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Correlation between randomly varying birefringence and random dispersion map in a dispersion-managed soliton system*

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The correlation between perturbations caused by randomly varying birefringence and a random dispersion map is considered in a dispersion-managed soliton system, and their effects on soliton propagation and interaction are investigated numerically. These perturbations lead to the disintegration of a soliton, and enhance the interaction between solitons. The correlation plays an important role, and reinforces these effects. Furthermore, there is a stochastic resonance between two perturbations in the system; here the effect is the largest, and the corresponding distance until disintegration is the shortest. Finally, nonlinear gain and a filter are introduced to effectively suppress these effects.

Keywords: correlation, dispersion-managed soliton system, randomly varying birefringence, random dispersion map, control

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1. Introduction

Optical solitons in fibres are formed by a balance of group dispersion and Kerr nonlinearity; numerical simulation and experiments have demonstrated that solitons can propagate an extended distance without distortion, so they may become the ideal message carrier in a long distance communication.[1]

Because of the asymmetry of the refractive index profile of the core, an optical fibre exhibits birefringence. If the birefringence is randomly varied during the soliton propagation, the travelling pulses will experience waveform distortions, and a dispersive wave radiation will appear in the system and result in decrease of transmission capacity.[2]

Recently, soliton propagation in a dispersion-managed system has been considered most attractive for long-haul optical communications, where the dispersion-managed soliton is a new type of optical soliton. For these dispersion-managed solitons, effects such as Gorden–Haus jitter, emission of radiation, and phase-match four-wave mixing are strongly reduced.[3] But perturbations in a dispersion-managed system do harm to soliton propagation for long-haul optical communications. For example, stochastic perturbations, caused by randomly varying birefringence or a random dispersion map, can lead to disintegration of solitons, and degrade the stability of soliton propagation.[4–6]

Though various noise perturbations are present simultaneously in some stochastic processes, noise perturbations are assumed to have different origins and are treated as independent random variables in most previous investigations. However, in a certain situation noise perturbations may have a common origin and thus may be correlated with each other as well. The experimental measurement and theoretical analysis of a soliton system show that the system must contain many noise perturbations, such as stochastic noise perturbations caused by stochastic dispersion, lump gain, distributing fibre loss, or stochastic variation in fibre radius.[7] Moreover, some noise perturbations are not white but coloured. When white noise perturbation and strong coloured noise perturbation are simultaneously present, their effects on the soliton system should contain their individual contributions and the correlated part. So it is necessary to perform

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a corrected numerical simulation in order to deduce the prediction of the proper soliton response in a system with multiple stochastic perturbations.[8,9]

2. Effects of correlated perturbations on soliton propagation

In a real transmission system, the envelope of a field becomes inhomogeneous because of possible variations in the fibre dispersion, the fibre loss and amplification, and then satisfies [2,3]

$$\begin{aligned}
&\mathrm{i}\frac{\partial u}{\partial Z}+\mathrm{i}\eta_1(Z)\frac{\partial u}{\partial \tau}+\frac{d(Z)}{2}[1+\eta_2(Z)]\frac{\partial^2 u}{\partial \tau^2}\\
&+\left(|u|^2+\frac{1}{3}|v|^2\right)u+\frac{1}{3}v^2u^*\exp(-4\mathrm{i}\beta Z)=0,\\
&\mathrm{i}\frac{\partial v}{\partial Z}-\mathrm{i}\eta_1(Z)\frac{\partial v}{\partial \tau}+\frac{d(Z)}{2}[1+\eta_2(Z)]\frac{\partial^2 v}{\partial \tau^2}\\
&+\left(|v|^2+\frac{1}{3}|u|^2\right)v+\frac{1}{3}u^2v^*\exp(4\mathrm{i}\beta Z)=0, \qquad (1)
\end{aligned}$$

where u and v are normalized elliptically polarized components along two orthogonal directions respectively. $Z=z/z_d$, $z_d=\tau_0^2/|\bar{d}|$, $\tau=t/\tau_0$. τ_0, z_d, $\bar{d}$ and $d(Z)$ are the pulse width, the dispersion length, the path-average dispersion and the dispersion map, respectively. Fluctuations of normalized time delay $\eta_1(Z)$ and dispersion magnitude $\eta_2(Z)$ are thought as the perturbations caused by randomly varying birefringence and random dispersion map.

