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Fractional Fourier transform for partially coherent beam in spatial-frequency domain*

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By using Fourier transform and the tensor analysis method, the fractional Fourier transform (FRT) in the spatial-frequency domain for partially coherent beams is derived. Based on the FRT in the spatial-frequency domain, an analytical transform formula is derived for a partially coherent twisted anisotropic Gaussian–Schell model (GSM) beam passing through the FRT system. The connections between the FRT formula and the generalized diffraction integral formulae for partially coherent beams through an aligned optical system and a misaligned optical system in the spatial-frequency domain are discussed, separately. By using the derived formula, the intensity distribution of partially coherent twisted anisotropic GSM beams in the FRT plane are studied in detail. The formula derived provide a convenient tool for analysing and calculating the FRTs of the partially coherent beams in spatial-frequency domain.

Keywords: fractional Fourier transform, partially coherent beam, spatial-frequency domain
PACC: 4230K, 4225K

1. Introduction

In recent years, beam propagation in the spatial-frequency domain has attracted more and more attention. For many systems, such as multiple spatial-filtering systems,[1] imagining-enhancement systems,[2] inertial confinement fusion systems,[3] and so on, it is more convenient to treat beam propagation and transformation in the spatial-frequency domain. The divergence and beam quality can be evaluated by the spatial-frequency spectrum of the complex amplitude in a transverse cross section of the beam.[4] The Collins formula which is a diffraction integral based on ray transfer matrices to treat beam propagation under the paraxial approximation has been converted into the spatial-frequency domain both for axially symmetrical and non-symmetrical systems.[5,6] The generalized Collins formulae for partially coherent beams passing through an axially non-symmetrical optical system were also derived both in spatial domain and spatial-frequency domain.[7,8]

On the other hand, since Ozaktas, Mendlovic and Lohmann first introduced FRT into optics in 1993,[9,10] a series of works have explored its properties, optical implementations and applications.[11−15] FRT has been used widely in signal processing,[11] beam shaping[12] etc. The connections between the Fresnel diffraction and FRT,[13] the fractional Hilbert transform,[14] fractional Hankel transform[15] and multi-fractional correlation[16] have also been defined. Recently, the FRT for the partially coherent beam was proposed,[17,18] an analytical transform formula for partially coherent twisted anisotropic Gaussian–Schell model (GSM) beams was derived, and transformation and spectrum properties of twisted anisotropic GSM beams in the FRT plane were also studied in detail.[19] But up to now, most efforts about the FRT have been confined to the spatial domain. Few efforts have been devoted to the study of FRT in the spatial-frequency domain.

In this paper, we study the FRT of partially coherent beam in the spatial-frequency domain by using the Fourier transform. An analytical formula is derived for the FRT of partially coherent twisted anisotropic GSM beams by means of the tensor method. The corresponding tensor $\boldsymbol{ABCD}$ law for partially coherent twisted anisotropic GSM beams in performing a FRT is obtained. The connections between the FRT formula and the generalized diffraction integral formulae for partially coherent beams through

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an aligned optical system and a misaligned optical system in the spatial-frequency domain are discussed in detail. An example of its application is given finally.

2. Fractional Fourier transform for partially coherent beams in spatial-frequency domain

The optical system for performing the FRT is shown in Fig.1. A FRT for the cross-spectral density

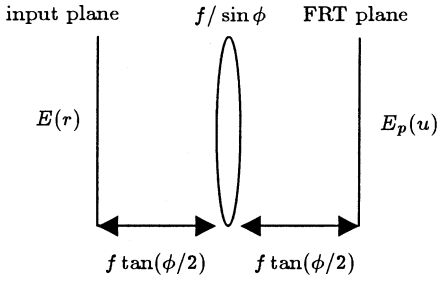


Fig.1. Optical system for performing the FRT.

of partially coherent beam in the spatial domain is described in the following form:[17,18]

$$W_p(\tilde{\boldsymbol{u}}) = \frac{1}{(\lambda f\sin\phi)^2}\iiiint W_0(\tilde{\boldsymbol{r}}) \times \exp\left[-\frac{\mathrm{i}\pi(|\boldsymbol{r}_1|^2+|\boldsymbol{u}_1|^2-|\boldsymbol{r}_2|^2-|\boldsymbol{u}_2|^2)}{\lambda f\tan\phi}\right] \times \exp\left[\frac{2\pi\mathrm{i}(\boldsymbol{r}_1\cdot\boldsymbol{u}_1-\boldsymbol{r}_2\cdot\boldsymbol{u}_2)}{\lambda f\sin\phi}\right]\mathrm{d}\tilde{\boldsymbol{r}}, \quad (1)$$

where $W_0(\tilde{\boldsymbol{r}})$ and $W_p(\tilde{\boldsymbol{u}})$ are respectively the cross-spectral densities in the input plane and in the FRT plane. $\tilde{\boldsymbol{r}}^{\mathrm{T}} = (\boldsymbol{r}_1^{\mathrm{T}}\ \boldsymbol{r}_2^{\mathrm{T}})$, $\tilde{\boldsymbol{u}}^{\mathrm{T}} = (\boldsymbol{u}_1^{\mathrm{T}}\ \boldsymbol{u}_2^{\mathrm{T}})$, $\tilde{\boldsymbol{r}}$ and $\tilde{\boldsymbol{u}}$ are spatial position vectors in the input plane and in the FRT plane, respectively. λ is the optical wavelength, f is the standard focal length. The fractional order of the FRT is given by $p = 2\phi/\pi$. When $p = 1$, the FRT reverts to the conventional Fourier transform for partially coherent beam. In Eq.(1), a factor $1/(\mathrm{i}\lambda f\sin\phi)^2$ in front of the integral ensures the energy conservation after FRT transform.

Taking the Fourier transformation of $W_p(\tilde{\boldsymbol{u}})$, we obtain

$$W_p(\boldsymbol{v}) = \frac{1}{4\pi^2}\iiiint W_p(\tilde{\boldsymbol{u}}) \times \exp(-\mathrm{i}2\pi\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{v}_1+\mathrm{i}2\pi\boldsymbol{u}_2^{\mathrm{T}}\boldsymbol{v}_2)\mathrm{d}\boldsymbol{u}_1\mathrm{d}\boldsymbol{u}_2 = \frac{1}{4\pi^2}\iiiint W_p(\tilde{\boldsymbol{u}})\exp(-\mathrm{i}2\pi\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{I}_f\tilde{\boldsymbol{v}})\mathrm{d}\tilde{\boldsymbol{u}}, \quad (2)$$

where $\tilde{\boldsymbol{v}}^{\mathrm{T}} = (\boldsymbol{v}_1^{\mathrm{T}}, \boldsymbol{v}_2^{\mathrm{T}})$, $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are the spatial-frequency position vectors in the output plane. $\boldsymbol{I}_f$ is a 4×4 matrix given by

$$\boldsymbol{I}_f = \begin{pmatrix} \boldsymbol{I} & 0 \\ 0 & -\boldsymbol{I} \end{pmatrix}, \quad (3)$$

$\boldsymbol{I}$ is a 2×2 unit matrix.

