Vol 13 No 7, July 2004
1009-1963/2004/13(07)/0996-04

Chinese Physics

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Synchronization of coupled dynamical chaotic networks via scalar controllers*

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(Received 2 December 2003; revised manuscript received 9 February 2004)

Some scalar linear controllers, which can ensure that the states of coupled chaotic dynamical networks asymptotically synchronize each other, are derived on the basis of high gain state feedback control. Numerical simulation is given to validate the proposed theoretical result.

Keywords: dynamical chaotic networks, asymptotic synchronization, scalar controller
PACC: 0545

1. Introduction

Chaos synchronization has been studied extensively in recent years. Since chaos synchronization is an important problem in nonlinear science, much effort has been devoted to investigate it and numerous advances [1−16] have been made. For example, chaos synchronization based on intermittent state observer,[14] the nonlinear observer theory method,[6] the neural network method,[5] the impulse control method,[4] the Lyapunov function method [16] and others are presented for synchronization of chaotic systems. It is well known that in communication systems, one would like to minimize the number of signals that are sent. Therefore, being able to synchronize with a scalar signal is beneficial to the design of chaotic communication systems. In particular, it would be desirable that the coupled chaotic network can be synchronized via some scalar controllers since these will minimize the number of controllers that need to be designed. In this paper, for a class of coupled dynamical network, some fairly simple scalar linear-state feedback controllers are indeed derived for the above-described task by exploiting the high gain state feedback control theory. Unlike other approaches,[17−19] where only local results were obtained, the coupled dynamical network is not linearized. Moreover, the synchronization between two coupled chaotic systems is only a special case of the present work. A typical example is finally given to verify the proposed theoretical result.

2. Problem formulation

Consider the dynamical network which consists of N coupled nodes with nonlinear uncertain chaos system

$$\begin{cases} \dot{x}_{i1} = x_{i2} + f_1(x_{i1}, x_{11}, x_{21}, \ldots, x_{N1}, t), \\ \dot{x}_{i2} = x_{i3} + f_2(x_{i1}, x_{i2}, x_{12}, x_{22}, \ldots, x_{N2}, t), \\ \vdots \\ \dot{x}_{in} = u_i + f_n(x_i, x_{1n}, x_{2n}, \ldots, x_{Nn}, t), \end{cases} \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \in R^n$, $i = 1, 2, \ldots, N$, represents the state vector of the system, f_i, $i = 1, 2, \ldots, n$ are unknown smooth nonlinear functions, and $u_i \in R$, $i = 1, 2, \ldots, N$ are the scalar control inputs to be determined later.

The objective is to find some smooth scalar controllers $u_i \in R$, $i = 1, 2, \ldots, N$ such that the solutions of systems (1) synchronize with each other, in the sense that

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \quad \text{for all} \quad i, j = 1, 2, \ldots, N. \tag{2}$$

*Project supported by the National Natural Science Foundation of China (Grant No 70371066) and by China Postdoctoral Science Foundation (Grant No 2003033307).

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Let $e_i = x_i - x_{i+1}$, $i = 1, 2, \ldots, N-1$. One can obtain the error dynamical systems

$$\begin{cases} \dot{e}_{i1} = e_{i2} + \tilde{f}_1(x_{i1}, x_{i+1,1}, x_{11}, x_{21}, \ldots, x_{N1}, t), \\ \dot{e}_{i2} = e_{i3} + \tilde{f}_2(x_{i1}, x_{i2}, x_{i+1,1}, x_{i+1,2}, \\ \qquad x_{12}, x_{22}, \ldots, x_{N2}, t), \\ \vdots \\ \dot{e}_{in} = \tilde{u}_i + \tilde{f}_n(x_i, x_{i+1}, x_{1n}, x_{2n}, \ldots, x_{Nn}, t), \end{cases} \tag{3}$$

where $\tilde{f}_k(x_{ik},\ x_{i+1,k}, t) = f_k(x_{i1}, x_{i2}, \ldots,\ x_{ik},\ x_{1k}, \ldots,\ x_{Nk}, t) - f_k(x_{i+1,1},\ x_{i+1,2}, \ldots,\ x_{i+1,k},\ x_{1k}, \ldots, x_{Nk}, t)$

$$\tilde{u}_i = u_i - u_{i+1}, i = 1, 2, \ldots, N-1.$$

For attaining the objective Eq.(2), we make the following assumption.

Assumption 1 Let

$$|\tilde{f}_k(x_{ik}, x_{i+1,k}, t)| \le \rho_i \sum_{j=1}^{k} |e_{ij}|,$$

$$i = 1, 2, \ldots, N-1, k = 1, 2, \ldots, n, \tag{4}$$

where ρ_i, i=1, 2, $\ldots, N-1$, are known non-negative constants.

