Vol 13 No 6, June 2004
1009-1963/2004/13(06)/0974-05

Chinese Physics

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Quantum statistical entropy for Kerr–de Sitter black hole*

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(Received 28 July 2003; revised manuscript received 12 December 2003)

Improving the membrane model by which the entropy of the black hole is studied, we study the entropy of the black hole in the non-thermal equilibrium state. To give the problem stated here widespread meaning, we discuss the $(n+2)$-dimensional de Sitter spacetime. Through discussion, we obtain that the black hole's entropy which contains two horizons (a black hole's horizon and a cosmological horizon) in the non-thermal equilibrium state comprises the entropy corresponding to the black hole's horizon and the entropy corresponding to the cosmological horizon. Furthermore, the entropy of the black hole is a natural property of the black hole. The entropy is irrelevant to the radiation field out of the horizon. This deepens the understanding of the relationship between black hole's entropy and horizon's area. A way to study the bosonic and fermionic entropy of the black hole in high non-thermal equilibrium spacetime is given.

Keywords: quantum statistics, brick-wall method, membrane model, entropy of black hole, higher-dimensional black hole

PACC: 9760L, 0420

1. Introduction

Entropy of the black hole is one of the important subjects in theoretical physics. Since entropy has statistical meaning, the understanding of entropy involves the sense of the microscopic essence of the black hole. Full understanding of it needs a good quantum gravitation theory. However, at present the work on that is not satisfactory. The statistical origin of the black hole is not solved as yet.[1] On the other hand, many papers give the result that the entropy of the black hole is proportional to the area of horizon.[2−9] The most frequently used method among them is the brick-wall method advanced by Hooft.[7] This method is used to study the statistical property of scalar field and Dirac field in various black holes [10−16] and it is found that if appropriate cutoff is chosen, the black hole's entropy is proportional to the area of its event horizon. However, our current research for non-equilibrium state in higher-dimensional spacetime has been challenged by calculating wave equations and has not made the expected achievement.

In this paper, the quantum statistics method and membrane model for studying the entropy of the black hole are generalized in a non-equilibrium higher-dimensional spacetime. The entropy corresponding to the black hole's horizon in non-equilibrium state and the entropy corresponding to the cosmological horizon are calculated. But our result shows that the entropy of a black hole is not what the brick-wall method indicates, which is an entropy of quantum field near the black hole's horizon. But the black hole's entropy is irrelevant to the radiation field out of the horizon. The entropy is the natural property of the black hole. This will deepen the understanding of the relationship between the entropy of black hole and the area of horizon.

2.Kerr–de Sitter black hole

The linear element in $(n+2)$-dimensional Kerr–de Sitter (KdS) spacetime is given by [17]

$$\begin{aligned} \mathrm{d}S^2 = &-\frac{\Delta_r}{\rho^2}\left(\mathrm{d}t-\frac{a}{\Xi}\sin^2\theta\mathrm{d}\phi\right)^2 \\ &+\frac{\rho^2}{\Delta_r}\mathrm{d}r^2+\frac{\rho^2}{\Delta_\theta}\mathrm{d}\theta^2 \\ &+\frac{\Delta_\theta\sin^2\theta}{\rho^2}\left(a\mathrm{d}t-\frac{r^2+a^2}{\Xi}\mathrm{d}\phi\right)^2 \\ &+r^2\cos^2\theta\mathrm{d}\Omega_{n-2}^2, \end{aligned} \tag{1}$$

*Project supported by the Natural Science Foundation of Shanxi Province, China (Grant No 20001009).

where

$$\begin{aligned}
&\Delta_r = (r^2+a^2)\left(1-\frac{r^2}{l^2}\right) - 2Mr^{3-n},\\
&\Delta_\theta = 1+\frac{a^2}{l^2}\cos^2\theta,\\
&\Xi = 1+\frac{a^2}{l^2},\\
&\rho^2 = r^2 + a^2\cos^2\theta.
\end{aligned} \tag{2}$$

Here l is the radius of curvature of $\mathrm{d}S$ space, $\mathrm{Vol}(S^n)$ denotes the volume of a unit n-sphere $\mathrm{d}\Omega_n^2$, and M is an integration constant, a is unit angular momentum.