Substituting Eq.(1) into Eq.(2) yields

$$W_p(\tilde{\boldsymbol{v}}) = \frac{1}{4\pi^2(\lambda f\sin\phi)^2}\iiiint W_0(\tilde{\boldsymbol{r}})\exp(-\mathrm{i}2\pi\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{v}_1+\mathrm{i}2\pi\boldsymbol{u}_2^{\mathrm{T}}\boldsymbol{v}_2) \times \iiiint \exp\left[-\frac{\mathrm{i}\pi(|\boldsymbol{r}_1|^2+|\boldsymbol{u}_1|^2-|\boldsymbol{r}_2|^2-|\boldsymbol{u}_2|^2)}{\lambda f\tan\phi}\right]\exp\left[\frac{2\pi\mathrm{i}(\boldsymbol{r}_1\cdot\boldsymbol{u}_1-\boldsymbol{r}_2\cdot\boldsymbol{u}_2)}{\lambda f\sin\phi}\right]\mathrm{d}\tilde{\boldsymbol{r}}\mathrm{d}\tilde{\boldsymbol{u}}, \quad (4)$$

where $W_0(\tilde{\boldsymbol{r}})$ is the inverse Fourier transformation of $W_0(\boldsymbol{f})$, given by

$$W_0(\tilde{\boldsymbol{r}}) = \frac{1}{4\pi^2}\iiiint W(\tilde{\boldsymbol{f}})\exp(\mathrm{i}2\pi\tilde{\boldsymbol{r}}^{\mathrm{T}}\boldsymbol{I}_f\tilde{\boldsymbol{f}})\mathrm{d}\tilde{\boldsymbol{f}}, \quad (5)$$

where $\tilde{\boldsymbol{f}}^{\mathrm{T}} = (\boldsymbol{f}_1^{\mathrm{T}}, \boldsymbol{f}_2^{\mathrm{T}})$, $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ being the spatial-frequency position vectors in the input plane.

Substituting Eq.(5) into Eq.(4), after tedious but straightforward integration, we obtain

$$W(\tilde{\boldsymbol{v}}) = \frac{\lambda^2f^2}{16\pi^4\sin^2\phi}\iiiint W(\tilde{\boldsymbol{f}}) \times \exp\left[\frac{\mathrm{i}\pi\lambda f(|\boldsymbol{v}_2|^2+|\boldsymbol{f}_2|^2-|\boldsymbol{v}_1|^2-|\boldsymbol{f}_1|^2)}{\tan\phi} - \frac{2\pi\lambda f\mathrm{i}(\boldsymbol{f}_2^{\mathrm{T}}\boldsymbol{v}_2-\boldsymbol{f}_1^{\mathrm{T}}\boldsymbol{v}_1)}{\sin\phi}\right]\mathrm{d}\tilde{\boldsymbol{f}}. \quad (6)$$

Equation (6) is the generalized FRT for partially coherent beam in the spatial-frequency domain.

After rearrangement, Eq.(6) can be expressed in tensor form as follows:

$$W(\tilde{\boldsymbol{v}}) = \frac{\lambda^2f^2}{16\pi^4\sin^2\phi}\iiiint W(\tilde{\boldsymbol{f}}) \times \exp[\mathrm{i}\pi\lambda(\tilde{\boldsymbol{f}}^{\mathrm{T}}\tilde{\boldsymbol{N}}_{11}\tilde{\boldsymbol{f}}+\tilde{\boldsymbol{v}}^{\mathrm{T}}\tilde{\boldsymbol{N}}_{11}\tilde{\boldsymbol{v}}) - 2\pi\lambda\mathrm{i}\tilde{\boldsymbol{f}}^{\mathrm{T}}\tilde{\boldsymbol{N}}_{12}\tilde{\boldsymbol{v}}]\mathrm{d}\tilde{\boldsymbol{f}}, \quad (7)$$

where

$$\tilde{\boldsymbol{N}}_{11} = \frac{f}{\tan\phi}\begin{pmatrix} -\boldsymbol{I} & 0 \\ 0 & \boldsymbol{I} \end{pmatrix},$$

$$\tilde{\boldsymbol{N}}_{12} = \frac{f}{\sin\phi}\begin{pmatrix} -\boldsymbol{I} & 0 \\ 0 & \boldsymbol{I} \end{pmatrix}. \quad (8)$$

Now we use Eq.(4) to study the FRT of the partially coherent twisted anisotropic GSM beams in the spatial-frequency domain. This kind of beam was first introduced by Simon *et al* in 1985.[20] Since then, many works have been done on the propagation and transformation properties of twisted anisotropic GSM beams.[21–27] The transformation invariants and the properties of the twist phase associated with the twisted anisotropic GSM beams have been studied in detail.[21–24] Coherent mode decomposition for these beams has also been obtained.[25,26] Friberg *et al* have verified several properties of twisted anisotropic GSM beams experimentally.[27] The conventional way used by many authors to treat the propagation and imaging of partially coherent GSM beams is the Wigner distribution function and related second-order moments. Recently, we have introduced the tensor method to treat the propagation of partially coherent beams. The generalized propagation formula for partially coherent beams through axially nonsymmetrical optical systems was derived. The corresponding tensor $\boldsymbol{ABCD}$ law for partially coherent twisted anisotropic GSM beams was also derived.[7]

The cross-spectral density of the most general partially coherent twisted anisotropic GSM beams can be expressed in tensor form as follows:[7]

$$W_0(\tilde{\boldsymbol{r}}) = G_0 \exp\left(-\frac{\mathrm{i}\pi}{\lambda} \tilde{\boldsymbol{r}}^{\mathrm{T}} \boldsymbol{M}_i^{-1} \tilde{\boldsymbol{r}} \right), \tag{9}$$

where $\boldsymbol{M}_i^{-1}$ is a 4×4 matrix, called the partially coherent complex curvature tensor, which takes the form

$$\boldsymbol{M}_i^{-1} = \begin{pmatrix} \boldsymbol{R}^{-1} - \dfrac{\mathrm{i}\lambda}{4\pi}(\boldsymbol{\sigma}_I^2)^{-1} - \dfrac{\mathrm{i}\lambda}{2\pi}(\boldsymbol{\sigma}_g^2)^{-1} & \dfrac{\mathrm{i}\lambda}{2\pi}(\boldsymbol{\sigma}_g^2)^{-1} + \mu \boldsymbol{J} \\ \dfrac{\mathrm{i}\lambda}{2\pi}(\boldsymbol{\sigma}_g^2)^{-1} + \mu \boldsymbol{J}^{\mathrm{T}} & -\boldsymbol{R}^{-1} - \dfrac{\mathrm{i}\lambda}{4\pi}(\boldsymbol{\sigma}_I^2)^{-1} - \dfrac{\mathrm{i}\lambda}{2\pi}(\boldsymbol{\sigma}_g^2)^{-1} \end{pmatrix}, \tag{10}$$

where σ_{I}^2 is a transverse spot-width matrix and σ_g^2 is a transverse coherence-width matrix. $\boldsymbol{R}^{-1}$ is a wave front curvature matrix. μ is a scalar real-valued twist factor. σ_{I}^2, σ_{g}^2 and $\boldsymbol{R}^{-1}$ are all 2×2 matrices with transpose symmetry, and $\boldsymbol{J}$ is a transpose antisymmetry matrix given by

$$\boldsymbol{J} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{11}$$

Taking the Fourier transformation of Eq.(7), we obtain

$$W_p(\tilde{\boldsymbol{f}}) = \frac{G_0 \lambda^2}{4\pi^2 [\det(\boldsymbol{M}_i^{-1})]^{1/2}} \exp[\mathrm{i}\pi\lambda \tilde{\boldsymbol{f}}^{\mathrm{T}} \boldsymbol{I}_f^{\mathrm{T}} \boldsymbol{M}_i \boldsymbol{I}_f \tilde{\boldsymbol{f}}], \tag{12}$$

where $\boldsymbol{M}_i$ is the inverse matrix of $\boldsymbol{M}_i^{-1}$, which may be called the 'partially coherent complex radius of curvature tensor'.

