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Spin current in double quantum dot*

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We have studied the dynamical behaviours of two electrons confined in a double quantum dot driven by rotating magnetic fields in terms of the theory of Lewis–Riesenfeld Hermitian invariants for the explicitly time-dependent Hamiltonian. The coherent spin oscillations in the dot provide a generation source for spin current. Exact solutions obtained allow us to investigate the dynamical properties of the spin localization for various initial localized states.

Keywords: spin current, quantum dot, time-dependent transformation
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1. Introduction

Spin-polarized transport through a quantum dot (QD) has been extensively investigated recently.[1−3] It has been shown theoretically [4,5] and demonstrated experimentally [6] that a QD system will function as a phase-coherent spin pump in the presence of sizable Zeeman splitting. A QD-based spin battery device has been proposed by use of a combination of a vertical magnetic field and an oscillating electric field.[7] Very recently, spin-polarized current has been detected from a quantum point contact (QPC) [8] and from Coulomb-blockaded QDs.[9] The effect of Kondo resonance on the charge current has been well studied in a QD,[10] its influence on the spin current and spin detection has also been investigated recently.[11]

In this paper, we describe the dynamical spin localization of two interacting electrons confined in the double QD driven by two rotating magnetic fields. The single-electron levels in the dot are split by an external magnetic field B_0, $(\varepsilon_{i\uparrow}-\varepsilon_{i\downarrow}=g\mu_{\rm B}B_0\,(i=1,2))$, where g is the effective electron gyromagnetic factor and $\mu_{\rm B}$ is the Bohr magneton. The two spin levels ($\varepsilon_{i\uparrow}$ and $\varepsilon_{i\downarrow}$) are coupled by a rotating magnetic field $(B_1\cos\omega t, B_1\sin\omega t)$, applied perpendicularly to the field B_0. The Hamiltonian seen reads as

$$\hat{H}\,(t)=\sum_k\left[\frac{P_k^2}{2m}+V\,(x-X_k)\right] + g\mu_{\rm B}B_1\left(\mathrm{e}^{\mathrm{i}\omega t}S^{+}+\mathrm{e}^{-\mathrm{i}\omega t}S^{-}\right)-g\mu_{\rm B}B_0S^{Z},$$

where k=1, 2 is the serial number of the QD. We use a two-site model of Hubbard-type to describe the dynamical system, which allows us to easily include the important effects of the electron correlations produced by the Coulomb interaction. With the time-evolution operator obtained by means of the generator of time-dependent $SU(2)$ spin coherent states, we are able to derive the analytic time-evolution formulae of spin current between the two QDs for various initial states.

2. Model and method

After a second quantization the model Hamiltonian under consideration can be written as

$$\begin{aligned}\hat{H}(t)=&\sum_{k,\sigma}2E_{k,\sigma}\hat{n}_{k,\sigma}\\&+\Omega\sum_k\left(\hat{a}^{+}_{k,\uparrow}\hat{a}_{k,\downarrow}\mathrm{e}^{\mathrm{i}\omega t}+\mathrm{H.C.}\right)\\&+\sum\sigma\left(\Omega_t\hat{a}^{+}_{1,\sigma}\hat{a}_{2,\sigma}+\mathrm{H.C.}\right),\end{aligned}\qquad(1)$$

where σ is the electron spin, $\uparrow$ and $\downarrow$ are used to indicated the up-spin and down-spin; $\hat{a}^{+}_{k,\sigma}(\hat{a}_{k,\sigma})$ is the creation (annihilation) operator for an electron of spin σ

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on dot k; the time-dependent terms describe the coupling between QD spin states $|k,\uparrow\rangle$ and $|k,\downarrow\rangle$ by the rotating magnetic fields, and

$$\Omega = g\mu_{\mathrm{B}}B_1;$$
$$\Omega_t = \int u_1^* \sum_k \left(\frac{P_k^2}{2m} + V\left(x - X_k\right)\right) u_2 \mathrm{d}x$$

is the tunnelling coefficient between the QDs 1 and 2; $\hat{n}_{k,\sigma} = \hat{a}_{k,\sigma}^+ \hat{a}_{k,\sigma}$ is the total occupation of dot k. In our case $\sum_{k,\sigma} \hat{n}_{k,\sigma}$=2.

Assuming $E_{1,\sigma} = E_{2,\sigma}$, $E_{k,\uparrow} = -E_{k,\downarrow}$, we can rewrite the Hamilton operator in terms of the generators of Lie algebra of $SU(2)$ as (apart from a trivial constant)

$$\hat{H}(t) = \sum_{l=1}^{4} \left[\varepsilon_l K_0^l + \Omega_l \left(\hat{K}_+^l \mathrm{e}^{\mathrm{i}\omega_l t} + \hat{K}_-^l \mathrm{e}^{-\mathrm{i}\omega_l t}\right)\right], \quad (2)$$

where

$$\varepsilon_1 = E_{1,\uparrow} - E_{1,\downarrow} = \varepsilon_2 = E_{2,\uparrow} - E_{2,\downarrow} = \varepsilon,$$
$$\varepsilon_3 = E_{1,\uparrow} - E_{2,\uparrow} = 0,$$
$$\varepsilon_4 = E_{2,\downarrow} - E_{1,\downarrow} = 0;$$
$$\Omega_1 = \Omega_2 = \Omega,$$
$$\Omega_3 = \Omega_4 = \Omega_t;$$
$$\omega_1 = \omega_2 = \omega,$$
$$\omega_3 = \omega_4 = 0;$$

