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Radiation pressure and laser cooling of a three-level atom in a ladder configuration*

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Radiation pressure and laser cooling of a moving three-level ladder-type atom in bichromatic travelling fields are considered. The dependence of the force on parameters such as detunings, Rabi frequencies and spontaneous decay rates is calculated numerically and shown graphically, and analytical expressions for the force are obtained for some special parameter values. It is shown that the radiation pressure shows Doppler-shifted resonance peaks resulting respectively from one-photon and two-photon transitions. Using the present scheme, Doppler cooling of sodium exploiting the $3\,^2S_{1/2}$–$3\,^2P_{3/2}$–$3\,^2D_{5/2}$ cascade transitions is investigated. It is found that temperatures lower than the Doppler limit can be achieved.

Keywords: three-level atom, ladder configuration, radiation pressure, Doppler cooling
PACC: 3280, 3280P, 4250, 4250V

1. Introduction

Laser cooling of neutral atoms and Bose–Einstein condensation [1] of ultracold atomic vapours have been one of the most exciting developments in physics in recent years. As is well known, the experimental realization of Bose–Einstein condensation hinges on an interplay of cooling and trapping of realistic atoms by optical forces. It is widely known that the simplest cooling mechanism, the so-called Doppler cooling, starts with a two-level atom interacting with an external laser field. The minimum temperature achieved, i.e. the Doppler limit, is determined by the natural linewidth of the excited state. However, it is realized that the multi-level structure of atoms can be exploited to achieve much lower temperature. The polarization gradient cooling scheme in optical molasses [2] exploits the interplay of the hyperfine and Zeeman structure of the ground state and the spatial variation of the laser field to achieve temperatures corresponding to tens of the recoil energy of the atom. Up to now, subrecoil cooling schemes, velocity-selective coherent population trapping [3] and Raman cooling,[4] have been proposed and demonstrated, both involving two-photon process in lambda-type three-level atoms.

The study of optical properties and laser cooling of a three-level atom is also interesting.[5–8] Allegrini and Arimondo [9] suggested Doppler cooling of hydrogen by two-photon π pulses on the 1S–3D transition, with the 2P state as the intermediate state for spontaneous cascade emission. Zehnlé and Garreau [10] proposed continuous-Doppler cooling of hydrogen by 1S–2S two-photon transition with a controllable quenching process coupling the 2S state to the 2P state. Rooijakkers *et al*[11] investigated Doppler cooling of metastable helium atoms using the combined action of two lasers exciting the 2^3S_1–$2\,^3P_2$–$3\,^3D_3$ cascade. Panat and Lawande [12] studied the optical forces on a three-level atom coupled to two laser beams in the intense field limit. Analytical expressions for the dipole force for special values of the detuning parameters are derived, while the dissipative force vanishes in a steady state. As pointed out by the authors themselves,[12] this is the artifact of the model approximations. Recently Magno *et al*[13] present a scheme for laser cooling of alkaline-earth-metal and ytterbium atoms using a two-photon $^1S_0$–$^1S_0$ transition. The Doppler limit of 123μK for C_a has been found numerically at optimum detunings and laser powers.

In the present paper, we study the steady-state

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radiation pressure on a ladder-type three-level atom with nonequispaced levels coupled to two external laser fields and its possible application to laser cooling. For a wide range of external laser strength, the dependence of forces on parameters such as detunings, Rabi frequencies and spontaneous decay rates is calculated numerically and shown graphically. For some special parameter values, simple analytical expressions for the force are obtained. The diffusion coefficient as well as the friction coefficient and the achievable equilibrium temperature are calculated. For the sake of simplicity, we will work with a one-dimensional model and assume that each of the laser field couples to one transition only with a small one-photon detuning.

The paper is organized as follows. The model is described and the expression for the radiation pressure is derived in Sec.2. In Sec.3, an analysis of the radiation force in a steady state is presented. Section 4 is devoted to the study of Doppler cooling of a ladder-type three-level atom, with sodium as an example. A brief summary is included in Sec.5.

2. Expression for the radiation pressure

Consider a moving three-level atom in a ladder configuration with states $|1\rangle|$, $|2\rangle$ and $|3\rangle$ having energies E_1, E_2 and E_3 ($E_3 > E_2 > E_1$) (see Fig.1) respectively. The atom is driven by two linearly polarized laser fields, E_a of wave-vector κ_a and frequency ω_a coupled to the 1–2 transition, and E_b of wave-vector κ_b and frequency ω_b coupled to the 2–3 transition. The 1–2 and 2–3 transition frequencies are respectively ω_{21} and ω_{32}, where $\hbar\omega_{ij} = E_i - E_j$. These transitions are associated with the electric dipole matrix elements μ_{12} and μ_{23} (assumed real for simplicity) respectively, whereas the 1–3 transition is dipole forbidden ($\mu_{13} = 0$). The Hamiltonian describing the interaction of the atom with the two laser fields and the vacuum is given in the electric dipole and rotating-wave approximation by