Substituting Eq.(12) into Eq.(7), we obtain

$$\begin{aligned} W_p(\tilde{\boldsymbol{v}}) =& \frac{G_0 \lambda^4 f^2}{64\pi^6 [\det(\boldsymbol{M}_i^{-1})]^{1/2} \sin^2\phi} \exp\{\mathrm{i}\pi\lambda \tilde{\boldsymbol{v}}^{\mathrm{T}} [\tilde{\boldsymbol{N}}_{11} - \tilde{\boldsymbol{N}}_{12}^{\mathrm{T}} (\boldsymbol{I}_f^{\mathrm{T}} \boldsymbol{M}_i \boldsymbol{I}_f + \tilde{\boldsymbol{N}}_{11})^{-1} \tilde{\boldsymbol{N}}_{12}] \tilde{\boldsymbol{v}}\} \\ & \times \iiiint \exp[\mathrm{i}\pi\lambda | (\boldsymbol{I}_f^{\mathrm{T}} \boldsymbol{M}_i \boldsymbol{I}_f + \tilde{\boldsymbol{N}}_{11})^{1/2} \tilde{\boldsymbol{f}} - (\boldsymbol{I}_f^{\mathrm{T}} \boldsymbol{M}_i \boldsymbol{I}_f + \tilde{\boldsymbol{N}}_{11})^{-1/2} \tilde{\boldsymbol{N}}_{12} \tilde{\boldsymbol{v}} |] \mathrm{d}\tilde{\boldsymbol{f}}. \end{aligned} \tag{13}$$

After vector integration, we obtain

$$\begin{aligned} W_p(\tilde{\boldsymbol{v}}) =& \frac{G_0 \lambda^2 f^2}{64\pi^6 [\det(\boldsymbol{M}_i^{-1})]^{1/2} [\det((\boldsymbol{I}_f^{\mathrm{T}} \boldsymbol{M}_i \boldsymbol{I}_f + \tilde{\boldsymbol{N}}_{11}))]^{1/2} \sin^2\phi} \\ & \times \exp\{\mathrm{i}\pi\lambda \tilde{\boldsymbol{v}}^{\mathrm{T}} [\tilde{\boldsymbol{N}}_{11} - \tilde{\boldsymbol{N}}_{12}^{\mathrm{T}} (\boldsymbol{I}_f^{\mathrm{T}} \boldsymbol{M}_i \boldsymbol{I}_f + \tilde{\boldsymbol{N}}_{11})^{-1} \tilde{\boldsymbol{N}}_{12}] \tilde{\boldsymbol{v}}\}. \end{aligned} \tag{14}$$

In the derivation of Eq.(14), the integral formula $\int_{-\infty}^{\infty} \exp(-ax^2)\mathrm{d}x = \sqrt{\pi/a}$ has been used. In order to simplify Eq.(14), we introduce four matrices $\bar{\boldsymbol{A}}$, $\bar{\boldsymbol{B}}$, $\bar{\boldsymbol{C}}$ and $\bar{\boldsymbol{D}}$, which take the following forms:

$$\bar{\boldsymbol{A}} = \tilde{\boldsymbol{N}}_{11} \tilde{\boldsymbol{N}}_{12}^{-1} = \cos\phi \begin{pmatrix} \boldsymbol{I} & 0 \\ 0 & \boldsymbol{I} \end{pmatrix},$$

$$\bar{\boldsymbol{B}} = \tilde{\boldsymbol{N}}_{11} \tilde{\boldsymbol{N}}_{12}^{-1} \tilde{\boldsymbol{N}}_{11} - \tilde{\boldsymbol{N}}_{12}^{\mathrm{T}} = f \sin\phi \begin{pmatrix} \boldsymbol{I} & 0 \\ 0 & -\boldsymbol{I} \end{pmatrix},$$

$$\bar{\boldsymbol{C}} = \tilde{\boldsymbol{N}}_{12}^{-1} = \frac{\sin\phi}{f} \begin{pmatrix} -\boldsymbol{I} & 0 \\ 0 & \boldsymbol{I} \end{pmatrix},$$

$$\bar{\boldsymbol{D}} = \tilde{\boldsymbol{N}}_{12}^{-1} \tilde{\boldsymbol{N}}_{11} = \cos\phi \begin{pmatrix} \boldsymbol{I} & 0 \\ 0 & \boldsymbol{I} \end{pmatrix}. \tag{15}$$

Here $\bar{\boldsymbol{A}}$, $\bar{\boldsymbol{B}}$, $\bar{\boldsymbol{C}}$ and $\bar{\boldsymbol{D}}$ are the sub-matrices of the optical system that is used for performing the FRT. We can find that they satisfy $\bar{\boldsymbol{A}} = \bar{\boldsymbol{D}}$, $-f\bar{\boldsymbol{C}} = \bar{\boldsymbol{B}}/f$. Substituting Eq.(15) into Eq.(14), we obtain

$$W_p(\tilde{\boldsymbol{v}}) = \frac{G_0\lambda^2}{64\pi^6[\det(\bar{\boldsymbol{C}}+\bar{\boldsymbol{D}}\boldsymbol{M}_1^{-1})]^{1/2}} \times \exp(\mathrm{i}\pi\lambda\tilde{\boldsymbol{v}}^{\mathrm{T}}\boldsymbol{I}_f^{\mathrm{T}}\boldsymbol{M}_p\boldsymbol{I}_f\tilde{\boldsymbol{v}}), \quad (16)$$