3. Main result

Introduce the transformation

$$\varepsilon_{ik} = \frac{e_{ik}}{L^{k-1}}, i = 1, 2, \ldots, N-1, k = 1, 2, \ldots, n, \tag{5}$$

and choose the controllers $\tilde{u}_i = -L^n a_1 e_{i1} - L^{n-1} a_2 e_{i2} - \cdots - L a_n e_{in}$, $i = 1, 2, \ldots, N-1$, where $L > 2$ is a gain parameter to be determined later, and $a_k > 0$, $k = 1, 2, \ldots, n$ are coefficients of the Hurwitz polynomial $p(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n$. By using Eq.(5), the system (3) can be transformed into

$$\begin{cases} \dot{\varepsilon}_{i1} = L\varepsilon_{i2} + \tilde{f}_1(x_{i1}, x_{i+1,1}, t), \\ \dot{\varepsilon}_{i2} = L\varepsilon_{i3} + \dfrac{\tilde{f}_2(x_{i1}, x_{i2}, x_{i+1,1}, x_{i+1,2}, t)}{L}, \\ \vdots \\ \dot{\varepsilon}_{in} = L(-a_1\varepsilon_{i1} - \cdots - a_n\varepsilon_{in}) + \dfrac{\tilde{f}_n(x_i, x_{i+1,1}, t)}{L^{n-1}}. \end{cases} \tag{6}$$

If $F_{ik} = \dfrac{\tilde{f}_k(x_{i1}, \ldots, x_{ik}, x_{i+1,1}, \ldots, x_{i+1,k})}{L^{k-1}}$, $k = 1, 2, \ldots, n$, the system (6) is rewritten as

$$\dot{\varepsilon}_i = LA\varepsilon_i + F_i, i = 1, 2, \ldots, N-1, \tag{7}$$

where $A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & -a_3 & \cdots & -a_n \end{bmatrix}$, $\varepsilon_i = (\varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{in})^{\mathrm{T}}$, $F_i = (F_{i1}, F_{i2}, \ldots, F_{in})^{\mathrm{T}}$.

Since A is a Hurwitz matrix, there exists a positive definite matrix P which satisfies $A^{\mathrm{T}}P + PA = -I$. Consider a Lyapunov function candidate $V = \sum_{i=1}^{N-1} \varepsilon_i^{\mathrm{T}} P \varepsilon_i$. The derivative of V along with the solutions of Eq.(7) is

$$\frac{\mathrm{d}V}{\mathrm{d}t} \le -L \sum_{i=1}^{N-1} \varepsilon_i^{\mathrm{T}} \varepsilon_i + 2 \sum_{i=1}^{N-1} \|\varepsilon_i\| \|P\| \left(\sum_{k=1}^{n} \|F_{ik}\| \right).$$

Since

$$\frac{L}{L-1} < 2, \quad \text{for} \quad L > 2,$$

and

$$\sum_{k=1}^{n} |F_{ik}| \le \sum_{k=1}^{n} \rho_i \sum_{j=1}^{k} \frac{|e_{ij}|}{L^{k-1}}$$
$$= \rho_i \sum_{j=1}^{n} |e_{ij}| \sum_{k=j}^{n} \frac{1}{L^{k-1}} < 2\rho_i \sum_{j=1}^{n} \frac{|e_{ij}|}{L^{j-1}},$$

one has

$$\frac{\mathrm{d}V}{\mathrm{d}t} \le -\sum_{i=1}^{N-1} (L - 4\sqrt{n}\rho_i \|P\|) \varepsilon_i^{\mathrm{T}} \varepsilon_i.$$

Choosing

$$L > \max_{1 \le i \le N-1} \{2, 4\sqrt{n}\rho_i \|P\|\},$$

one has

$$\lim_{t \to \infty} \|\varepsilon_i\| = 0, i = 1, \ldots, N-1.$$

It follows that $\lim_{t \to \infty} \|e_i\| = 0, i = 1, 2, \ldots, N-1$. Therefore, the following result is obtained.

Theorem 1 Let Assumption 1 hold. Then, there exist the scalar linear controllers

$$\begin{aligned} u_i = & u_1 + L^n a_1 (x_{11} - x_{i1}) \\ & + L^{n-1} a_2 (x_{12} - x_{i2}) + \cdots \\ & + L a_n (x_{1n} - x_{in}), i = 2, \ldots, N, \end{aligned} \tag{8}$$

such that the solutions to system (1) satisfies Eq.(2).

Proof: For any $i, j = 1, \ldots, N$, since $\|x_i - x_j\| = \|(x_i - x_{i+1}) + (x_{i+1} - x_{i+2}) + \cdots (x_{j-1} - x_j)\| \le \sum_{k=i}^{j-1} \|e_k\|$, along with $\lim_{t \to \infty} \|e_k\| = 0, k = 1, 2, \ldots, N-1$, it follows that the solutions to system (1) satisfy Eq.(2).