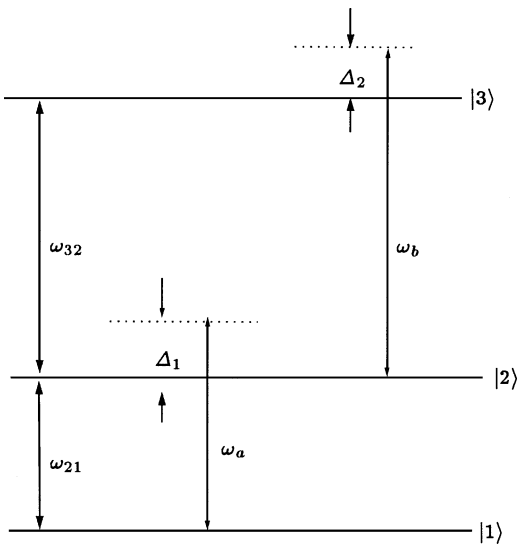


Fig.1. A three-level atom in a ladder configuration with different transition frequencies driven by two laser fields of frequencies ω_a and ω_b detuned from the atomic transition frequency by $\Delta_1 = \omega_a - \omega_{21}$ and $\Delta_2 = \omega_b - \omega_{32}$.

$$
\begin{aligned}
H =& \frac{P^2}{2M} + \sum_{i=1}^{3} E_i|i\rangle\langle i| + V_{\rm I}(R) + V_{\rm S}(R),\\
V_{\rm I}(R) =& -\frac{1}{2}{\rm i}\hbar\Omega_a(|2\rangle\langle 1|{\rm e}^{{\rm i}(K_a R-\omega_a t)}\\
&-|1\rangle\langle 2|{\rm e}^{-{\rm i}(K_a R-\omega_a t)})\\
&-\frac{1}{2}{\rm i}\hbar\Omega_b(|3\rangle\langle 2|{\rm e}^{{\rm i}(K_b R-\omega_b t)}\\
&-|2\rangle\langle 3|{\rm e}^{-{\rm i}(K_b R-\omega_b t)}),\\
V_{\rm S}(R) =& -\frac{1}{2}{\rm i}\hbar\sum_l(\Omega_l^{(1)}b_l|2\rangle\langle 1|{\rm e}^{{\rm i}(K_l R-\omega_l t)}\\
&-(\Omega_l^{(1)})^* b_l^+|1\rangle\langle 2|{\rm e}^{-{\rm i}(K_l R-\omega_l t)})\\
&-\frac{1}{2}{\rm i}\hbar\sum_l(\Omega_l^{(2)}b_l|3\rangle\langle 2|{\rm e}^{-{\rm i}(K_l R-\omega_l t)}\\
&-(\Omega_l^{(2)})^* b_l^+|2\rangle\langle 3|{\rm e}^{-{\rm i}(K_l R-\omega_l t)}),
\end{aligned}
\tag{1}
$$

where P and R are the momentum and position operators of the atom. $V_{\rm I}(R)$ and $V_{\rm S}(R)$ describe the interaction of the atom with the laser field and the vacuum field. M is the atomic mass and $\Omega_a = \mu_{12}E_{a0}/\hbar$ and $\Omega_b = \mu_{23}E_{b0}/\hbar$ are the Rabi frequencies determined by the laser amplitude E_{a0} (E_{b0}) and the corresponding atomic dipole moment. b_l and b_l^+ are bosonic operators for mode l of the vacuum field. $\Omega_l^{(1)} = (\mu_{12}\hat{e}_l)\left(\frac{2\omega_l}{\hbar\varepsilon_0 V}\right)^{1/2}$ and $\Omega_l^{(2)} = (\mu_{23}\hat{e}_l)\left(\frac{2\omega_l}{\hbar\varepsilon_0 V}\right)^{1/2}$ are the usual Rabi frequencies of the vacuum field, and $\hat{e}_l$ is the unit polarization vector of mode l.

The Heisenberg equations for R and P are written as

$$
\begin{aligned}
\dot{R} =& \frac{1}{{\rm i}\hbar}[R, H] = \frac{P}{M},\\
\dot{P} =& \frac{1}{{\rm i}\hbar}[P, H] = -\nabla V_{\rm I}(R) - \nabla V_{\rm S}(R).
\end{aligned}
\tag{2}
$$

The force operator on the atom is

$$
F = F_{\rm I} + F_{\rm S} = \dot{P} = -\nabla V_{\rm I}(R) - \nabla V_{\rm S}(R). \tag{3}
$$

The radiation pressure f is equal to the ensemble average of the force operator F, i.e. $f =< F >$. As

$< b_l >=< b_l^+ >= 0$, $< F_s >= 0$ and we need to consider the average of $F_{\rm I}$ only. Nevertheless, the fluctuation of $F_{\rm S}$ contributes to the momentum diffusion of the atom, and will be given later.

For atoms of very slow velocity v, the de Broglie wavelength of the atom, $\lambda_{\rm DB} = h/Mv$, is much smaller than the optical wavelength. It is possible to construct atomic wave packets having very small dimensions in comparison with the optical wavelength and is legitimate to replace the operator R by its mean value r. The radiation pressure is then given by

$$f = \langle F \rangle = {\rm Tr}(\rho F) = -\frac{1}{2}\left\{ \begin{array}{l} \hbar k_a \Omega_a [\rho_{12} {\rm e}^{{\rm i}(K_a r - \omega_a t)} + \rho_{21} {\rm e}^{-{\rm i}(K_a r - \omega_a t)}] \\ +\hbar k_b \Omega_b [\rho_{23} {\rm e}^{{\rm i}(K_b r - \omega_b t)} + \rho_{32} {\rm e}^{-{\rm i}(K_b r - \omega_b t)}] \end{array} \right\}, \tag{4}$$

where ρ is the reduced density operator for the atom, and $\rho_{ij} = \langle i|\rho|j\rangle$, its matrix elements. The evolution of ρ in the external laser driving fields along with the effects of spontaneous emission is governed by the master equation [14,15]