where

$$\begin{aligned}\boldsymbol{I}_f^{\mathrm{T}}\boldsymbol{M}_p\boldsymbol{I}_f =& \bar{\boldsymbol{A}}\bar{\boldsymbol{C}}^{-1} - \bar{\boldsymbol{C}}^{-1\mathrm{T}}(\boldsymbol{I}_f^{\mathrm{T}}\boldsymbol{M}_i\boldsymbol{I}_f + \bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{D}})^{-1}\bar{\boldsymbol{C}}^{-1}\\ =& \bar{\boldsymbol{A}}\bar{\boldsymbol{C}}^{-1} - \bar{\boldsymbol{C}}^{-1\mathrm{T}}(\bar{\boldsymbol{C}}\boldsymbol{I}_f^{\mathrm{T}}\boldsymbol{M}_i\boldsymbol{I}_f + \bar{\boldsymbol{D}})^{-1}\\ =& [\bar{\boldsymbol{A}}\boldsymbol{I}_f^{\mathrm{T}}\boldsymbol{M}_i\boldsymbol{I}_f + \bar{\boldsymbol{A}}\bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{D}} - \bar{\boldsymbol{C}}^{-1\mathrm{T}}]\\ &\times(\bar{\boldsymbol{C}}\boldsymbol{I}_f^{\mathrm{T}}\boldsymbol{M}_i\boldsymbol{I}_f + \boldsymbol{D})^{-1}\\ =& (\bar{\boldsymbol{A}}\boldsymbol{I}_f^{\mathrm{T}}\boldsymbol{M}_i\boldsymbol{I}_f + \boldsymbol{I}_f^{\mathrm{T}}\bar{\boldsymbol{B}}\boldsymbol{I}_f)\\ &\times(\bar{\boldsymbol{C}}\boldsymbol{I}_f^{\mathrm{T}}\boldsymbol{M}_i\boldsymbol{I}_f + \boldsymbol{I}_f^{\mathrm{T}}\bar{\boldsymbol{D}}\boldsymbol{I}_f)^{-1}\\ =& (\boldsymbol{I}_f^{\mathrm{T}}\bar{\boldsymbol{A}}\boldsymbol{M}_i\boldsymbol{I}_f + \boldsymbol{I}_f^{\mathrm{T}}\bar{\boldsymbol{B}}\boldsymbol{I}_f)\\ &\times(\boldsymbol{I}_f^{\mathrm{T}}\bar{\boldsymbol{C}}\boldsymbol{M}_i\boldsymbol{I}_f + \boldsymbol{I}_f^{\mathrm{T}}\bar{\boldsymbol{D}}\boldsymbol{I}_f)^{-1}\\ =& \boldsymbol{I}_f^{\mathrm{T}}(\bar{\boldsymbol{A}}\boldsymbol{M}_i + \bar{\boldsymbol{B}})\boldsymbol{I}_f\boldsymbol{I}_f^{-1}(\bar{\boldsymbol{C}}\boldsymbol{M}_i + \bar{\boldsymbol{D}})^{-1}\boldsymbol{I}_f^{-1\mathrm{T}}\\ =& \boldsymbol{I}_f^{\mathrm{T}}(\bar{\boldsymbol{A}}\boldsymbol{M}_i + \bar{\boldsymbol{B}})(\bar{\boldsymbol{C}}\boldsymbol{M}_i + \bar{\boldsymbol{D}})^{-1}\boldsymbol{I}_f. \quad (17)\end{aligned}$$

Therefore we have

$$\boldsymbol{M}_p = (\bar{\boldsymbol{A}}M_i + \bar{\boldsymbol{B}})(\boldsymbol{C}\bar{\boldsymbol{M}}_i + \bar{\boldsymbol{D}})^{-1}. \quad (18)$$

In the derivation of Eqs.(16) and (17), the relations $\boldsymbol{I}_f^{\mathrm{T}}\bar{\boldsymbol{D}}\boldsymbol{I}_f = \bar{\boldsymbol{D}}$, $\boldsymbol{I}_f^{\mathrm{T}}\bar{\boldsymbol{B}}\boldsymbol{I}_f = \bar{\boldsymbol{B}}$, $\boldsymbol{I}_f = \boldsymbol{I}_f^{-1} = \boldsymbol{I}_f^{\mathrm{T}}$ and $\bar{\boldsymbol{B}} = \bar{\boldsymbol{A}}\bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{D}} - \bar{\boldsymbol{C}}^{-1\mathrm{T}}$ have been used. $\boldsymbol{M}_i$ and $\boldsymbol{M}_p$ are the partially coherent complex radius-of-curvature tensors in the input and FRT planes, respectively. Equation (18) may be called the tensor $\boldsymbol{ABCD}$ law for the partially coherent twisted anisotropic GSM beam to perform the FRT in the spatial-frequency domain. Equations (16) and (18) provide a convenient tool in treating the FRT of partially coherent twisted anisotropic GSM beams in the spatial-frequency domain. For example, if we want to treat the FRT of twisted anisotropic GSM beams for n times in the spatial-frequency domain, we can easily obtain the partially coherent complex radius-of-curvature tensor in the FRT plane as follows:

$$\boldsymbol{M}_{np}^{-1} = (\bar{\boldsymbol{C}}_n + \bar{\boldsymbol{D}}_n\boldsymbol{M}_i^{-1})(\bar{\boldsymbol{A}}_n + \bar{\boldsymbol{B}}_n\boldsymbol{M}_i^{-1})^{-1}, \quad (19)$$

where $\bar{\boldsymbol{A}}_n$, $\bar{\boldsymbol{B}}_n$, $\bar{\boldsymbol{C}}_n$, $\bar{\boldsymbol{D}}_n$ are given by

$$\begin{pmatrix} \bar{\boldsymbol{A}}_n & \bar{\boldsymbol{B}}_n \\ \bar{\boldsymbol{C}}_n & \bar{\boldsymbol{D}}_n \end{pmatrix} = \begin{pmatrix} \bar{\boldsymbol{A}} & \bar{\boldsymbol{B}} \\ \bar{\boldsymbol{C}} & \bar{\boldsymbol{D}} \end{pmatrix}^n. \quad (20)$$

3. Connections between the FRT formula and the generalized diffraction integral formulae for partially coherent beams in the spatial-frequency domain

Similar to that in the spatial domain,[18] there should be a connection between the FRT formula and the generalized Collins formula for partially coherent beams in the spatial-frequency domain. From Eq.(15), we have

$$\begin{aligned}\tilde{\boldsymbol{N}}_{11} &= \bar{\boldsymbol{A}}\bar{\boldsymbol{C}}^{-1} = \bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{D}},\\ \tilde{\boldsymbol{N}}_{12} &= \bar{\boldsymbol{C}}^{-1} = \bar{\boldsymbol{A}}\bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{D}} - \bar{\boldsymbol{B}}. \quad (21)\end{aligned}$$

Substituting Eq.(21) into Eq.(7), we have

$$\begin{aligned}W_p(\tilde{\boldsymbol{v}}) =& \frac{\lambda^2}{16\pi^4[\det(\bar{\boldsymbol{C}})]^{1/2}}\iiiint W(\tilde{\boldsymbol{f}})\\ &\times\exp[\mathrm{i}\pi\lambda(\tilde{\boldsymbol{f}}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{D}}f + \tilde{\boldsymbol{v}}^{\mathrm{T}}\bar{\boldsymbol{A}}\bar{\boldsymbol{C}}^{-1}\tilde{\boldsymbol{v}}\\ &- 2\tilde{\boldsymbol{f}}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\tilde{\boldsymbol{v}})]\mathrm{d}\tilde{\boldsymbol{f}}, \quad (22)\end{aligned}$$

where $\bar{\boldsymbol{A}}$, $\bar{\boldsymbol{B}}$, $\bar{\boldsymbol{C}}$ and $\bar{\boldsymbol{D}}$ are the sub-matrices for the FRT system given by Eq.(15). Equation (22) is the same, in form, as the generalized Collins formula for partially coherent beams in the spatial-frequency domain.[8] Therefore, Eq.(21) is the condition for a system to perform FRT. Any aligned optical system whose transfer matrices satisfy Eq.(21) can implement the FRT for partially coherent beams.