Although Eq.(1) has a Hamiltonian structure, it is not exactly integrable because of the inhomogeneity (Z-dependent coefficient). Such a behaviour of u may be obtained as its response averaged over both $\eta_1(Z)$ and $\eta_2(Z)$, which results in path-averaged solitons. However, taking a simple average of both $\eta_1(Z)$ and $\eta_2(Z)$ fails to provide the proper response because of the correlations between variations of u, $\eta_1(Z)$ and $\eta_2(Z)$.

In fact, both $\eta_1(Z)$ and $\eta_2(Z)$ change around zero in a practical soliton system, such as a soliton system with random dispersion map. Furthermore, $\eta_1(Z)$ and $\eta_2(Z)$ are assumed to be the stochastic variables, and satisfy

$$\begin{aligned}
&\langle\eta_i(Z)\rangle=0, \qquad i=1,2\\
&\langle\eta_i(Z)\eta_i(Z')\rangle=D_i\delta(Z-Z'),
\end{aligned} \qquad (2)$$

where D_i is the standard deviation.

We assume that one noise perturbation is not white but coloured, such as the perturbation in dispersion amplitude due to the random dispersion map. When white noise perturbations and strong coloured noise perturbations are simultaneously present, their effects on the soliton system should contain their individual contributions and the correlated part. Most previous investigations have been based on the assumption [10]

$$\begin{aligned}
&\langle\eta_1(Z)\eta_2(Z')\rangle\\
&=\langle\eta_2(Z)\eta_1(Z')\rangle=k\sqrt{D_1D_2}\delta(Z-Z'), \quad |k|\le 1, \qquad (3)
\end{aligned}$$

where k is the strength of correlation between two stochastic perturbations.

It should be pointed out that multiplicative perturbations are all assumed to be independent white Gaussian perturbations in previous models, such as in Ref.[7]. However, more realistic physical models of soliton systems require considering the case of coloured perturbations, especially when these perturbations are of the same origin, since the presence of correlation may change the dynamics of the soliton system. When they are entirely or partially of the same origin, the strength of correlation is introduced, and Eqs.(2) and (3) are adopted to demonstrate their correlation.[8]

Numerical simulations of pulse propagation are performed in the optical fibre link with randomly varying birefringence and random dispersion map. The fibre segments z_+ and z_- are 50km, and their dispersions are 2.446ps/km·nm and –2.258ps/km·nm, respectively. For the soliton of a pulse-width 10ps, the dispersion length is 270km corresponding to the average dispersion $\bar{d}$=0.094/km·nm.[3] The random modulation with Gaussian random deviation is used in both normalized time delay and dispersion magnitude of every fibre segment. Uniformly distributed random numbers from the intervals θ_1, θ_2, $\theta\in(0,1)$ are used, and random numbers are produced by the built-in generator Random(x) from the software Matlab. Transformation of uniformly distributed random numbers from the intervals θ_1, θ_2, $\theta\in(0,1)$ into Gaussian random numbers is performed using the following relation:[11]

$$\begin{aligned}
\eta_i=&\sqrt{2D_i}\times[\sqrt{1-k^2}\sin(2\pi\theta_i)\times\sqrt{-2\ln\theta_i}\\
&+k\sin(2\pi\theta)\times\sqrt{-2\ln\theta}], \quad (i=1,2). \qquad (4)
\end{aligned}$$

As usual, an average of hundreds of different sequences for random numbers has been used, and the fast-varying terms (the last left-terms) of Eq.(1) are neglected in the numerical simulations below.

Most previous investigations have shown that these noise perturbations caused by randomly varying

birefringence or random dispersion map degrade the stability of soliton propagation, but the effect of correlation has not been considered. Figure 1 shows the pulse-width, compared with the initially input pulse-width of soliton, versus propagation distance for different standard deviations and correlation strengths. Figure 2 demonstrates the normalized intensity of soliton versus the propagation distance for different correlation strengths, with the standard deviation of every perturbation being the same, namely $D_i = D$ (i=1, 2). The initially input pulse is $A \times \mathrm{sech}(\tau)$, where A=1.2. These perturbations result in the widening of pulse-width, lead to the disintegration of the soliton, and the distance until soliton disintegration is related to the deviations of perturbations and the correlation strength. The correlation reinforces their effects, quickens the disintegration of the soliton, and shortens the distance until soliton disintegration. Furthermore, there is a stochastic resonance between two noise perturbations in the system; here their effect is the largest due to the maximum correlation (k=1), and the corresponding distance until soliton disintegration is the shortest.