$$\frac{\partial \rho}{\partial t} = -\frac{\rm i}{\hbar}[H,\rho] + \frac{1}{2}\sum_{(i,j)=(a,b)} \Gamma_{ij}\{[S_i^-, \rho S_j^+] + [S_i^- \rho, S_j^+]\}, \tag{5}$$

where $S_a^+ = |2\rangle\langle 1|$, $S_a^- = |1\rangle\langle 2|$, and $S_b^+ = |3\rangle\langle 2|$, $S_b^- = |2\rangle\langle 3|$. The parameters Γ_{aa} and Γ_{bb} are the rates for spontaneous emission from state $|2\rangle$ to state $|1\rangle$ and from state $|3\rangle$ to state $|2\rangle$ respectively. For brevity we shall denote Γ_{aa} and Γ_{bb} by Γ_a and Γ_b respectively later. The coherence transfer rate Γ_{ab} coupling the 1–2 and 2–3 atomic coherences is given by

$$\Gamma_{ab} = \Gamma_{ba} = \frac{\mu_{12}\mu_{23}}{6\pi\varepsilon_0 \hbar c^3}(\omega_{21}^3 + \omega_{32}^3). \tag{6}$$

Γ_{ab} is zero when μ_{12} and μ_{23} are orthogonal. In the case of nonequispaced levels Γ_{ab} can be ignored via the application of the rotating-wave approximation of the second kind (adiabatic approximation),[15,16] since they couple coherences whose frequencies differ by much more than an optical frequency.

As is well known, the time scales for the evolution of the internal and external degrees of freedom of the atom are highly distinct. The former, of the order of the natural life of the excited states, is much shorter than the latter, of the order of $\hbar/E_{\rm rec}$, where $E_{\rm rec}$ is the recoil energy of the atom. The large difference between the two time scales results in the fact that the internal degrees of freedom have time to reach the steady state before the external degrees of freedom have changed appreciably. This allows us to leave out the kinetic energy term $P^2/2M$ in the Hamiltonian in the master equation (5).

From the master equation, we obtain equations of motion for the matrix elements ρ_{ij}, which are linearly coupled equations with explicit time-dependent factors of the complex exponential type. These factors can be removed by the following transformation:

$$\begin{aligned} \sigma_{ii} &= \rho_{ii}, i = 1,2,3, \\ \sigma_{12} &= \rho_{12}{\rm e}^{{\rm i}(k_a r - \omega_a t)} = \sigma_{21}^*, \\ \sigma_{23} &= \rho_{23}{\rm e}^{{\rm i}(k_b r - \omega_b t)} = \sigma_{32}^*, \\ \sigma_{13} &= \rho_{13}{\rm e}^{{\rm i}(k_a r - \omega_a t) + {\rm i}(k_b r - \omega_b t)} = \sigma_{31}^*. \end{aligned} \tag{7}$$

The equations of motion for the density matrix elements σ_{ij} are as follows:

$$\begin{aligned} \frac{\partial \sigma_{11}}{\partial \tau} &= \frac{1}{2}\xi_a(\sigma_{12} + \sigma_{21}) + \gamma_a \sigma_{22}, \\ \frac{\partial \sigma_{22}}{\partial \tau} &= -\frac{1}{2}\xi_a(\sigma_{12} + \sigma_{21}) \\ &\quad + \frac{1}{2}\xi_b(\sigma_{23} + \sigma_{32}) + \gamma_b \sigma_{33} - \gamma_a \sigma_{22}, \\ \frac{\partial \sigma_{33}}{\partial \tau} &= -\frac{1}{2}\xi_b(\sigma_{23} + \sigma_{32}) - \gamma_b \sigma_{33}, \\ \frac{\partial \sigma_{12}}{\partial \tau} &= -\frac{1}{2}[\gamma_a + {\rm i}(\Delta - \delta)]\sigma_{12} + \frac{1}{2}\xi_a(\sigma_{22} - \sigma_{11}) \\ &\quad + \frac{1}{2}\xi_b \sigma_{13} + {\rm i}\sigma_{12} k_a v, \\ \frac{\partial \sigma_{13}}{\partial \tau} &= -[\frac{1}{2}\gamma_b + {\rm i}\Delta]\sigma_{13} + \frac{1}{2}(\xi_a \sigma_{23} - \xi_b \sigma_{12}) \\ &\quad + {\rm i}\sigma_{13}(k_a v + k_b v), \\ \frac{\partial \sigma_{23}}{\partial \tau} &= -\frac{1}{2}[1 + {\rm i}(\Delta + \delta)]\sigma_{23} - \frac{1}{2}\xi_a \sigma_{13} \\ &\quad + \frac{1}{2}\xi_b(\sigma_{33} - \sigma_{22}) + {\rm i}\sigma_{23} k_b v, \end{aligned} \tag{8}$$

where we have introduced the following suitable scaled variables:

$$\begin{aligned} \tau &= (\Gamma_a + \Gamma_b)t, \\ \gamma_a &= \Gamma_a/(\Gamma_a + \Gamma_b), \\ \gamma_b &= \Gamma_b/(\Gamma_a + \Gamma_b), \\ \xi_a &= \Omega_a/(\Gamma_a + \Gamma_b), \\ \xi_b &= \Omega_b/(\Gamma_a + \Gamma_b), \\ \Delta &= (\Delta_1 + \Delta_2)/(\Gamma_a + \Gamma_b), \\ \delta &= (\Delta_2 - \Delta_1)/(\Gamma_a + \Gamma_b). \end{aligned} \tag{9}$$

Thus τ is the scaled time, γ_a and γ_b are scaled relaxation rates, ξ_a and ξ_b are scaled Rabi frequencies, Δ

is the scaled two-photon detuning, δ is the difference between the scaled one photon detunings, and v is the scaled velocity of the atom $v = \mathrm{d}r/\mathrm{d}\tau$. The equations for σ_{21}, σ_{32}, and σ_{31} can be obtained from Eqs.(8) by complex conjugation.