Now we discuss the relation between the FRT formula and the generalized diffraction integral formula for partially coherent beams through misaligned optical systems in the spatial-frequency domain. Let us consider a misaligned optical system with multi-elements as shown in Fig.2, in which the system is characterized

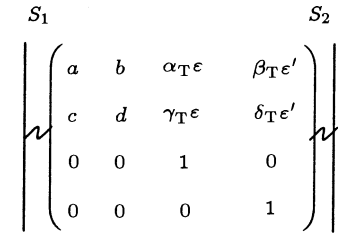


Fig.2. A misaligned optical system.

by the 4×4 augmented matrix, and S_1 and S_2 represent the input and output reference planes, respectively. The generalized diffraction integral formula for the coherent beam through misaligned optical systems is given by [28,29]

$$\begin{aligned}E_2(\rho_{1x},\rho_{1y}) =& -\frac{\mathrm{i}}{\lambda b}\iint E_1(x_1,y_1)\\ &\times\exp\Big\{-\frac{\mathrm{i}\pi}{\lambda b}[(x_1^2+y_1^2)\\ &-2(x_1\rho_{1x}+y_1\rho_{1y}) + d(\rho_{1x}^2+\rho_{1y}^2)\\ &+ex_1+fy_1+g\rho_{1x}+h\rho_{1y}]\Big\}\mathrm{d}x_1\mathrm{d}y_1. \quad (23)\end{aligned}$$

In Eq.(23), the phase factor $\exp(\mathrm{i}kl_0)$ along the axis between the two reference planes has been omitted. a, b, c and d are the transfer matrix elements of the aligned optical system. The parameters e, f, g and h take the following form:

$$e = 2(\alpha_{\mathrm{T}}\varepsilon_x + \beta_{\mathrm{T}}\varepsilon_x', \tag{24}$$

$$f = 2(\alpha_{\mathrm{T}}\varepsilon_y + \beta_{\mathrm{T}}\varepsilon_y', \tag{25}$$

$$g = 2(b\gamma_{\mathrm{T}} - d\alpha_{\mathrm{T}})\varepsilon_x + 2(b\delta_{\mathrm{T}} - d\beta_{\mathrm{T}})\varepsilon_x', \tag{26}$$

$$h = 2(b\gamma_{\mathrm{T}} - d\alpha_{\mathrm{T}})\varepsilon_y + 2(b\delta_{\mathrm{T}} - d\beta_{\mathrm{T}})\varepsilon_y', \tag{27}$$

re ε_x, ε_x', ε_y and ε_y' denote the two-dimensional alignment parameters: ε_x and ε_y are the displacement elements in the x and y directions respectively, nd ε_y' are the tilting angle of the element in the nd y directions respectively; α_{T}, β_{T}, γ_{T} and δ_{T} resent the misaligned matrix elements determined

$$\alpha_{\mathrm{T}} = 1 - a, \quad \beta_{\mathrm{T}} = l - b,$$
$$\gamma_{\mathrm{T}} = -c, \quad \delta_{\mathrm{T}} = \pm 1 - d, \tag{28}$$

re l is the axial distance from the input plane and ut plane. For forward-going optical elements, δ_{T} hosen for '+' sign; for backward-going ones, δ_{T} is sen for '-' sign.

Equation (23) can be rearranged into a more com- form by using the tensor method as follows:

$$\boldsymbol{\rho}_1) = -\frac{\mathrm{i}}{\lambda[\det(\boldsymbol{B})]^{1/2}} \iint E(\boldsymbol{r}_1) \times \exp\left[-\frac{\mathrm{i}\pi}{\lambda}(\boldsymbol{r}_1^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{A}\boldsymbol{r}_1 - 2\boldsymbol{r}_1^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{\rho}_1 + \boldsymbol{\rho}_1^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}^{-1}\boldsymbol{\rho}_1^{\mathrm{T}})\right] \times \exp\left[-\frac{\mathrm{i}\pi}{\lambda}(\boldsymbol{r}_1^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{e}_f + \boldsymbol{\rho}_1^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{g}_h)\right]\mathrm{d}\boldsymbol{r}_1, \tag{29}$$

where $\boldsymbol{r}_1^{\mathrm{T}} = (x_1 \quad y_1)$, $\boldsymbol{r}_2^{\mathrm{T}} = (x_2 \quad y_2)$, $\boldsymbol{e}_f = (e \quad f)$, $\boldsymbol{g}_h = (g \quad h)$; A, B, C and D take the following form:

$$\boldsymbol{A} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix},$$

$$\boldsymbol{C} = \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix}, \quad \boldsymbol{D} = \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix}. \tag{30}$$

Assume that the optical fields at the two arbitrary points $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ in the incident plane are $E(\boldsymbol{r}_1)$ and $E(\boldsymbol{r}_2)$, the optical fields at the two arbitrary points ρ_1 and ρ_2 in the output plane are $E(\rho_1)$ and $E(\rho_2)$, respectively. The cross-spectral densities in the incident and output planes will be

$$W(\tilde{\boldsymbol{r}}) = \langle E(\boldsymbol{r}_1)E^*(\boldsymbol{r}_2)\rangle,$$
$$W(\tilde{\boldsymbol{\rho}}) = \langle E(\boldsymbol{\rho}_1)E^*(\boldsymbol{\rho}_2)\rangle, \tag{31}$$

where '< >' denotes ensemble average, $\tilde{\boldsymbol{r}}^{\mathrm{T}} = (\boldsymbol{r}_1^{\mathrm{T}} \quad \boldsymbol{r}_2^{\mathrm{T}})$, $\tilde{\boldsymbol{\rho}}^{\mathrm{T}} = (\boldsymbol{\rho}_1^{\mathrm{T}} \quad \boldsymbol{\rho}_2^{\mathrm{T}})$. Using Eq.(29), we can obtain the propagation formula for the cross-spectral density of partially coherent beams through misaligned optical systems as follows:

$$W(\tilde{\boldsymbol{\rho}}) = \frac{1}{\lambda^2[\det(\bar{\boldsymbol{B}})]^{1/2}} \iint W(\tilde{\boldsymbol{r}}) \times \exp\left[-\frac{\mathrm{i}\pi}{\lambda}(\tilde{\boldsymbol{r}}^{\mathrm{T}}\bar{\boldsymbol{B}}^{-1}\bar{\boldsymbol{A}}\tilde{\boldsymbol{r}} - 2\tilde{\boldsymbol{r}}^{\mathrm{T}}\bar{\boldsymbol{B}}^{-1}\tilde{\boldsymbol{\rho}} + \tilde{\boldsymbol{\rho}}^{\mathrm{T}}\bar{\boldsymbol{D}}\bar{\boldsymbol{B}}^{-1}\tilde{\boldsymbol{\rho}})\right] \times \exp\left[-\frac{\mathrm{i}\pi}{\lambda}(\tilde{\boldsymbol{r}}^{\mathrm{T}}\bar{\boldsymbol{B}}^{-1}\bar{\boldsymbol{e}}_f + \tilde{\boldsymbol{\rho}}^{\mathrm{T}}\bar{\boldsymbol{B}}^{-1}\bar{\boldsymbol{g}}_h)\right]\mathrm{d}\tilde{\boldsymbol{r}}, \tag{32}$$

