\mathrm{i}\eta(\hat{a}^+\mathrm{e}^{\mathrm{i}\nu t}+\hat{a}\mathrm{e}^{-\mathrm{i}\nu t}) + \mathrm{H.c.}\}. \quad (6)$$

We define the new atomic basis [14]

$$|+_\phi\rangle = \frac{1}{\sqrt{2}}(|e\rangle + \mathrm{e}^{\mathrm{i}\phi}|g\rangle), \quad (7)$$

$$|-_\phi\rangle = \frac{1}{\sqrt{2}}(|e\rangle - \mathrm{e}^{\mathrm{i}\phi}|g\rangle). \quad (8)$$

This corresponds to

$$|e\rangle = \frac{1}{\sqrt{2}}(|+_\phi\rangle + |-_\phi\rangle), \quad (9)$$

$$|g\rangle = \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\phi}(|+_\phi\rangle - |-_\phi\rangle). \quad (10)$$

Then we can rewrite $\hat{\sigma}^+$ and $\hat{\sigma}^-$ as

$$\hat{\sigma}^+ = \mathrm{e}^{\mathrm{i}\phi}[\hat{S}_{z,\phi} + (\hat{S}^-_\phi - \hat{S}^+_\phi)/2], \quad (11)$$

and

$$\hat{\sigma}^- = \mathrm{e}^{-\mathrm{i}\phi}[\hat{S}_{z,\phi} - (\hat{S}^-_\phi - \hat{S}^+_\phi)/2], \quad (12)$$

where $\hat{S}_{z,\phi} = \frac{1}{2}(|+_\phi\rangle\langle+_\phi| - |-_\phi\rangle\langle-_\phi|)$, $\hat{S}^+_\phi = |+_\phi\rangle\langle-_\phi|$ and $\hat{S}^-_\phi = |-_\phi\rangle\langle+_\phi|$. Inserting Eqs.(11) and (12) into Eq.(6), we obtain

$$\hat{H}'_{\mathrm{I},\phi} = \hat{H}_{\mathrm{I},0} + \hat{H}_{\mathrm{i},\phi}, \quad (13)$$

where

$$\hat{H}_{\mathrm{I},0} = \Omega\hat{S}_z, \quad (14)$$

$$\hat{H}_{\mathrm{i},\phi} = \frac{1}{2}\mathrm{i}\eta\Omega(\hat{S}^- - \hat{S}^+)(a^+\mathrm{e}^{\mathrm{i}\nu t} + \mathrm{H.c.}). \quad (15)$$

The time evolution of this system is decided by Schrödinger's equation:

$$\mathrm{i}\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = \hat{H}'_{\mathrm{I},\phi}|\psi(t)\rangle. \quad (16)$$

We perform the unitary transformation

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}\hat{H}_{\mathrm{I},0}t}|\psi'(t)\rangle. \quad (17)$$

Then we have

$$\mathrm{i}\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = \hat{H}_{\mathrm{I},0}|\psi(t)\rangle + \mathrm{i}\mathrm{e}^{-\mathrm{i}\hat{H}_{\mathrm{I},0}t}\frac{\mathrm{d}|\psi'(t)\rangle}{\mathrm{d}t}. \quad (18)$$

Using Eqs.(13) and (16) we obtain

$$\mathrm{i}\mathrm{e}^{-\mathrm{i}\hat{H}_{\mathrm{I},0}t}\frac{\mathrm{d}|\psi'(t)\rangle}{\mathrm{d}t} = \hat{H}_{\mathrm{i},\phi}|\psi(t)\rangle = \hat{H}_{\mathrm{i},\phi}\mathrm{e}^{-\mathrm{i}\hat{H}_{\mathrm{I},0}t}|\psi' t)\rangle. \quad (19)$$

Then we obtain

$$\mathrm{i}\frac{\mathrm{d}|\psi'(t)\rangle}{\mathrm{d}t} = \hat{H}'_{\mathrm{i},\phi}|\psi'(t)\rangle, \quad (20)$$

where

$$\hat{H}'_{\mathrm{i},\phi} = \mathrm{e}^{\mathrm{i}\hat{H}_{0,\phi}t}\hat{H}_{\mathrm{i},\phi}\mathrm{e}^{-\mathrm{i}\hat{H}_{0,\phi}t} \mathrm{i}\eta\frac{\Omega}{2}[\hat{S}^-_\phi\hat{a}^+\mathrm{e}^{\mathrm{i}\delta t} - \hat{S}^+_\phi\hat{a}\mathrm{e}^{-\mathrm{i}\delta t} - \hat{S}^+_\phi\hat{a}^+\mathrm{e}^{\mathrm{i}(\nu+\Omega)t} + \hat{S}^-_\phi\hat{a}\mathrm{e}^{-\mathrm{i}(\nu+\Omega)t}], \quad (21)$$