In terms of σ_{ij} and the scaled variables, the radiation force (4) can be rewritten as

$$f = -\frac{1}{2}[\Gamma_a + \Gamma_b](\hbar k_a \xi_a(\sigma_{12} + \sigma_{21}) + \hbar k_b \xi_b(\sigma_{23} + \sigma_{32})). \quad (10)$$

Equating the first three equations of Eqs.(8) to zero, we obtain from Eq.(10) the force in steady state:

$$f_{\mathrm{st}} = \hbar k_a \Gamma_a \sigma_{22} + \hbar k_b \Gamma_b \sigma_{33}. \quad (11)$$

It begins to appear that Eq.(11) is a simple extension of the well-known result for a two-level atom.[17] But, as there are more parameters for a three-level system, it displays rather rich characteristics. Equation (11) can also be written as

$$f_{\mathrm{st}} = [\Gamma_a + \Gamma_b]\frac{1}{1+\eta}(\hbar k_a \sigma_{22} + \eta \hbar k_b \sigma_{33}), \quad (12)$$

where $\eta = \gamma_2/\gamma_1$ is the ratio of the two spontaneous decay rates.

By setting the left-hand sides of Eqs.(8) to zero and using matrix inversion techniques, numerical results for the steady-state radiation force can be found.

3. Analysis of the dissipative force

Simple analytic expressions for the force can be obtained provided that the two Rabi frequencies are equal $\Omega_a = \Omega_b = \Omega$, i.e. $\xi_a = \xi_b = \xi$. Then, in the weak field limit ($\xi \ll 1$), the force is given by

$$f_{\mathrm{st}} = \hbar\Gamma_a k_a \frac{\xi^2}{\gamma_a^2 + (\Delta - \delta - 2k_a v)^2} + \hbar\Gamma_b k_b \frac{\xi^4}{[\gamma_b^2 + 4(\Delta - k_a v - k_b v)^2][\gamma_a^2 + (\Delta - \delta - 2k_a v)^2]}. \quad (13)$$

The two terms are associated with the steady-state populations of the intermediate state and the upper state respectively. The first term shows resonance behaviour when $\Delta - 2k_a v - \delta = 0$, i.e. $\Delta_1 = k_a v$, corresponding to the one-photon Doppler-shifted resonance in the transition $|1\rangle \rightarrow |2\rangle$. The second term, however, shows resonant behaviour when $\Delta = (k_a + k_b)v$ and $\Delta - 2k_a v - \delta = 0$. The first case corresponds to two-photon Doppler-shifted resonance, the second case to resonance in the intermediate state population through which a stepwise excitations to the upper state occurs.

At somewhat higher incident intensity, the force is given by

$$f_{\mathrm{st}} = \hbar k_a \Gamma_a \left(\frac{\xi^2}{\gamma_a^2 + 2\xi^2 + (\Delta - \delta - 2k_a v)^2} + \frac{\xi^4}{[\gamma_b^2 + 4(\Delta - k_a v - k_b v)^2][\gamma_a^2 + 2\xi^2 + (\Delta - \delta - 2k_a v)^2]} \right) + \hbar k_b \Gamma_b \frac{\xi^4}{[\gamma_b^2 + 4(\Delta - k_a v - k_b v)^2][\gamma_a^2 + 2\xi^2 + (\Delta - \delta - 2k_a v)^2]}. \quad (14)$$

The first term is associated with the steady-state population in the intermediate state, while the second term is that in the upper state. It shows that the one-photon resonance at $\Delta_1 = k_a v$ in both the intermediate and upper states is power broadened by the term $2\xi^2$. At the same time, both the upper and the intermediate states show two-photon Doppler-shifted resonance at $\Delta = (k_a + k_b)v$. The latter effect is due to simultaneous absorption of two photons to cause the transition $|1\rangle \rightarrow |3\rangle$ followed by a spontaneous emission of a photon associated with the $|3\rangle \rightarrow |2\rangle$ transition.

In a very strong-field regime, we have

$$f_{\mathrm{st}} = \hbar k_a \Gamma_a \frac{\gamma_b}{\gamma_a + 2\gamma_b} + \hbar k_b \Gamma_b \frac{\gamma_a + \gamma_b}{2\gamma_a + 4\gamma_b}, \quad (15)$$

where $\gamma_b/(\gamma_a + 2\gamma_b)$and $(\gamma_a + \gamma_b)/(2\gamma_a + 4\gamma_b)$ are the saturated values of the population of the intermediate and the upper states respectively. When $\Gamma_a = \Gamma_b = \Gamma$, we have $\sigma_{22} = \sigma_{33} = 1/3$, indicating that all three atomic states are equally populated. In this case, the radiation force is $f_{\mathrm{st}} = \frac{1}{3}(\hbar k_a + \hbar k_b)\Gamma$.

In what follows we will use the previously described numerical methods to study the dependence of the force on the detuning, the spontaneous decay

rate and the Rabi frequency. The force is normalized to $\hbar|k_a|(\Gamma_a + \Gamma_b)/2$.

