Vol 13 No 7, July 2004
1009-1963/2004/13(07)/0988-02

Chinese Physics

© 2004 Chin. Phys. Soc.
and IOP Publishing Ltd

Generation of Bell states in two-component Bose–Einstein condensates*

Zhang Guo-Feng(张国锋)[a)b)†], Yin Wen(殷雯)[a)b)],
Liang Jiu-Qing(梁九卿)[a)], and Yan Qi-Wei(严启伟)[b)]

[a)] *Institute of Theoretical Physics, Shanxi University, Taiyuan* 030006, *China*
[b)] *State Key Laboratory of Magnetism, Institute of Physics and Centre for Condensed Matter Physics, Chinese Academy of Sciences, Beijing* 100080, *China*

(Received 3 November 2003; revised manuscript received 1 December 2003)

We examine analytically the generation of Bell state in Bose condensates of two interacting species trapped in a double well configuration. The density of probability for finding the entangled Bell state is given. The effect of the tunnelling rate and the interspecies interaction strength on the generation of Bell state is discussed. We find that the oscillation amplitude of the density of probability for finding the entangled Bell state becomes greater as the tunnelling rate Ω increases, and the self-interaction strength of the component A(B) has no effect on it.

Keywords: quantum entanglement, Bose-Einstein condensates
PACC: 0365, 7450

1. Introduction

Quantum entanglement is of fundamental interest since many of the most basic aspects of quantum theory require its successful generation and manipulation. In particular, the Bell maximally entangled states is the starting point for fundamental discussions such as the violation of Bell's inequalities [1] and the nonlocality problem,[2,3,4] as well as for teleportation [5,6,7] and quantum cryptography.[8] Here we will show how to generate maximally entangled states of two qubits of the form $|\Psi_{\rm Bell}\rangle=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$ in Bose–Einstein condensates (BECs) of two interacting species. The quantum tunnelling of BECs has been well studied.[9,10] Recently, a theoretical model is proposed to examine the quantum tunnelling in BEC of two interacting species trapped in a double well configuration (see Fig.1). The atoms of the component A(B) are initially prepared in the left (right) potential well.

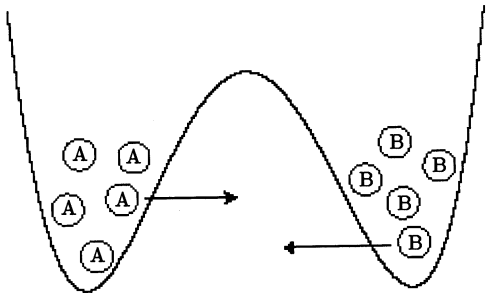


Fig.1. A sketch of the double-well tunnelling system. Initially, the atoms of the component A are trapped in the left side and the atoms of component B are trapped in the right side.

2. The model and the solution

The Hamilton operator of the system in the two-mode approximation is given by [11]

$$\begin{aligned} H =& \frac{\Omega}{2}(a_{\rm L}^{+}a_{\rm R}+a_{\rm R}^{+}a_{\rm L}+b_{\rm L}^{+}b_{\rm R}+b_{\rm R}^{+}b_{\rm L}) \\ &+\frac{k_{\rm a}}{2}[(a_{\rm L}^{+}a_{\rm L})^2+(a_{\rm R}^{+}a_{\rm R})^2] \\ &+\frac{k_{\rm b}}{2}[(b_{\rm L}^{+}b_{\rm L})^2+(b_{\rm R}^{+}b_{\rm R})^2] \\ &+k(a_{\rm L}^{+}a_{\rm L}b_{\rm L}^{+}b_{\rm L}+a_{\rm R}^{+}a_{\rm R}b_{\rm R}^{+}b_{\rm R}), \end{aligned} \tag{1}$$

where the subscripts L and R respectively denote the localized modes in left and right potential wells. The model in fact consists of four mode operators. We use $\widehat{a}_j^{+}$ and $\widehat{b}_j^{+}$ (j=L, R) to denote the creation operators of the component A and B respectively. The parameters $k_{\rm a}$, $k_{\rm b}$, and k describe the self-interaction strength of the components A,

*Project supported by the National Natural Science Foundation of China (Grant No 10075032).
†E-mail: gf1978zhang@sohu.com

B and the interspecies interaction strength, respectively, and Ω describes the tunnelling rate. We consider the exactly solvable case with only one A atom in the left well and one B atom in the right well. In this case the system is spanned by four basis vectors: $|1,0\rangle_{\rm A}|1,0\rangle_{\rm B}$,$|1,0\rangle_{\rm A}|0,1\rangle_{\rm B}$,$|0,1\rangle_{\rm A}|1,0\rangle_{\rm B}$ and $|0,1\rangle_{\rm A}|0,1\rangle_{\rm B}$ where $|p,q\rangle$ denotes the state with p atoms of species S (S=A, B) in the left well and q atoms of species S in the right well. So we may express the state vector as $|\Psi(t)\rangle = \sum_{n=0}^{1}\sum_{m=0}^{1} C_{n,m}(t)|n,1-n\rangle_{\rm A}|m,1-m\rangle_{\rm B}$. The amplitudes and $C_{n,m}(t)$ are governed by the Schrödinger equation: $\mathrm{i}\dfrac{\partial|\Psi(t)\rangle}{\partial t} = H|\Psi(t)\rangle$ ($\hbar=1$) with the initial condition $C_{n,m}(0)=\delta_{n,1}\delta_{m,0}$. We obtain the following results:

$$
\begin{aligned}
C_{1,1}(t) =& C_{0,0}(t) = \frac{\Omega\exp\left[-\frac{\mathrm{i}\lambda t}{2}\right](-1+\exp[\mathrm{i}\Omega_0 t])}{2\Omega_0},\\
C_{1,0}(t) =& \frac{\exp\left[-\frac{\mathrm{i}\lambda t}{2}\right]\left((-1+\exp[\mathrm{i}\Omega_0 t])k+\left(1+\exp[\mathrm{i}\Omega_0 t]+2\exp\left[\frac{\mathrm{i}(\Omega_0+k)t}{2}\right]\right)\Omega_0\right)}{4\Omega_0},\\
C_{0,1}(t) =& \frac{\exp\left[-\frac{\mathrm{i}\lambda t}{2}\right]\left((-1+\exp[\mathrm{i}\Omega_0 t])k+\left(1+\exp[\mathrm{i}\Omega_0 t]-2\exp\left[\frac{\mathrm{i}(\Omega_0+k)t}{2}\right]\right)\Omega_0\right)}{4\Omega_0},
\end{aligned}
\tag{2}
$$

where $\Omega_0 = \sqrt{k^2+4\Omega^2}$, $\lambda = k+\Omega_0+k_{\rm a}+k_{\rm b}$. Similarly, the state $|\Phi_{\rm Bell}\rangle = \dfrac{1}{\sqrt{2}}(|1,0\rangle_{\rm A}|0,1\rangle_{\rm B} + |0,1\rangle_{\rm A}|1,0\rangle_{\rm B})$ is a form of Bell state that is maximally entangled in the two-particle two-mode subspace. We centre our attention on the discussion of finding the maximally entangled Bell states. To achieve this, we project the state $|\Phi_{\rm Bell}\rangle$ over the wavefunction $|\Psi(t)\rangle$ to obtain the density of probability, Γ(Bell), for finding the entangled Bell state in this coupled system, that is, $\Gamma(\mathrm{Bell})=|<\Phi\mid\Psi(t)>|^2=\dfrac{1}{2}|C_{1,0}(t)+C_{0,1}(t)|^2$. Thus we obtain

$$
\Gamma(\mathrm{Bell}) = \frac{k^2+\Omega^2+2\Omega^2\cos[\Omega_0 t]}{2\Omega_0^2}. \tag{3}
$$

From the equation, we can see that the density of probability is independent of the self-interaction strength of component A(B), and only depends on the interspecies interaction strength k and the tunnelling rate Ω. In Fig.2, we can see that when Ω is weak, the Γ(Bell) almost stays at the initial value. As Ω increases, Γ(Bell) has a bigger oscillation amplitude.

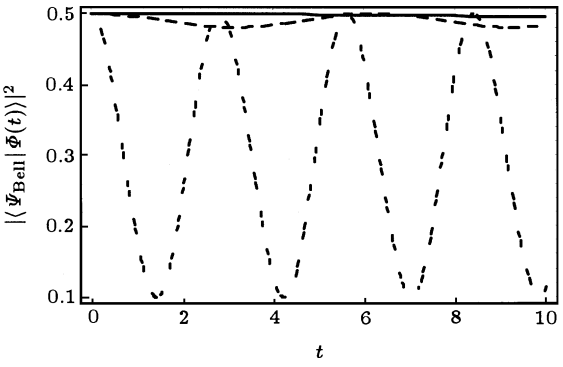


Fig.2. Density of probability for finding the entangled Bell state versus time for $\frac{k}{\Omega}$=10, Ω=0.01 (solid curve), Ω=0.1 (dashed curve) and Ω=1 (double-dashed curve).

3. Conclusion

To conclude, we study analytically the generation of Bell state in Bose condensates of two interacting species trapped in a double well configuration. This result provides some suggestion for obtaining the maximally entangled state. We studied the case with only one A atom in the left well and one B atom in the right well; other cases will be studied in our future work.

References

[1] Aspect A, Dalibard J and Roger G 1982 *Phys. Rev. Lett.* **49** 1804

[2] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[3] Zheng Y Z *et al* 2002 *Chin. Phys.* **12** 529

[4] Wen S C and Fan D Y 2001 *Chin. Phys.* **11** 1032

[5] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wotters W K 1993 *Phys. Rev. Lett.* **70** 1895

[6] Ye L, Yao C M and Guo G C 2001 *Chin. Phys.* **11** 1001

[7] Wang G Z and Fang M F 2002 *Acta Phys. Sin.* **51** 411 (in Chinese)

[8] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661

[9] Liu W M, Wu B and Niu Q 2000 *Phys. Rev. Lett.* **84** 2294

[10] Liu W M, Fan W B, Zheng W M, Liang J Q and Chui S T 2002 *Phys. Rev. Lett.* **88** 170408

[11] Ng H T, Law C K and Leung P T cond-mat/0209631