The thermodynamic quantities associated with the cosmological horizon are [17]

$$\begin{aligned}
T_{\mathrm{c}} =& \frac{r_{\mathrm{c}}}{4\pi(r_{\mathrm{c}}^2+a^2)}\left(-(n-1)\left(1-\frac{a^2}{l^2}\right)\right.\\
&\left.-(n-3)\frac{a^2}{r_{\mathrm{c}}^2}+(n+1)\frac{r_{\mathrm{c}}^2}{l^2}\right),\\
J_{\mathrm{c}} =& \frac{a\mathrm{Vol}(S^n)}{8G\pi\Xi^2}r_{\mathrm{c}}^{n-3}(r_{\mathrm{c}}^2+a^2)\left(1-\frac{r_{\mathrm{c}}^2}{l^2}\right),\\
\Omega_{\mathrm{c}} =& -\frac{a\Xi}{r_{\mathrm{c}}^2+a^2},
\end{aligned} \tag{3}$$

where J is the angular momentum of the KdS solution in the Balasubranmanian, de Boer and Minic (BBM) [18] prescription and Ω is the chemical potential conjugate to J.

The thermodynamic quantities associated with the black hole horizon are [17]

$$\begin{aligned}
T_+ =& \frac{r_+}{4\pi(r_+^2+a^2)}\left((n-1)\left(1-\frac{a^2}{l^2}\right)\right.\\
&\left.+(n-3)\frac{a^2}{r_+^2}-(n+1)\frac{r_+^2}{l^2}\right),\\
J_+ =& \frac{a\mathrm{Vol}(S^n)}{8G\pi\Xi^2}r_+^{n-3}(r_+^2+a^2)\left(1-\frac{r_+^2}{l^2}\right),\\
\Omega_+ =& \frac{a\Xi}{r_+^2+a^2},
\end{aligned} \tag{4}$$

where r_{c} and r_+ are respectively the locations of the cosmological horizon and the black hole's horizon. T_{c} and T_+ are respectively the Hawking radiation temperatures of the cosmological horizon and black hole's horizon, G is the gravitational constant in $(n+2)$ dimensions.

3. The bosonic entropy

According to generalized relativity theory the frequency shift of the particle which is from the surface of a star obtained by the observer at rest at an infinite distance is as follows:

$$\nu = \nu_0\sqrt{-\tilde{g}_{tt}}, \tag{5}$$

where ν_0 is the natural frequency of an atom on the surface of the star, and ν is the natural frequency of this particle out of the surface of the star at random point r, which is obtained by the observer at rest at an infinite distance.

From Refs.[18,19], the natural radiation temperature observed by the observer at rest at an infinite distance is as follows:

$$T = \frac{T_+}{\sqrt{-\tilde{g}_{tt}}}, \tag{6}$$

where T_+ is the equilibrium temperature near the black hole and

$$\begin{aligned}
\tilde{g}_{tt} =& \frac{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}{g_{\varphi\varphi}}\\
=& -\frac{\Delta_r\Delta_\theta(r^2+a^2\cos^2\theta)}{\Delta_\theta(r^2+a^2)^2-\Delta_r a^2\sin^2\theta}.
\end{aligned} \tag{7}$$

For bosonic gas, we calculate the partition function Z

$$\ln Z = -\sum_i g_i\ln(1-\mathrm{e}^{-\beta\varepsilon_i}). \tag{8}$$

In unit volume, the number of quantum states with an energy between ε and $\varepsilon+\mathrm{d}\varepsilon$ or the frequency between ν and $\nu+\mathrm{d}\nu$ is as follows:

$$g(\nu)\mathrm{d}\nu = j\mathrm{Vol}(S^n)\nu^n\mathrm{d}\nu, \tag{9}$$

where j is the spinning degeneracy of particles. Since the spacetime (1) is axially symmetric, the area of the curved surface at a random point r is