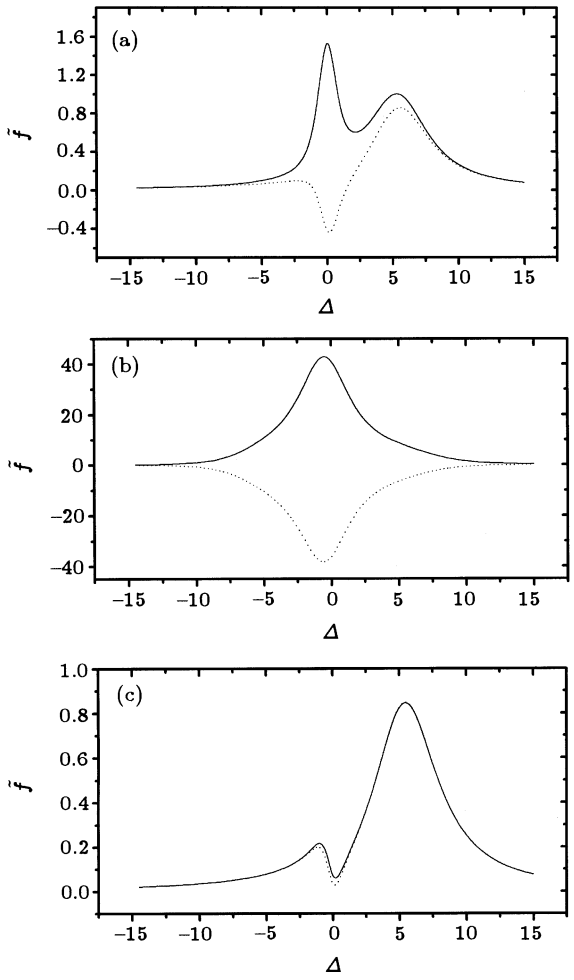


Fig.2. $\tilde{f} = \dfrac{2f_{\rm st}}{\hbar|k_a|(\Gamma_a + \Gamma_b)}$, see the text.

Figure 2 shows for equal Rabi frequencies $\xi_a = \xi_b = 2$, the radiation force versus the two-photon detuning Δ for detuning $\delta = 5$, Doppler-shifted frequency $k_a v = 0.02$ and $k_b v = 0.04$ (solid line). For equal spontaneous decay rates $\gamma_a = \gamma_b = 0.5$, two resonance are seen, associated respectively with one-photon and two-photon absorptions, as described above. This is illustrated in Fig.2(a). For $\eta = \gamma_b/\gamma_a = 10^2$, only one resonance remains, indicating that the one-photon process is suppressed due to the long lifetime of the intermediate state, and the two-photon process makes a dominant contribution to the force, as can be realized from Eq.(11). This is illustrated in Fig.2(b). For $\eta = \gamma_b/\gamma_a = 10^{-2}$, Eq.(11) shows that the term with σ_{22} dominates. A pronounced resonance and a dip at $\Delta \approx k_a v + k_b v$ are seen as in Fig.2(c). This resonance is associated with one-photon absorption. The emergence of the dip is due to the absorption of two photons from the driving field so that a population trapping in a coherent superposition of the upper state $|3\rangle$ and the lower state $|1\rangle$ results owing to the long lifetime of state $|3\rangle$, with only a small population in state $|2\rangle$. When the two laser waves are counter-propagating, the average momentum gained by the atom will be the algebraic sum of the contributions from the two transitions. The radiation force for $k_a v = 0.02$, $k_b v = -0.04$ (dotted line) is also shown in Fig.2.

Figure 3 shows for different values of the two Rabi frequencies, the dissipative force versus two-photon detuning for $\delta = 5$, $k_a v = 0.02$, $k_b v = 0.04$. Results for $\xi_a = 1$, $\xi_b = 2$ (solid line) and $\xi_a = 2$, $\xi_b = 1$ (dotted line) are shown. For comparison the result for $\xi_a = \xi_b = 2$ (dashed line) is also plotted in Fig.3: (a) $\eta = 1$, (b) $\eta = 10^2$, (c) $\eta = 10^{-2}$. A notable feature is that in Fig.3(b) the solid line is enhanced while the dotted line is diminished, due to the enhancement of the two-photon process. The contrary is seen in Fig.3(c), due to the domination of the one-photon process.