where $\bar{\boldsymbol{e}}_f^{\mathrm{T}} = (\boldsymbol{e}_f^{\mathrm{T}} \quad \boldsymbol{e}_f^{\mathrm{T}})$, $\bar{\boldsymbol{g}}_h^{\mathrm{T}} = (\boldsymbol{g}_h^{\mathrm{T}} \quad \boldsymbol{g}_h^{\mathrm{T}})$; $\bar{\boldsymbol{A}}$, $\bar{\boldsymbol{B}}$, $\bar{\boldsymbol{C}}$ and $\bar{\boldsymbol{D}}$ are defined as:

$$\bar{\boldsymbol{A}} = \begin{pmatrix} \boldsymbol{A} & 0 \\ 0 & \boldsymbol{A} \end{pmatrix}, \quad \bar{\boldsymbol{B}} = \begin{pmatrix} \boldsymbol{B} & 0 \\ 0 & -\boldsymbol{B} \end{pmatrix},$$

$$\bar{\boldsymbol{C}} = \begin{pmatrix} \boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{C} \end{pmatrix}, \quad \bar{\boldsymbol{D}} = \begin{pmatrix} \boldsymbol{D} & 0 \\ 0 & \boldsymbol{D} \end{pmatrix}. \tag{33}$$

Taking the Fourier transformation of $W(\tilde{\boldsymbol{\rho}})$, we have

$$W(\tilde{\boldsymbol{v}}) = \frac{1}{4\pi^2} \iiiint W(\tilde{\boldsymbol{\rho}}) \times \exp(-\mathrm{i}2\pi\boldsymbol{\rho}_1^{\mathrm{T}}\boldsymbol{v}_1 + \mathrm{i}2\pi\boldsymbol{\rho}_2^{\mathrm{T}}\boldsymbol{v}_2)\mathrm{d}\boldsymbol{\rho}_1\mathrm{d}\boldsymbol{\rho}_2 = \frac{1}{4\pi^2} \iiiint W(\tilde{\boldsymbol{\rho}})\exp(-\mathrm{i}2\pi\tilde{\boldsymbol{\rho}}^{\mathrm{T}}\boldsymbol{I}_f\tilde{\boldsymbol{v}})\mathrm{d}\tilde{\boldsymbol{\rho}}, \tag{34}$$

where $\tilde{\boldsymbol{v}}^{\mathrm{T}} = (\boldsymbol{v}_1^{\mathrm{T}}, \boldsymbol{v}_2^{\mathrm{T}})$, $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are the spatial-frequency position vectors in the output plane.

Substituting Eq.(32) into Eq.(34), we obtain

$$W(\tilde{\boldsymbol{v}}) = \frac{1}{4\pi^2\lambda^2[\det(\bar{\boldsymbol{B}})]^{1/2}} \iiiint W(\tilde{\boldsymbol{r}}) \times \exp(-\mathrm{i}2\pi\boldsymbol{\rho}_1^{\mathrm{T}}\boldsymbol{v}_1 + \mathrm{i}2\pi\boldsymbol{\rho}_2^{\mathrm{T}}\boldsymbol{v}_2) \times \iiiint \exp\left[-\frac{\mathrm{i}\pi}{\lambda}(\tilde{\boldsymbol{r}}^{\mathrm{T}}\bar{\boldsymbol{B}}^{-1}\bar{\boldsymbol{A}}\tilde{\boldsymbol{r}} - 2\tilde{\boldsymbol{r}}^{\mathrm{T}}\bar{\boldsymbol{B}}^{-1}\tilde{\boldsymbol{\rho}} + \tilde{\boldsymbol{\rho}}^{\mathrm{T}}\bar{\boldsymbol{D}}\bar{B}^{-1}\tilde{\boldsymbol{\rho}})\right] \times \exp\left[-\frac{\mathrm{i}\pi}{\lambda}(\tilde{\boldsymbol{r}}^{\mathrm{T}}\bar{\boldsymbol{B}}^{-1}\bar{\boldsymbol{e}}_f + \tilde{\boldsymbol{\rho}}^{\mathrm{T}}\bar{\boldsymbol{B}}^{-1}\bar{\boldsymbol{g}}_h)\right]\mathrm{d}\tilde{\boldsymbol{r}}\mathrm{d}\tilde{\boldsymbol{\rho}}, \tag{35}$$

where $W(\tilde{\boldsymbol{r}})$ is the inverse Fourier transformation of $W(\tilde{\boldsymbol{f}})$, given by

$$W(\tilde{\boldsymbol{r}}) = \frac{1}{4\pi^2}\iiiint W(\tilde{\boldsymbol{f}})\exp(\mathrm{i}2\pi\tilde{\boldsymbol{r}}^{\mathrm{T}}\boldsymbol{I}_f\tilde{\boldsymbol{f}})\mathrm{d}\tilde{\boldsymbol{f}}, \quad (36)$$

where $\tilde{\boldsymbol{f}}^{\mathrm{T}} = (\boldsymbol{f}_1^{\mathrm{T}}, \boldsymbol{f}_2^{\mathrm{T}})$, $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ being the spatial-frequency position vectors in the input plane.

Substituting Eq.(36) into Eq.(35), after tedious but straightforward integration, we obtain

$$\begin{aligned} W(\tilde{\boldsymbol{v}}) =& \frac{\lambda^2}{16\pi^4[\det(\bar{\boldsymbol{C}})]^{1/2}}\iiiint W(\boldsymbol{f}) \\ & \times \exp\Big[\mathrm{i}\pi\lambda\tilde{\boldsymbol{f}}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{D}}\tilde{f} - 2\boldsymbol{f}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\tilde{\boldsymbol{v}} \\ & + \tilde{\boldsymbol{v}}^{\mathrm{T}}\bar{\boldsymbol{A}}\bar{\boldsymbol{C}}^{-1}\tilde{\boldsymbol{v}} + \frac{1}{\lambda}\boldsymbol{f}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{B}}\bar{\boldsymbol{D}}^{-1}\tilde{\boldsymbol{e}}_f \\ & + \frac{1}{\lambda}\boldsymbol{f}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{B}}^{-1}\tilde{\boldsymbol{g}}_h - \frac{1}{\lambda}\boldsymbol{v}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{B}}^{-1}\tilde{\boldsymbol{e}}_f \\ & - \frac{1}{\lambda}\boldsymbol{v}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{B}}^{-1}\bar{\boldsymbol{A}}\tilde{g}_h)\Big]\mathrm{d}\boldsymbol{f}, \end{aligned} \quad (37)$$

where $\tilde{\boldsymbol{e}}_f^{\mathrm{T}} = (\boldsymbol{e}_f^{\mathrm{T}} \quad -\boldsymbol{e}_f^{\mathrm{T}})$ and $\tilde{\boldsymbol{g}}_h^{\mathrm{T}} = (\boldsymbol{g}_h^{\mathrm{T}} \quad -\boldsymbol{g}_h^{\mathrm{T}})$. Equation (37) is the generalized integral formula for the partially coherent beam through misaligned optical system in the spatial-frequency domain.