$$A(r) = \int\sqrt{g}\mathrm{d}\theta\mathrm{d}\varphi(r\cos\theta)^{n-2}\mathrm{Vol}(S^{n-2}), \tag{10}$$

where $\mathrm{Vol}(S^{n-2})$ is the volume of the $(n-2)$-dimensional sphere $\mathrm{d}\Omega_{n-2}^2$ with unit radius, $g = \begin{vmatrix} g_{\theta\theta} & g_{\theta\varphi} \\ g_{\varphi\theta} & g_{\varphi\varphi} \end{vmatrix} = g_{\theta\theta}g_{\varphi\varphi}$. Then near the black hole the volume of the lamella at a random point r outside the horizon is

$$\mathrm{d}V = A(r)\sqrt{g_{rr}}\mathrm{d}r. \tag{11}$$

So, the partition function of the system at the lamella with random thickness at point r outside the horizon is

$$\begin{aligned}\ln Z =& \int A(r)\sqrt{g_{rr}}\mathrm{d}r\sum_i g_i\sum_{l=1}\frac{1}{l}\mathrm{e}^{-l\beta\varepsilon_i} = j\int A(r)\mathrm{Vol}(S^n)\sqrt{g_{rr}}\mathrm{d}r\sum_{l=1}\frac{1}{l}\int_0^\infty \mathrm{e}^{-(lh\nu)/T}\nu^n\mathrm{d}\nu\\ =&j\int A(r)\mathrm{Vol}(S^n)\sqrt{g_{rr}}\mathrm{d}r\varsigma(n+2)\left(\frac{T}{h}\right)^{n+1}n!\\ =&j\int A(r)\mathrm{Vol}(S^n)\sqrt{g_{rr}}\mathrm{d}r\frac{\varsigma(n+2)n!}{(2\pi)^{n+1}\beta_0^{n+1}}\frac{1}{[-\tilde{g}_{tt}]^{(n+1)/2}}\\ =&j\int \mathrm{d}\varphi\int (r\cos\theta)^{n-2}\mathrm{Vol}(S^n)\mathrm{Vol}(S^{n-2})\frac{\varsigma(n+2)n!}{(2\pi)^{n+1}\beta_0^{n+1}}\\ &\times\frac{[\Delta_\theta(r^2+a^2)^2-\Delta_r a^2\sin^2\theta]^{(n+2)/2}}{(\Delta_\theta\Delta_r)^{(n+2)/2}[r^2+a^2\cos^2\theta]^{n/2}}\frac{\sin\theta}{\Xi}\mathrm{d}r\mathrm{d}\theta\\ =&A\frac{\varsigma(n+2)n!}{(2\pi)^n\beta_0^{n+1}}\int_0^\pi\int\frac{[\Delta_\theta(r^2+a^2)^2-\Delta_r a^2\sin^2\theta]^{(n+2)/2}(r\cos\theta)^{n-2}}{(\Delta_\theta\Delta_r)^{(n+2)/2}[r^2+a^2\cos^2\theta]^{n/2}}\frac{\sin\theta}{\Xi}\mathrm{d}r\mathrm{d}\theta,\end{aligned}\tag{12}$$

where $\varsigma(n+2)=\sum\limits_{l=1}^{\infty}\frac{1}{l^{n+2}}$ is a Riemann function, $\frac{1}{\beta}=T$, $\beta=\beta_0\sqrt{-\tilde{g}_{tt}}$, and $A=j\mathrm{Vol}(S^n)\mathrm{Vol}(S^{n-2})$. Using the relation between entropy and partition function

$$S=\ln Z-\beta_0\frac{\partial\ln Z}{\partial\beta_0},\tag{13}$$

we can obtain the entropy corresponding to the bosonic system:

$$\begin{aligned}S_\mathrm{b} =&j\int\mathrm{d}\varphi\int(r\cos\theta)^{n-2}\mathrm{Vol}(S^n)\mathrm{Vol}(S^{n-2})\\ &\times\frac{\varsigma(n+2)n!}{(2\pi)^{n+1}\beta_0^{n+1}}(n+2)\\ &\times\frac{[\Delta_\theta(r^2+a^2)^2-\Delta_r a^2\sin^2\theta]^{(n+2)/2}}{(\Delta_\theta\Delta_r)^{(n+2)/2}[r^2+a^2\cos^2\theta]^{n/2}}\frac{\sin\theta}{\Xi}\mathrm{d}r\mathrm{d}\theta.\end{aligned}\tag{14}$$