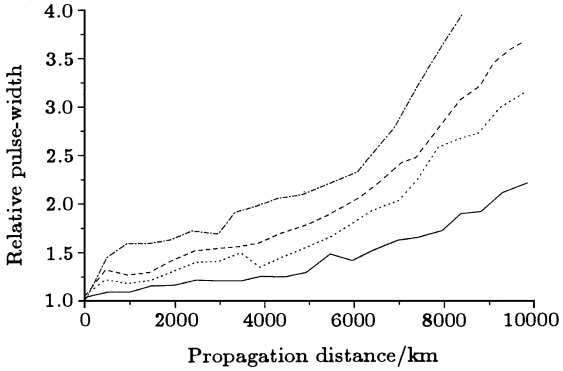


Fig.1. The relative pulse-width versus propagation distance. Solid line: D_1=0.5, D_2=0; dotted line: D_1=0.0, D_2=0.5; dashed line: $D_1 = D_2$=0.5, k=0.0; dot-dashed line: $D_1 = D_2$=0.5, k=1.0.

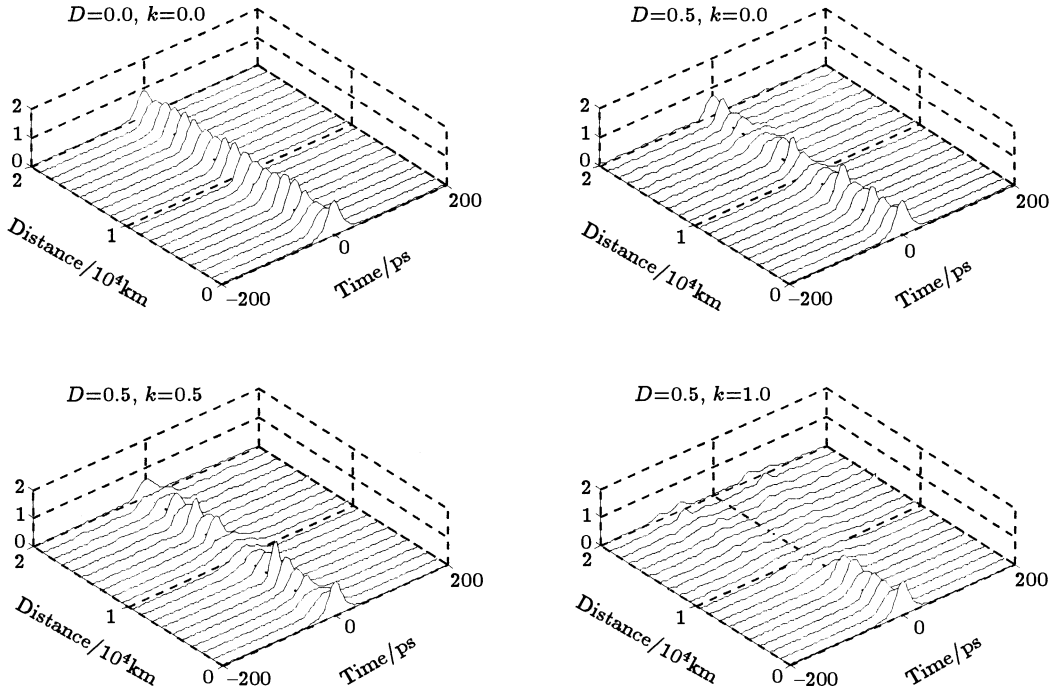


Fig.2. The normalized intensity of the soliton versus the propagation distance for different correlation strengths.

3. Effects of correlated perturbations on soliton interaction

Interaction between neighbouring solitons may induce crosstalk between two communication channels and the time jitter, and thus increase the bit-error-rate of the transmission system. When an initial separation between solitons is larger than five times the soliton width in a general soliton transmission system, the interaction can be suppressed effectively.[1] But in a soliton system with randomly varying birefringence and random dispersion map, the situation is very different.

Figure 3 shows the normalized intensity of two solitons versus the propagation distance for different standard deviations and correlation

strengths. The initially input soliton pulse is $A\times$ $[\mathrm{sech}(\tau-\Delta/2)+\mathrm{sech}(\tau+\Delta/2)]$, where A=1.2 and the separation of solitons is Δ=10 (about six times the soliton width). The standard deviation of every perturbation is the same, namely $D_i = D$ (i=1, 2). In the soliton system, we can see that these noise perturbations enhance the interaction between solitons even if the separation is large enough: for example, if the separation is larger than five times the pulse-width. Their effects are related to the standard deviation, the correlation strength and the separation between solitons. Perturbations amplify the linear wave with small amplitude which affects neighbouring solitons, further correlates the two solitons and then enhances their interaction. We find that the correlation also plays an important role, enhancing interaction, in the system where there is a stochastic resonance at which the effect of these noise perturbations on soliton interaction is the largest.