By using variable transformation

$$\boldsymbol{v}_{1e} = \boldsymbol{v}_1 - \frac{1}{2\lambda}\boldsymbol{B}^{-1}\boldsymbol{D}\boldsymbol{e}_f - \frac{1}{2\lambda}\boldsymbol{B}^{-1}\boldsymbol{g}_h, \quad (38)$$

$$\boldsymbol{v}_{2e} = \boldsymbol{v}_2 - \frac{1}{2\lambda}\boldsymbol{B}^{-1}\boldsymbol{D}\boldsymbol{e}_f - \frac{1}{2\lambda}\boldsymbol{B}^{-1}\boldsymbol{g}_h, \quad (39)$$

$$\tilde{\boldsymbol{v}}_e = \tilde{\boldsymbol{v}} - \frac{1}{2\lambda}\bar{\boldsymbol{B}}^{-1}\bar{\boldsymbol{D}}\tilde{e}_f - \frac{1}{2\lambda}\bar{\boldsymbol{B}}^{-1}\tilde{\boldsymbol{g}}_h. \quad (40)$$

Eq.(37) can be transformed into the following form:

$$\begin{aligned} W(\tilde{\boldsymbol{v}}) =& \frac{\lambda^2}{16\pi^4[\det(\bar{\boldsymbol{C}})]^{1/2}} \\ & \times \exp[\mathrm{i}\pi\tilde{\boldsymbol{v}}^{\mathrm{T}}\tilde{\boldsymbol{e}}_f]\iiiint W(\tilde{\boldsymbol{f}}) \\ & \times \exp[\mathrm{i}\pi\lambda(\tilde{\boldsymbol{f}}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{D}}\tilde{f} - 2\tilde{\boldsymbol{f}}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\tilde{\boldsymbol{v}} \\ & + \tilde{\boldsymbol{v}}^{\mathrm{T}}\bar{\boldsymbol{A}}\bar{\boldsymbol{C}}^{-1}\tilde{\boldsymbol{v}})]\mathrm{d}\tilde{\boldsymbol{f}}. \end{aligned} \quad (41)$$

Comparing Eq.(22) with Eq.(41), we can find that the main difference is only the term $\exp[-\mathrm{i}\pi\tilde{\boldsymbol{f}}^{\mathrm{T}}\tilde{\boldsymbol{e}}_f]$ in front of the integral in Eq.(41), which is irrelevant with the intensity distribution of the partially coherent beams in the FRT plane in the spatial-frequency domain. If we omit this term, then Eq.(41) can be expressed as

$$\begin{aligned} W(\tilde{\boldsymbol{v}}) =& \frac{\lambda^2}{16\pi^4[\det(\bar{\boldsymbol{C}})]^{1/2}}\iiiint W(\tilde{\boldsymbol{f}}) \\ & \times \exp[\mathrm{i}\pi\lambda(\tilde{\boldsymbol{f}}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\bar{\boldsymbol{D}}\tilde{f} - 2\tilde{\boldsymbol{f}}^{\mathrm{T}}\bar{\boldsymbol{C}}^{-1}\tilde{\boldsymbol{v}} \\ & + \tilde{\boldsymbol{v}}^{\mathrm{T}}\bar{\boldsymbol{A}}\bar{\boldsymbol{C}}^{-1}\tilde{\boldsymbol{v}})]\mathrm{d}\tilde{\boldsymbol{f}}. \end{aligned} \quad (42)$$

If we assume that $\bar{\boldsymbol{A}}$, $\bar{\boldsymbol{B}}$, $\bar{\boldsymbol{C}}$ and $\bar{\boldsymbol{D}}$ in Eq.(42) satisfy Eq.(15), then the generalized diffraction integral formula for partially coherent beams passing through misaligned optical systems can be described in the form of the FRT formula in the spatial-frequency domain. But compared with the case of an aligned optical system, the cross-spectral density of partially coherent beam passing through a misaligned optical system for performing FRT has displacements

$$\boldsymbol{v}_{1e} = \boldsymbol{v}_1 - \frac{1}{2\lambda}\boldsymbol{B}^{-1}\boldsymbol{D}\boldsymbol{e}_f - \frac{1}{2\lambda}\boldsymbol{B}^{-1}\boldsymbol{g}_h,$$

$$\boldsymbol{v}_{2e} = \boldsymbol{v}_2 - \frac{1}{2\lambda}\boldsymbol{B}^{-1}\boldsymbol{D}\boldsymbol{e}_f - \frac{1}{2\lambda}\boldsymbol{B}^{-1}\boldsymbol{g}_h,$$

and

$$\tilde{\boldsymbol{v}}_e = \tilde{\boldsymbol{v}} - \frac{1}{2\lambda}\bar{\boldsymbol{B}}^{-1}\bar{\boldsymbol{D}}\tilde{e}_f - \frac{1}{2\lambda}\bar{\boldsymbol{B}}^{-1}\tilde{\boldsymbol{g}}_h$$

in the spatial-frequency domain.

4. Intensity distribution properties of partially coherent twisted anisotropic GSM beams in the FRT plane in the spatial-frequency domain

As an example of the application, in this section we study the intensity distribution properties of partially coherent twisted anisotropic GSM beams in the FRT plane in the spatial-frequency domain. Assume the cross-spectral density and the corresponding partially coherent complex curvature tensor in the input plane take the form of Eq.(9) and Eq.(10), respectively. Applying Eqs.(15)–(18), we can easily study the intensity distribution properties of twisted anisotropic GSM beams in the FRT plane with different optical system parameters in the spatial-frequency domain.

Figure 3 shows the contour graphs of the intensity distribution of the twisted anisotropic GSM beam in the FRT plane with different fractional order in the spatial-frequency domain. The initial parameters used in the calculation are

$$(\boldsymbol{\sigma}_I^2)^{-1} = \begin{pmatrix} 1 & 0.2 \\ 0.2 & 0.5 \end{pmatrix}(\mathrm{mm})^{-2},$$

$$(\boldsymbol{\sigma}_{\mathrm{g}}^2)^{-1} = \begin{pmatrix} 0.5 & 0.2 \\ 0.2 & 0.33 \end{pmatrix}(\mathrm{mm})^{-2},$$

$$\boldsymbol{R}^{-1} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}(\mathrm{mm})^{-1},$$

$\mu = 0.00001(\mathrm{mm})^{-1}$, f=200mm. From Fig.3, we can find that the beam shape of twisted anisotropic GSM beam in the FRT plane in the spatial-frequency domain is invariant with the change of the fractional order p, which is very different from the result in the spatial domain, as shown in Ref.[19]. In the spatial domain, both the beam shape and orientation of the beam spot vary with the change of the fractional order. But the beam spot size of twisted anisotropic GSM beam in the FRT plane in the spatial-frequency domain is closely related to the fractional order p.