In Eq.(14), we should take integration and use the membrane model.[2,12] Near the horizon, we take the integral region $[r_+ + \delta,\ r_+ + N\delta]$, where δ is a small non-negative quantity, and N is a constant larger than one. Then

$$\begin{aligned}S_\mathrm{b} =&j\int\mathrm{d}\varphi\int_{r_++\delta}^{r_++N\delta}(r\cos\theta)^{n-2}\mathrm{Vol}(S^n)\mathrm{Vol}(S^{n-2})\\ &\times\frac{\varsigma(n+2)n!}{(2\pi)^{n+1}\beta_0^{n+1}}(n+2)\\ &\times\frac{[\Delta_\theta(r^2+a^2)^2-\Delta_r a^2\sin^2\theta]^{(n+2)/2}}{(\Delta_\theta\Delta_r)^{(n+2)/2}[r^2+a^2\cos^2\theta]^{n/2}}\frac{\sin\theta}{\Xi}\mathrm{d}r\mathrm{d}\theta.\end{aligned}\tag{15}$$

Because the integral region that we have taken is near the horizon, we have $r^2>a^2\cos^2\theta$, then

$$\begin{aligned}S_\mathrm{b}\approx&j\int\mathrm{d}\varphi\int_{r_++\delta}^{r_++N\delta}(r\cos\theta)^{n-2}\mathrm{Vol}(S^n)\mathrm{Vol}(S^{n-2})\frac{\varsigma(n+2)n!}{(2\pi)^{n+1}\beta_0^{n+1}}(n+2)\\ &\times\frac{[\Delta_\theta(r^2+a^2)^2-\Delta_r a^2\sin^2\theta]^{(n+2)/2}}{(\Delta_\theta\Delta_r)^{(n+2)/2}r^n}\frac{\sin\theta}{\Xi}\mathrm{d}r\mathrm{d}\theta\\ =&j\int\mathrm{d}\varphi\int_{r_++\delta}^{r_++N\delta}(r_+\cos\theta)^{n-2}\mathrm{Vol}(S^n)\mathrm{Vol}(S^{n-2})\frac{\varsigma(n+2)n!}{(2\pi)^{n+1}\beta_0^{n+1}}(n+2)\\ &\times\frac{(r_+^2+a^2)^{n+2}}{(r-r_+)^{(n+2)/2}(\Delta_r')_{r=r_+}^{(n+2)/2}r_+^n}\frac{\sin\theta}{\Xi}\mathrm{d}r\mathrm{d}\theta+F(r_+,N,\delta)\\ =&j\frac{\varsigma(n+2)n!}{(2\pi)^{n+1}\beta_0^{n+1}}\frac{r_+^{n-2}\mathrm{Vol}(S^n)}{r_+^n(\Delta_r')_{r=r_+}^{(n+2)/2}}\frac{n+2}{\Xi}(r_+^2+a^2)^{n+2}\frac{2}{n}\\ &\times\left[\frac{N^{n/2}-1}{(N\delta)^{n/2}}\right]\int\mathrm{Vol}(S^{n-2})\cos^{n-2}\theta\,\mathrm{d}\varphi\,\sin\theta\,\mathrm{d}\theta+F(r_+,N,\delta)\\ =&j\frac{\varsigma(n+2)n!}{2^{2n+3}\pi^{(3/2)n+2}}\frac{n+2}{\beta_0^{n/2}}\frac{(r_+^2+a^2)^{(n+2)/2}}{\Xi}\frac{r_+^{n-2}}{r_+^n}\frac{2}{n}\left[\frac{N^{n/2}-1}{(N\delta)^{n/2}}\right][\mathrm{Vol}(S^n)]^2+F(r_+,N,\delta),\end{aligned}\tag{16}$$