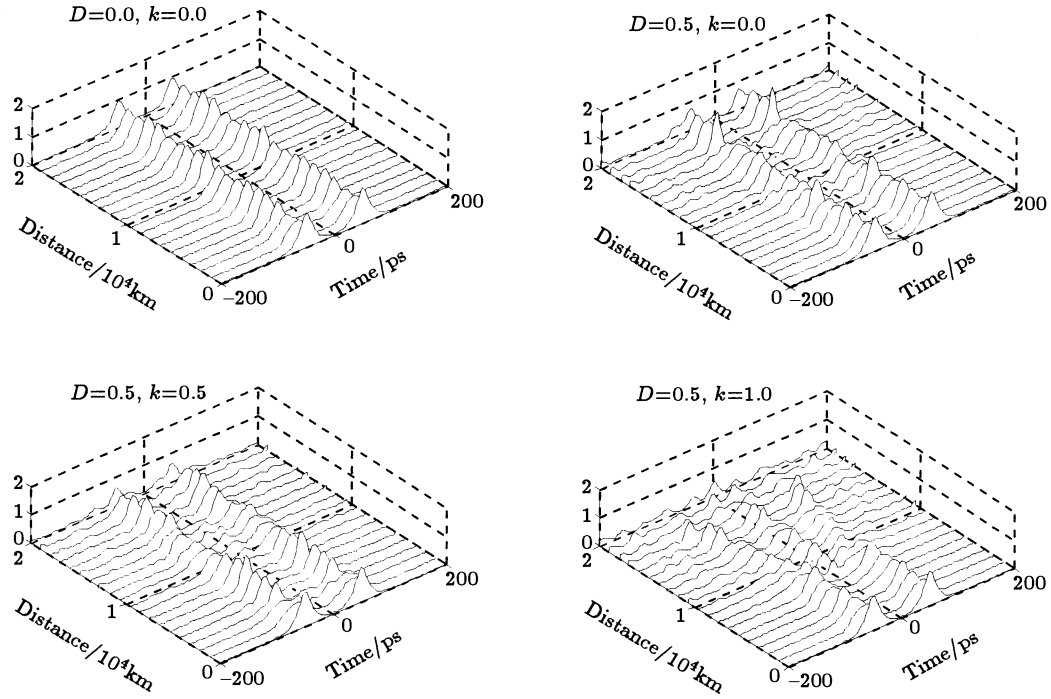


Fig.3. The normalized intensity of double soliton versus the propagation distance for different correlation strengths.

4. Control for correlated perturbations

Combined with a filter, the nonlinear gain with higher-order terms amplifies perfectly the solitons with large amplitudes while the linear waves with small amplitudes are unamplified or attenuated, so the instability of the soliton system is suppressed. When propagating in a fibre exhibiting the correlated noise perturbations and the nonlinear gain with the filter, the soliton pulses can be described by the modified nonlinear Schrödinger equation

$$\begin{aligned}
&\mathrm{i}\frac{\partial u}{\partial Z}+\mathrm{i}\eta_1(Z)\frac{\partial u}{\partial \tau}+\frac{d(Z)}{2}[1+\eta_2(Z)]\frac{\partial^2 u}{\partial \tau^2}+\left(|u|^2+\frac{1}{3}|v|^2\right)u+\frac{1}{3}v^2u^*\exp(-4\mathrm{i}\beta Z)\\
&=\mathrm{i}\gamma_1 u+\mathrm{i}\gamma_2|u|^2u+\mathrm{i}\gamma_3|u|^4u+\mathrm{i}\beta\frac{\partial^2 u}{\partial \tau^2},\\
&\mathrm{i}\frac{\partial v}{\partial Z}-\mathrm{i}\eta_1(Z)\frac{\partial v}{\partial \tau}+\frac{d(Z)}{2}[1+\eta_2(Z)]\frac{\partial^2 v}{\partial \tau^2}+\left(|v|^2+\frac{1}{3}|u|^2\right)v+\frac{1}{3}u^2v^*\exp(4\mathrm{i}\beta Z)\\
&=\mathrm{i}\gamma_1 v+\mathrm{i}\gamma_2|v|^2v+\mathrm{i}\gamma_3|v|^4v+\mathrm{i}\beta\frac{\partial^2 v}{\partial \tau^2},
\end{aligned}\tag{5}$$

where γ_1, γ_2 and γ_3 are the coefficients of the linear and the nonlinear gains respectively, and β is the coefficient of the filter. These parameters are chosen on the following conditions:[12,13] $\gamma_1 < 0$, $\gamma_3 < 0$, $\beta < 2\gamma_2$, and $15\gamma_1 - 5(\beta - 2\gamma_2) + 8\gamma_3 = 0$.