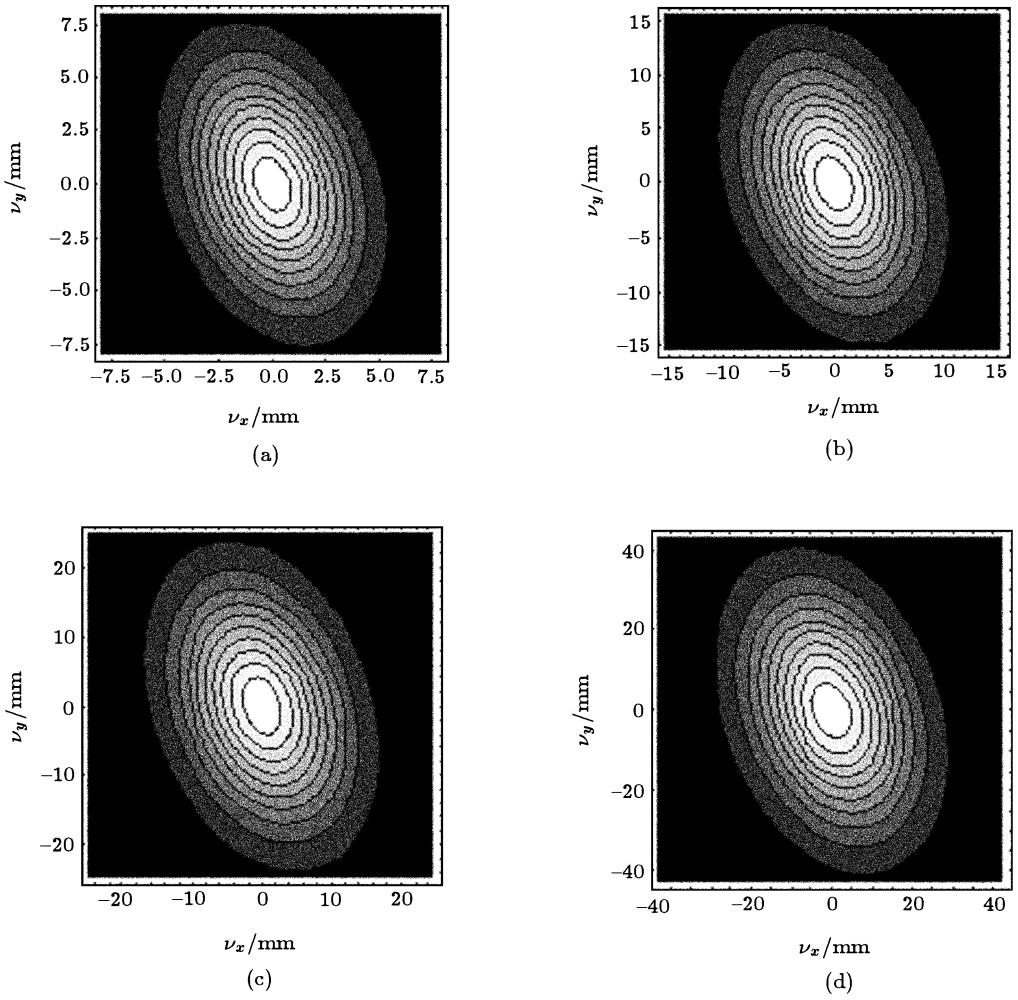


Fig.3. Contour graphs of the intensity distribution of a partially coherent twisted anisotropic GSM beam in the FRT plane with different fractional order in the spatial-frequency domain. (a) $p = 0.2$, (b) $p = 0.4$, (c) $p = 0.6$, (d) $p = 0.8$.

To learn more about the influence of the fractional order p on the intensity distribution of twisted anisotropic GSM beam in the FRT plane in the spatial-frequency domain, we have calculated the evolution of the on-axis intensity of a partially coherent twisted anisotropic GSM beam in the FRT plane with the fractional order p in the spatial-frequency domain; the result is shown in Fig.4. It is clear from Fig.4 that the evolution of on-axis intensity in the FRT plane with the fractional order p is periodic, and the period is 2. When $p = 2n$, the on-axis intensity has a maximum value, indicating that the beam spot size has a minimum value. When $p = 2n + 1$, the on-axis intensity has a minimum value. The result is also different from that in the spatial domain as shown in Ref.[19]. In the spatial domain, the beam spot size takes the maximum value at $p = 2n$, and takes the minimum value at $p = 2n + 1$.

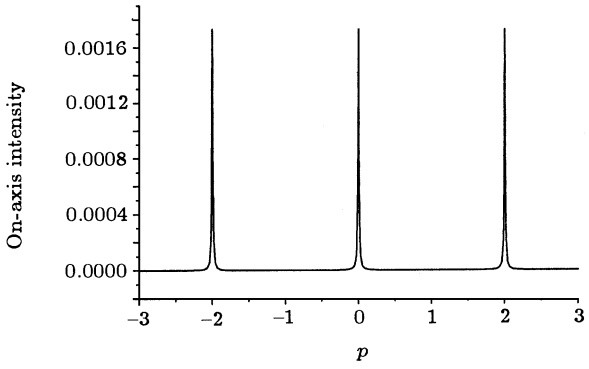


Fig.4. Evolution of the on-axis intensity of a partially coherent twisted anisotropic GSM beam in the FRT plane with fractional order p in the spatial-frequency domain.

From what has been discussed above, we can find that the intensity distribution properties of partially coherent twisted anisotropic GSM beams in the FRT plane in the spatial-frequency domain are closely related to the fractional order, and the relation between the intensity distribution and the fractional order is very different from that in the spatial domain. We can easily conclude that we can conveniently control the intensity distribution of partially coherent twisted anisotropic GSM beams in the FRT plane in the spatial-frequency domain by properly choosing the fractional order of the FRT system. The derived results have applications in material thermal processing by laser beam, where special intensity distribution is required. The results are also useful in the design of optical systems for beam shaping, which manipulates the intensity distribution and the wave front of a laser beam.

5. Conclusions

By use of a new tensor method, the FRT for partially coherent beams is extended to the spatial-frequency domain. Based on the FRT in spatial-frequency domain, the tensor $\boldsymbol{ABCD}$ law for partially coherent twisted anisotropic GSM beam performing the FRT in the spatial-frequency domain is derived. The connections between the FRT formula and the generalized diffraction integral formulae for partially coherent beams passing through the aligned optical system and misaligned optical system in the spatial-frequency domain are discussed. The derived formulae provide a powerful tool for analysing and calculating the FRT of partially coherent twisted anisotropic GSM beams in the spatial-frequency domain. As an application example, the intensity distribution properties of partially coherent twisted anisotropic GSM beams in the FRT plane are studied, and compared with the results in the spatial domain. The results show that the intensity properties of partially coherent beam in the FRT plane in the spatial-frequency domain are closely related to the parameters of the FRT system.

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