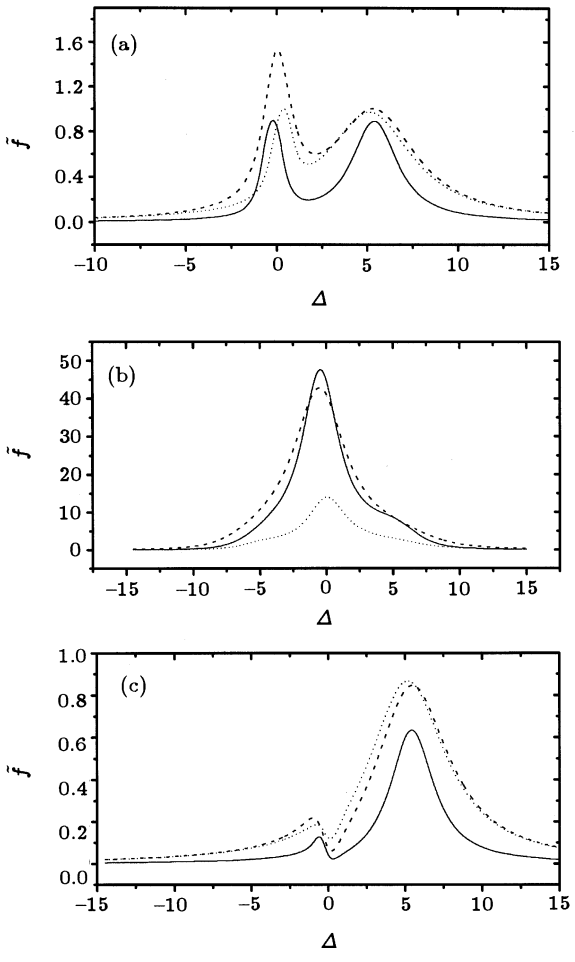


Fig.3. $\tilde{f} = \dfrac{2f_{\rm st}}{\hbar|k_a|(\Gamma_a + \Gamma_b)}$, see the text for explanation.

4. Investigation of Doppler cooling of a ladder-type three-level atom

We now turn to the investigation of force fluctuations, which ultimately determine the equilibrium temperature achieved. The diffusion coefficient of the atomic momentum may be defined as [17]

$$\begin{aligned} D =& \int_0^\infty [\langle F(0)F(t)\rangle - \langle F(0)\rangle\langle F(0)\rangle]\mathrm{d}t \\ =& \int_0^\infty [\langle F_{\mathrm{I}}(0)F_{\mathrm{I}}(t)\rangle - \langle F_{\mathrm{I}}(0)\rangle\langle F_{\mathrm{I}}(0)\rangle]\mathrm{d}t \\ &+ \int_0^\infty [\langle F_{\mathrm{S}}(0)F_{\mathrm{S}}(t)\rangle - \langle F_{\mathrm{S}}(0)\rangle\langle F_{\mathrm{S}}(0)\rangle]\mathrm{d}t \\ =& D_{\mathrm{I}} + D_{\mathrm{S}}, \end{aligned} \tag{16}$$

where F is given by Eq.(3) and $\langle F\rangle$ by Eq.(4). Before inserting Eqs.(3) and (4) into Eq.(16), we introduce the following transformation for convenience of mathematical operation:

$$\begin{aligned} \langle u_a\rangle =& \sigma_{12} + \sigma_{21}, \\ \langle u_b\rangle =& \sigma_{23} + \sigma_{32}, \\ \langle u_c\rangle =& \sigma_{13} + \sigma_{31}, \\ \langle v_a\rangle =& (-\mathrm{i})(\sigma_{12} - \sigma_{21}), \\ \langle v_b\rangle =& (-\mathrm{i})(\sigma_{23} - \sigma_{32}), \\ \langle v_c\rangle =& (-\mathrm{i})(\sigma_{13} - \sigma_{31}), \\ \langle w_a\rangle =& \sigma_{22} - \sigma_{11}, \\ \langle w_b\rangle =& \sigma_{33} - \sigma_{22}; \end{aligned} \tag{17}$$

the optical Bloch equations (8) may then be changed into a different form containing only real variables,

$$\frac{\partial\langle\lambda\rangle}{\partial\tau} = A\langle\lambda\rangle + b \tag{18}$$

with

$$\lambda \equiv (u_a, u_b, u_c, v_a, v_b, v_c, w_a, w_b),$$

$$A \equiv \begin{bmatrix} -\frac{1}{2}\gamma_a & 0 & \frac{1}{2}\xi_b & -\beta_1 & 0 & 0 & \xi_a & 0 \\ 0 & -\frac{1}{2} & -\frac{1}{2}\xi_a & 0 & -\beta_3 & 0 & 0 & \xi_b \\ -\frac{1}{2}\xi_b & \frac{1}{2}\xi_a & -\frac{1}{2}\gamma_b & 0 & 0 & -\beta_2 & 0 & 0 \\ \beta_1 & 0 & 0 & -\frac{1}{2}\gamma_a & 0 & \frac{1}{2}\xi_b & 0 & 0 \\ 0 & \beta_3 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2}\xi_a & 0 & 0 \\ 0 & 0 & \beta_2 & -\frac{1}{2}\xi_a & \frac{1}{2}\xi_b & -\frac{1}{2}\gamma_b & 0 & 0 \\ -\xi_a & \frac{1}{2}\xi_b & 0 & 0 & 0 & 0 & \frac{1}{3}(\gamma_b - 2\gamma_a) & \frac{2}{3}(\gamma_b + \gamma_a) \\ \frac{1}{2}\xi_a & -\xi_b & 0 & 0 & 0 & 0 & \frac{1}{3}(\gamma_a - 2\gamma_b) & -\frac{1}{3}(\gamma_a + 4\gamma_b) \end{bmatrix}$$

$$b \equiv (0,0,0,0,0,0,\frac{1}{3}(\gamma_b - 2\gamma_a), \frac{1}{3}(\gamma_a - 2\gamma_b)), \tag{19}$$

where $\beta_1 = \frac{1}{4}(\Delta - \delta - 2k_a v)$, $\beta_2 = \frac{1}{2}(\Delta - k_a v - k_b v)$, $\beta_3 = \frac{1}{4}(\Delta + \delta - 2k_b v)$. The inhomogeneous equation (18) can be written as a homogeneous one for $\langle\tilde{\lambda}\rangle = \langle\lambda\rangle - \lambda_{\mathrm{st}}$, where λ_{st} is the steady-state solution of Eq.(18), and

$$\frac{\partial\langle\tilde{\lambda}\rangle}{\partial\tau} = A\langle\tilde{\lambda}\rangle. \tag{20}$$