Figure 4 demonstrates the normalized intensity of single and double solitons versus the propagation distance for different standard deviations and correlation strengths, but with the controllable nonlinear gain and the filter. The coefficients of the linear and the nonlinear gains and the filter are $\gamma_1 = -0.05$, $\gamma_2 = 0.5$, $\gamma_3 = -0.3432$ and $\beta = 0.30$ respectively, and the input pulses are the same as those in Figs.2 and 3. We can see that all the ideal soliton properties are recovered as if the stochastic perturbations do not exist when there is the control of the nonlinear gain with a filter in the transmission line. In other words, the use of the nonlinear gain combined with a filter can effectively suppress the effects of correlated perturbations on the soliton propagation and interaction even if the propagation distance is long enough, and the introduction of the control does not lead to any harmful effects on the dispersion-managed soliton system.

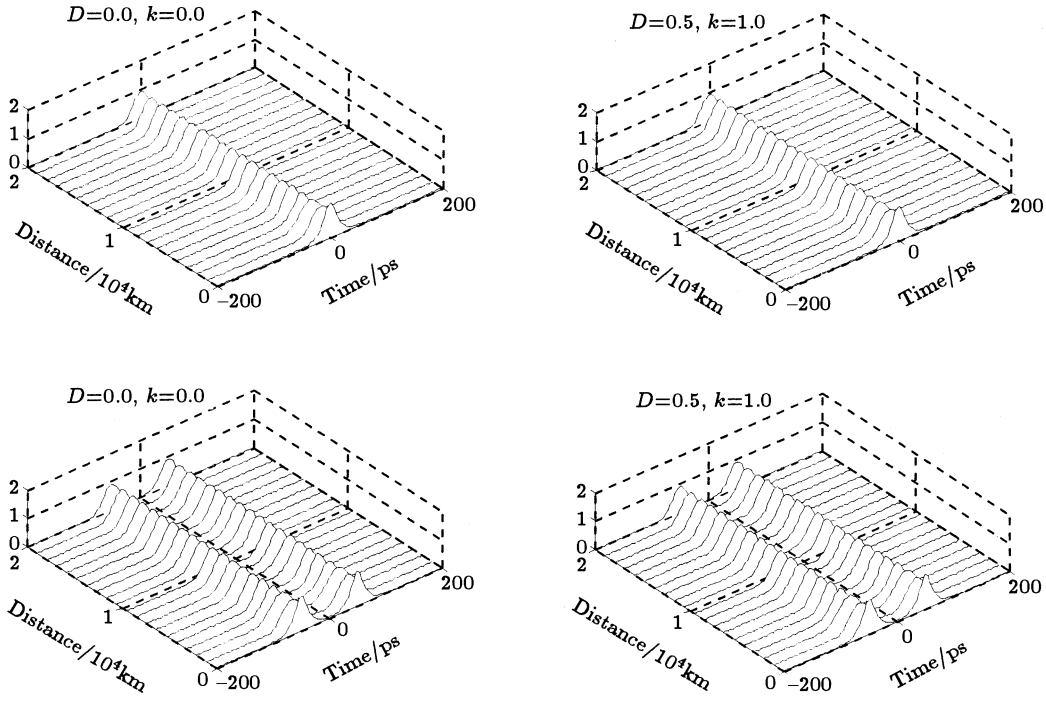


Fig.4. The normalized intensity of single and double soliton versus the propagation distance with the correlation nonlinear gain and the filter.

5. Conclusion

Effects of correlation between perturbations caused by randomly varying birefringence and the random dispersion map on soliton propagation and interaction have been investigated using numerical simulation. When the correlated randomly varying birefringence and random dispersion map are simultaneously present, their effects on the soliton system should contain their individual contributions and the correlated part. It is found that these perturbations result in widening, lead to disintegration and enhance the interaction of solitons. The correlation plays an important role, and boosts these effects. Furthermore, there is a stochastic resonance between two perturbations in the system; at this point their effect on the soliton propagation and interaction is the largest, and the corresponding distance until disintegration is the shortest. Finally, the nonlinear gain and the filter are introduced to effectively suppress these effects.

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