where

$$\begin{aligned}F(r_+,N,\delta)=&j\frac{\varsigma(n+2)n!(n+2)}{(2\pi)^{n+1}\beta_0^{n+1}\Xi}\mathrm{Vol}(S^n)\\&\times\int\cos^{n-2}\theta\,\mathrm{Vol}(S^{n-2})\,\sin\theta\,\mathrm{d}\varphi\mathrm{d}\theta\\&\times\sum_{k=1}\frac{1}{K!}H^{(k)}(r_+)(r-r_+)^k\mathrm{d}r\\H(r)=&\frac{[\Delta_\theta(r^2+a^2)^2-\Delta_r a^2\sin^2\theta]^{(n+2)/2}}{(\Delta_\theta\Delta_r/(r-r_+))^{(n+2)/2}\,r^2},\\H^{(k)}(r_+)=&\left.\frac{\mathrm{d}^kH}{\mathrm{d}r^k}\right|_{r=r_+},\\\Delta_r'=&\frac{\mathrm{d}\Delta_r}{\mathrm{d}r},\\\frac{1}{\beta_0}=&T_+=\left.\frac{\Delta_r'}{4\pi(r_+^2+a^2)}\right|_{r=r_+}.\end{aligned}$$

Now we discuss the problem in terms of the membrane model. In Eq.(16) N and δ are two independent arbitrary parameters, and the entropy of the black hole that we have discussed should be the natural property of the black hole. So the entropy should have nothing to do with the parameters introduced in the calculation. To make our result irrelevant to the parameters, we let

$$\delta^{n/2}=\frac{N^{n/2}-1}{N^{n/2}}C, \tag{17}$$

where the constant C is

$$C=\frac{(n+2)(n-1)!\varsigma(n+2)\mathrm{Vol}(S^n)}{2^{2n+2}\pi^{\frac{3}{2}n+2}r_+^n}GT_+^{n/2}. \tag{18}$$

So the entropy of the black hole can be written as

$$S_{\mathrm{b}}=j\frac{r_+^{n-2}\mathrm{Vol}(S^n)}{4G\Xi}(r_+^2+a^2)+F(r_+,\delta,N). \tag{19}$$

From Eq.(17), when $N\to1$, $\delta\to0$, and $N\delta\to0$, i.e. both the upper and the lower bounds of integration approach the surface of the horizon of the black hole. The thickness of membrane turns to zero, and the membrane clings to the surface of the horizon of the black hole. That is, when $N\to1$, the limit of membrane is the surface of the horizon. Furthermore, we can prove the following result:

$$\lim_{N\to1}F(r_+,\delta,N)\to0. \tag{20}$$

So the entropy of the bosonic field corresponding to the surface of the black hole's horizon is as follows:

$$S_{\mathrm{b}}^+=j\frac{r_+^{n-2}\mathrm{Vol}(S^n)}{4G\Xi}(r_+^2+a^2). \tag{21}$$

Similarly, near the cosmological horizon, we take the integration region $[r_{\mathrm{c}}+\delta,\ r_{\mathrm{c}}+N\delta]$, and the entropy of the bosonic field corresponding to the cosmological horizon is given by

$$\begin{aligned}S_{\mathrm{b}}^{\mathrm{c}}=&j\int\mathrm{d}\varphi\int_{r_{\mathrm{c}}-N\delta}^{r_{\mathrm{c}}-\delta}(r_{\mathrm{c}}\cos\theta)^{n-2}\mathrm{Vol}(S^n)\mathrm{Vol}(S^{n-2})\\&\times\frac{\varsigma(n+2)n!}{(2\pi)^{n+1}\beta_0^{n+1}}(n+2)\\&\times\frac{(r_{\mathrm{c}}^2+a^2)^{n+2}}{(r-r_{\mathrm{c}})^{(n+2)/2}(\Delta_r')_{r=r_{\mathrm{c}}}^{(n+2)/2}r_{\mathrm{c}}^n}\\&\times\frac{\sin\theta}{\Xi}\mathrm{d}r\mathrm{d}\theta+F(r_{\mathrm{c}},N,\delta),\end{aligned}$$

that is,

$$S_{\mathrm{b}}^{\mathrm{c}}=j\frac{r_{\mathrm{c}}^{n-2}\mathrm{Vol}(S^n)}{4G\Xi}(r_{\mathrm{c}}^2+a^2). \tag{22}$$