Then the diffusion coefficient D_{I} may be expressed as

$$D_{\mathrm{I}} = \frac{1}{4}\hbar^2 k_a^2(\Gamma_a + \Gamma_b)\mathrm{Re}\int_0^\infty \begin{bmatrix} \xi_a^2\langle\tilde{u}_a(0)\tilde{u}_a(\tau)\rangle + \beta\xi_a\xi_b\langle\tilde{u}_a(0)\tilde{u}_b(\tau)\rangle \\ +\beta\xi_b\xi_a\langle\tilde{u}_b(0)\tilde{u}_a(\tau)\rangle + \beta^2\xi_b^2\langle\tilde{u}_b(0)\tilde{u}_b(\tau)\rangle \end{bmatrix}\mathrm{d}\tau, \tag{21}$$

where $\beta = k_b/k_a$. We can determine the correlation function $\langle\tilde{\lambda}_i(0)\tilde{\lambda}_j(\infty)\rangle$ from the quantum regression theorem [18,19]

$$\frac{\partial}{\partial\tau}\langle\tilde{\lambda}(0)\tilde{\lambda}(t)\rangle = A\langle\tilde{\lambda}(0)\tilde{\lambda}(t)\rangle. \tag{22}$$

By integrating Eq.(22), we obtain

$$\int_0^\infty \langle\tilde{\lambda}(0)\tilde{\lambda}(t)\rangle \mathrm{d}\tau = -A^{-1}\langle\tilde{\lambda}^2\rangle \equiv G, \tag{23}$$

where we have taken into account $\langle\tilde{\lambda}_i(0)\tilde{\lambda}_j(\infty)\rangle = \langle\tilde{\lambda}_i(0)\rangle\langle\tilde{\lambda}_j(\infty)\rangle = 0$. The diffusion coefficient D_{I} can then be reduced to

$$\begin{aligned} D_{\mathrm{I}} =& \frac{1}{4}\hbar^2 k_a^2(\Gamma_a + \Gamma_b) \\ & \times (\xi_a^2 G_{11} + \beta\xi_a\xi_b G_{12} \\ & + \beta\xi_a\xi_b G_{21} + \beta^2\xi_b^2 G_{22}). \end{aligned} \tag{24}$$

As $\langle F_{\mathrm{S}}\rangle = 0$, the diffusion due to vacuum fluctuation is given by

$$D_{\mathrm{S}} = \mathrm{Re}\int_0^\infty \langle F_{\mathrm{S}}(0)F_{\mathrm{S}}(t)\rangle \mathrm{d}t. \tag{25}$$

Inserting of F_{S} into Eq.(25) yields

$$D_{\mathrm{S}} = \frac{1}{10}(\hbar k_a)^2\Gamma_a\sigma_{22} + \frac{1}{10}(\hbar k_b)^2\Gamma_b\sigma_{33}. \tag{26}$$

The diffusion coefficient is a function of the atomic velocity; we shall take $k_a v = 0.05$. At the same time, the mean force $\langle F(t)\rangle$ is a nonlinear function of the atomic velocity. For a slowly moving atom, we may neglect the higher-order terms of v and retain only the first-order term, i.e. $\langle F(t)\rangle = F_0 - \alpha v$, with α the friction coefficient, which determines the time it takes to cool the atom to its final state. The final temperature attained by an atom may be calculated by the Einstein relation

$$k_{\mathrm{B}}T = \frac{D}{\alpha}. \tag{27}$$

Numerical results for the $3\,^2\mathrm{S}_{1/2} - 3\,^2\mathrm{P}_{3/2} - 3\,^2\mathrm{D}_{5/2}$ cascade in sodium (wavelengths 589 and 820nm, $\Gamma_a = 59.1\times10^6\mathrm{s}^{-1}$ and $\Gamma_b = 49.5\times10^6\mathrm{s}^{-1}$) [20] are plotted in Figs.4 and 5, where the friction coefficient and diffusion coefficient are normalized to $\hbar k_a^2/4$ and $\hbar^2 k_a^2\Gamma_a/4$ respectively. Temperature is scaled to the two-level Doppler limit $\hbar\Gamma_a/2$. Friction, diffusion, and the equilibrium temperature versus two-photon detuning Δ for $\xi_a = 1$, $\xi_b = 3$ and different detunings $\delta = 10$ (dotted line), $\delta = 15$ (dashed line) and $\delta = 20$ (solid line) are shown in Fig.4. It is shown that the diffusion coefficient shows a dip at two-photon transition, implying that the correlation of the fluctuation of the radiation pressure is reduced in simultaneous absorption of the two photons. Friction, diffusion, and the equilibrium temperature versus the difference of the two one-photon detunings δ for $\Delta = -(1-\beta)k_a v$ and different Rabi frequencies $\xi_a = 1$, $\xi_b = 2$ (dotted line) and $\xi_a = 1$, $\xi_b = 3$ (dashed line), $\xi_a = 1$, $\xi_b = 4$ (solid line) are shown in Fig.5. It can be seen from Figs.4 and 5 that the equilibrium temperature may be cooled below the Doppler limit. The result may be explained as follows. As there are more excitation channels in a three-level atom, the friction coefficient α is much higher than that for the two-level one, so that although the momentum diffusion coefficient D for a three-level system is higher than that of the two-level counterpart, the ratio D/α, i.e. the equilibrium temperature, is lower in the former case. It can also be seen from Fig.5 that a wide range of parameter values may be used for laser cooling of a ladder-type three-level atom.