$$\delta = \nu - \Omega. \quad (22)$$

Assume that $\nu + \Omega \gg \eta\frac{\Omega}{2}$. Then the terms $\frac{\eta\Omega}{2}[-\hat{S}^+_\phi\hat{a}^+\mathrm{e}^{\mathrm{i}(\nu+\Omega)t} + \hat{S}^-_\phi\hat{a}\mathrm{e}^{-\mathrm{i}(\nu+\Omega)t}]$ describe the far off-resonant coupling between the states $|+_\phi, n\rangle$ and $|-_\phi, n-1\rangle$ and thus can be discarded.[12,13] $H'_{\mathrm{i},\phi}$ reduces to

$$\hat{H}'_{\mathrm{i},\phi} = \mathrm{i}\eta\frac{\Omega}{2}(\hat{S}^-_\phi\hat{a}^+\mathrm{e}^{\mathrm{i}\delta t} - \hat{S}^+_\phi\hat{a}\mathrm{e}^{-\mathrm{i}\delta t}). \quad (23)$$

In the case $\delta = 0$ we obtain

$$\hat{H}'_{\mathrm{i},\phi} = \mathrm{i}\eta\frac{\Omega}{2}(\hat{S}^-_\phi\hat{a}^+ - \hat{S}^+_\phi\hat{a}). \quad (24)$$

First we assume that $\phi = 0$ and the ion is initially in the state $|+_0\rangle$. The vibrational mode is initially in the vacuum state $|0\rangle$. After the above-mentioned laser–ion interaction we obtain

$$|\psi'_1(\tau_1)\rangle = \cos(g\tau_1)|+_0\rangle|0\rangle + \sin(g\tau_1)|-_0\rangle|1\rangle, \quad (25)$$

where

$$g = \eta\frac{\Omega}{2}. \quad (26)$$

With the choice $g\tau_1 = \pi/2$ we obtain

$$|\psi'_1(\tau_1)\rangle = |-_0\rangle|1\rangle. \quad (27)$$

Now we shift the phase of the laser field by $\pi$. It is easily shown that $|+_{j,\pi}\rangle = |-_{j,0}\rangle$ and $|-_{j,\pi}\rangle = |+_{j,0}\rangle$. In this case the Hamiltonian is

$$H'_{\mathrm{i},\pi} = \mathrm{i}\eta\frac{\Omega}{2}(S_\pi^- a^+ - S_\pi^+ a) = \mathrm{i}\eta\frac{\Omega}{2}(S_0^+ a^+ - S_0^- a). \quad (28)$$

Thus we obtain the time evolution of the system

$$|\psi_2'(\tau_2)\rangle = \cos(\sqrt{2}g\tau_2)|-_0\rangle|1\rangle + \sin(\sqrt{2}g\tau_2)|+_0\rangle|2\rangle. \quad (29)$$

With the choice $\sqrt{2}g\tau_2 = \pi/2$ we obtain

$$|\psi_2'(\tau)\rangle = |+_0\rangle|2\rangle. \quad (30)$$

In this way we obtain a Fock state with two quanta for the vibrational mode. If we shift the phase of the laser field $N$-1 times and the length $\tau_n$ of the $n$th interval satisfies $\sqrt{n}g\tau_n = \pi/2$, we obtain the Fock state $|N\rangle$ for the vibrational mode.

Now we consider another case. The length of the first interval satisfies $g\tau_1 = \pi/4$. Then we have

$$|\psi_1'(\tau_1)\rangle = \frac{1}{\sqrt{2}}[|+_0\rangle|0\rangle + |-_0\rangle|1\rangle]. \quad (31)$$

Then the phase is shifted by $\pi$. The time evolution of the system is

$$|\psi_2'(\tau_2)\rangle = \frac{1}{\sqrt{2}}\{|+_0\rangle|0\rangle + [\cos(\sqrt{2}g\tau_2)|-_0\rangle|1\rangle + \sin(\sqrt{2}g\tau_2)|+_0\rangle|2\rangle]\}. \quad (32)$$

With the choice $\sqrt{2}g\tau_2 = \pi/2$ we obtain

$$|\psi_2'(\tau)\rangle = \frac{1}{\sqrt{2}}[|0\rangle + |2\rangle]|+_0\rangle. \quad (33)$$

In this way we obtain the superposition of the vacuum state and the two-photon state. Such states are important for studying decoherence.[15]

In conclusion, we have proposed a scheme to prepare the vibrational motion of a trapped ion in a Fock state based on the excitation of the ion by a classical laser field tuned to the ion transition. In comparison with the previous schemes,[6,11] the distinct advantage of the present scheme is that the Rabi frequency of the laser field is equal to the vibrational frequency. Therefore, the time needed to complete the procedure is greatly shortened, which is important in view of decoherence. Superpositions of the vacuum state and Fock state with two quanta can also be generated.

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