Remarks: In the designing of controllers $u_i \in R, i = 1, \ldots, N$, if one chooses $u_k = 0$, then $u_1 = L^n a_1(x_{k1} - x_{11}) + L^{n-1}a_2(x_{k2} - x_{12}) + \cdots + La_n(x_{kn} - x_{1n})$. If one chooses $u_1 = 0$, controllers $u_i \in R, i = 2, \ldots, N$ can be constructed according to Eq.(8).

4. Simulation

Consider the Rössler system

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + ay, \\ \dot{z} &= b + z(x - c). \end{aligned} \tag{9}$$

It has been proved that Eqs.(9) present chaos when $a = b$=0.2, c=5.7. Introducing linear transformation $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and using Eqs.(9) as nodes, we obtain the following coupled dynamical network:

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} + ax_{i1} + \sum_{j=1}^{N} c_j x_{j1}, \\ \dot{x}_{i2} &= x_{i3} - x_{i1}, \\ \dot{x}_{i3} &= u_i - b + x_{i3}(x_{i2} - c), i = 1, 2, \ldots, N, \end{aligned} \tag{10}$$

where $c_j = 1$, $j = 1, 2, \ldots, N$.

According to formula (8) of Theorem 1, some scalar linear-state feedback controllers are constructed. In simulation, we choose N=50, n=3, and

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -5 \end{bmatrix},$$

$$P = \begin{bmatrix} 9.6667 & -0.5000 & -1.8333 \\ -0.5000 & 1.8333 & -0.5000 \\ -1.8333 & -0.5000 & 0.6667 \end{bmatrix},$$

$$L = 2087.8.$$

Figures 1–3 clearly show that the states of the dynamical network (10) synchronize globally in the sense of Eq.(2).

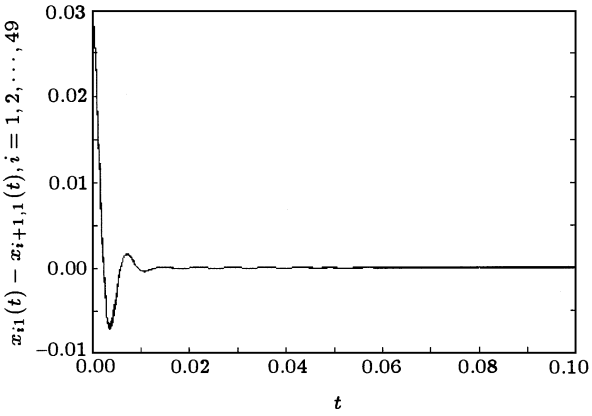


Fig.1. The synchronization error-states $x_{i1}(t) - x_{i+1,1}(t)$, $i = 1, 2, \ldots, 49$.

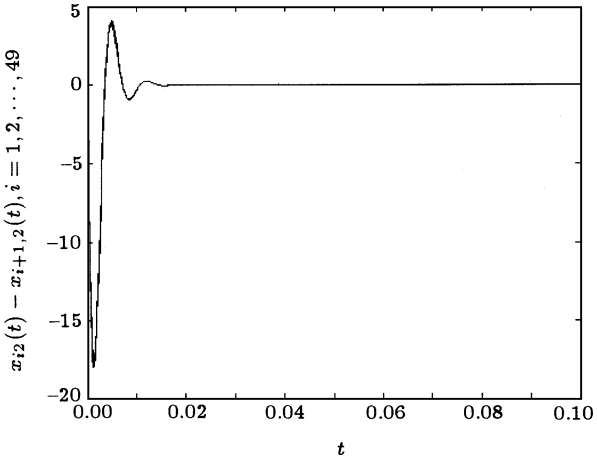


Fig.2. The synchronization error-states $x_{i2}(t) - x_{i+1,2}(t)$, $i = 1, 2, \ldots, 49$.

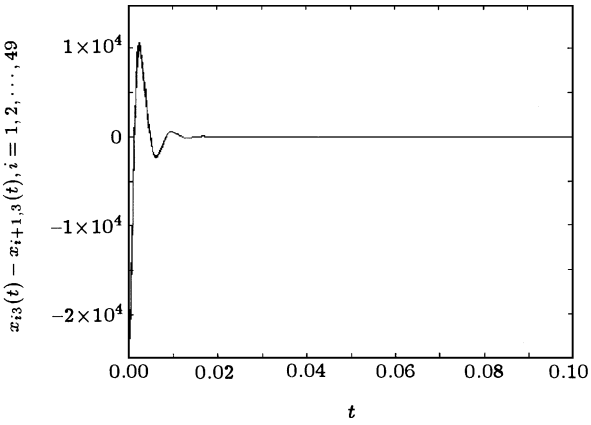


Fig.3. The synchronization error-states $x_{i3}(t) - x_{i+1,3}(t)$, $i = 1, 2, \ldots, 49$.

5. Conclusion

For a class of coupled dynamical chaotic networks, some fairly simple scalar linear-state feedback controllers are obtained based on the high gain state feedback control theory. Simulation has verified the proposed theoretical result.

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