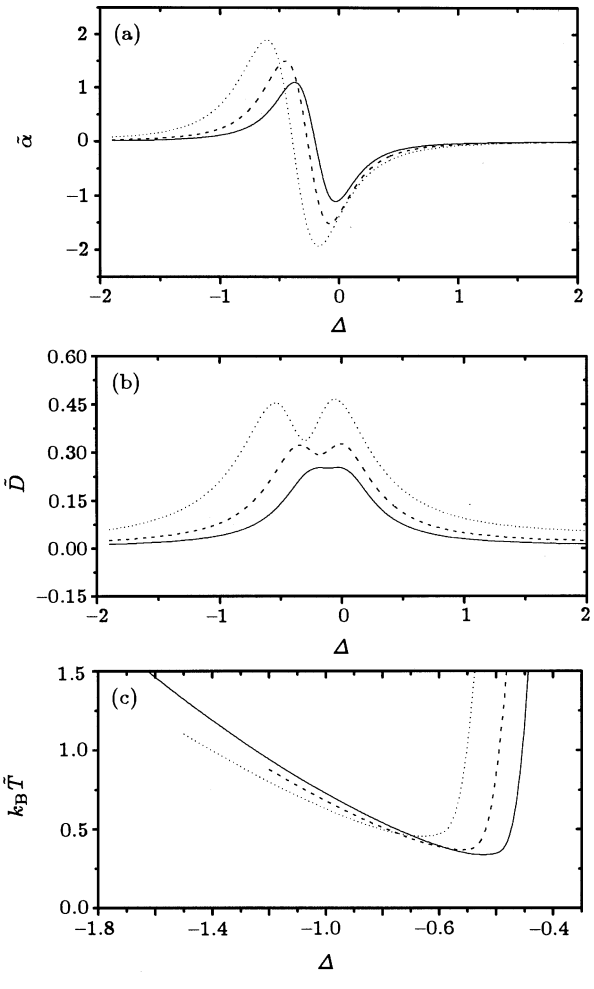


Fig.4. $\tilde{\alpha} = \dfrac{\alpha}{\frac{1}{4}\hbar k_a^2}$, $\tilde{D} = \dfrac{D}{\frac{1}{4}\hbar^2 k_a^2\Gamma_a}$, $k_{\mathrm{B}}\tilde{T} = \dfrac{k_{\mathrm{B}}T}{\frac{1}{2}\hbar\Gamma_a}$, see the text for explanation.

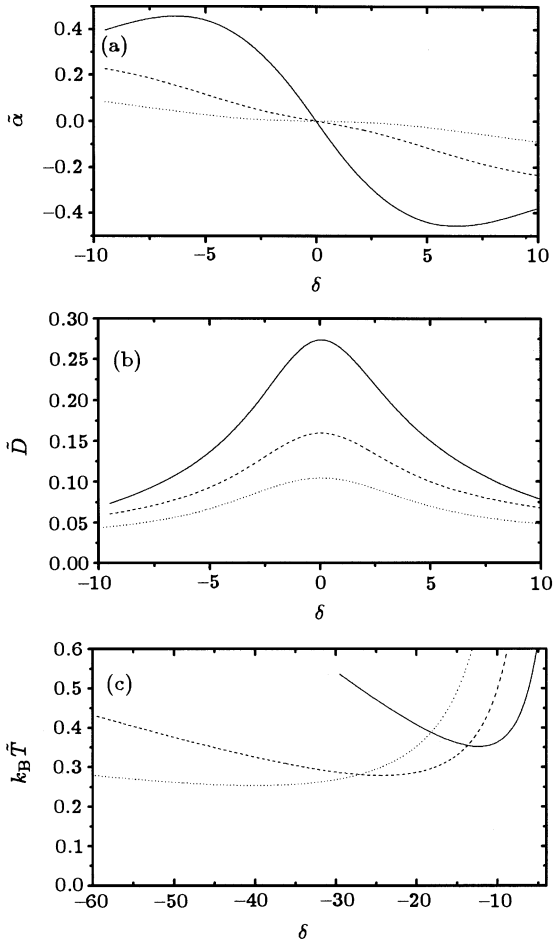


Fig.5. $\tilde{\alpha} = \dfrac{\alpha}{\frac{1}{4}\hbar k_a^2}$, $\tilde{D} = \dfrac{D}{\frac{1}{4}\hbar^2 k_a^2 \Gamma_a}$, $k_B\tilde{T} = \dfrac{k_B T}{\frac{1}{2}\hbar\Gamma_a}$, see the text.

5. Summary

In this paper, we have discussed the radiation force and its application for laser cooling on a moving ladder-type three-level atom with nonequispaced levels for a wide range of external field strength. The atom is coupled to two travelling laser fields. Heisenberg equations for the motion of the centre of the atomic wave packet and master equation for the evolution of the internal states of the atom are shown. As the time scales for the evolution of internal and external degrees of freedom are highly distinct, the mean radiation force in a steady state is derived. Simple analytical expressions for the force are obtained in special cases. The dependence of the force on various parameters is described numerically and shown graphically. It is seen that the one-photon and two-photon processes play their respective role under different conditions. The diffusion coefficient is derived from the quantum regression theory, and the Einstein relation is employed to determine the equilibrium temperature of the atom. It is shown that for a suitable choice of the controllable parameters in our scheme, it is possible to cool the three-level atoms down below the Doppler limit.

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