4. The fermionic entropy

For fermionic gas, the partition function is

$$\ln Z=\sum_i g_i\ln(1+\mathrm{e}^{-\beta\varepsilon_i}). \tag{23}$$

From Eq.(9), we obtain

$$\begin{aligned}\ln Z=&\int A(r)\sqrt{g_{rr}}\mathrm{d}r\sum_i g_i\sum_{l=1}\frac{(-1)^{l-1}}{l}\mathrm{e}^{-l\beta\varepsilon_i}\\=&\omega\int A(r)\mathrm{Vol}(S^n)\sqrt{g_{rr}}\mathrm{d}r\\&\times\sum_{l=1}\frac{(-1)^{l-1}}{l}\int_0^\infty\mathrm{e}^{-(lh\nu)/T}\nu^n\mathrm{d}\nu,\end{aligned} \tag{24}$$

where ω is the spinning degeneracy of fermionic particles. Using the result in Section 3, we can obtain the fermionic entropy as follows:

$$S_{\mathrm{f}}^+=\omega\frac{1}{\varsigma(n+2)}\sum_{l=1}^\infty\frac{(-1)^n}{l^{n+2}}\frac{r_+^{n-2}\mathrm{Vol}(S^n)}{4G\Xi}(r_+^2+a^2). \tag{25}$$

The entropy of fermionic field corresponding to the cosmological horizon is given by

$$S_{\mathrm{b}}^{\mathrm{c}}=\omega\frac{1}{\varsigma(n+2)}\sum_{l=1}^\infty\frac{(-1)^n}{l^{n+2}}\frac{r_{\mathrm{c}}^{n-2}\mathrm{Vol}(S^n)}{4G\Xi}(r_{\mathrm{c}}^2+a^2). \tag{26}$$

5. Conclusion

In this paper, the partition functions of various fields in $(n+2)$-dimensional de Sitter black hole are derived directly by using the quantum statistical method. We do not use the approximation method

in the calculations, thus we have avoided the difficulty in solving wave equations. Since we used the improved brick-wall method and the membrane model to calculate the entropy of various fields, in the calculation process we let the thin membrane approach the surface of the horizon, and as a result the problem that the state density is divergent around the horizon does not exist any more. When $N \to 1$, $\delta \to 0$, $N\delta \to 0$, the outer and inner 'brick wall' both approach the outer horizon of the black hole. From Eqs.(21) and (25), we know that the left-out term and the divergent logarithmic term in the original brick-wall method no longer exist. The obtained entropy is proportional to the area of the black hole's horizon, so it can be taken as the black hole's entropy.

By the above analysis, we obtain that by using the statistical and membrane model method to calculate the entropy of the black hole, the doubt as to whether the entropy of the scalar or Dirac field outside the event horizon is the entropy of the black hole in the original brick-wall method does not exist and the complicated approximations in the solution are avoided. In the whole process, the physical idea is clear, the calculation is simple and the result is reasonable. We also take into account the influence of the spinning degeneracy of particles on the entropy. To calculate the entropy in various spacetimes, we only need to change the red-shift factor, while other factors remain the same. Especially for complicated spacetime, we can directly derive the entropy of various quantum particles without solving the complicated wave equations. Through discussion, we obtain that the black hole's entropy which contains two horizons (a black hole's horizon and a cosmological horizon) in the non-thermal equilibrium state comprises the entropy corresponding to the black hole's horizon and the entropy corresponding to the cosmological horizon. Furthermore, the entropy of the black hole is a natural property of the black hole. The entropy is irrelevant to the radiation field out of the horizon. This will deepen the understanding of the relationship between black hole's entropy and horizon's area. We think this is a reasonable way to study the bosonic and fermionic entropy of the black hole in high-dimensional non-thermal equilibrium spacetime.

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