and

$$\hat{K}_0^1 = \frac{1}{2}\left(\hat{n}_{1,\uparrow} - \hat{n}_{1,\downarrow}\right),$$
$$\hat{K}_+^1 = \hat{a}_{1,\uparrow}^+ \hat{a}_{1,\downarrow}, \quad \hat{K}_-^1 = \hat{a}_{1,\downarrow}^+ \hat{a}_{1,\uparrow},$$
$$\hat{K}_0^2 = \frac{1}{2}\left(\hat{n}_{2,\uparrow} - \hat{n}_{2,\downarrow}\right),$$
$$\hat{K}_+^2 = \hat{a}_{2,\uparrow}^+ \hat{a}_{2,\downarrow}, \quad \hat{K}_-^2 = \hat{a}_{2,\downarrow}^+ \hat{a}_{2,\uparrow},$$
$$\hat{K}_0^3 = \frac{1}{2}\left(\hat{n}_{1,\uparrow} - \hat{n}_{2,\uparrow}\right),$$
$$\hat{K}_+^3 = \hat{a}_{1,\uparrow}^+ \hat{a}_{2,\uparrow}, \quad \hat{K}_-^3 = \hat{a}_{2,\uparrow}^+ \hat{a}_{1,\uparrow},$$
$$\hat{K}_0^4 = \frac{1}{2}\left(\hat{n}_{2,\downarrow} - \hat{n}_{1,\downarrow}\right),$$
$$\hat{K}_+^4 = \hat{a}_{2,\downarrow}^+ \hat{a}_{1,\downarrow}, \quad \hat{K}_-^4 = \hat{a}_{1,\downarrow}^+ \hat{a}_{2,\uparrow}, \quad (3)$$

where $\hat{K}_0^l$ is a Hermitian operator, while $\hat{K}_+^l = \left(\hat{K}_-^l\right)^\dagger$ satisfies the $SU(2)$ commutation relations

$$\left[\hat{K}_0^l, K_\pm^l\right] = \pm\hat{K}_\pm^l,$$
$$\left[\hat{K}_+^l, \hat{K}_-^l\right] = 2\hat{K}_0^l$$

for l=1,2,3,4.

We start with a generalized gauge transformation [12] in terms of the time-dependent unitary transformation [13] defined by

$$\hat{\mathcal{R}}(t) = \prod_{l=1}^{4} \hat{R}_l(t)$$
$$= \exp\left[\sum_{l=1}^{4} \frac{\lambda_l}{2}\left(\hat{K}_+^l \mathrm{e}^{-\mathrm{i}\varphi_l(t)} - \hat{K}_-^l \mathrm{e}^{\mathrm{i}\varphi_l(t)}\right)\right]. \quad (4)$$

The time-dependent Schrödinger equation is covariant under the gauge transformation [12] such that

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\left|\psi'(t)\right\rangle = \hat{H}'\left|\psi'(t)\right\rangle,$$

where

$$\hat{H}' = \hat{\mathcal{R}}^\dagger \hat{H} \hat{\mathcal{R}} - \mathrm{i}\hat{\mathcal{R}}^\dagger \frac{\partial}{\partial t}\hat{\mathcal{R}},$$
$$\left|\psi(t)\right\rangle = \hat{\mathcal{R}}\left|\psi'(t)\right\rangle;$$

and the state $|\psi(t)\rangle$ is assumed to be the solution of the original Schrödinger equation

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\left|\psi(t)\right\rangle = \hat{H}\left|\psi(t)\right\rangle.$$

The auxiliary parameter λ_l, which is time-dependent in general, is to be determined by requiring that the Hamilton operator $\hat{H}'$ be diagonal in the K_0^l representation. Using the following relationship:[13]

$$\hat{R}_l^\dagger(t)\,\hat{K}_0^l\,\hat{R}_l(t) = \hat{K}_0^l \cos\lambda_l + \frac{1}{2}\left(\hat{K}_+^l \mathrm{e}^{-\mathrm{i}\varphi_l(t)} + \hat{K}_-^l \mathrm{e}^{\mathrm{i}\varphi_l(t)}\right)\sin\lambda_l, \quad (5)$$

$$\hat{R}_l^\dagger(t)\,\hat{K}_+^l\,\hat{R}_l(t) = \hat{K}_+^l \cos^2\frac{\lambda_l}{2} - \hat{K}_-^l \mathrm{e}^{\mathrm{i}2\varphi_l(t)} \sin^2\frac{\lambda_l}{2} - \hat{K}_0^l \mathrm{e}^{\mathrm{i}\varphi_l(t)} \sin\lambda_l, \quad (6)$$

$$\hat{R}_l^\dagger(t)\,\hat{K}_-^l\,\hat{R}_l(t) = \hat{K}_-^l \cos^2\frac{\lambda_l}{2} - \hat{K}_+^l \mathrm{e}^{-\mathrm{i}2\varphi_l(t)} \sin^2\frac{\lambda_l}{2} - \hat{K}_0^l \mathrm{e}^{-\mathrm{i}\varphi_l(t)} \sin\lambda_l, \quad (7)$$

$$\mathrm{i}\hat{R}_l^\dagger(t)\frac{\partial}{\partial t}\hat{R}_l(t) = -2\hat{K}_0^l \frac{\mathrm{d}\varphi_l}{\mathrm{d}t}\sin^2\frac{\lambda_l}{2} + \frac{1}{2}\hat{K}_+^l \mathrm{e}^{-\mathrm{i}\varphi_l(t)}\left(\mathrm{i}\frac{\mathrm{d}\lambda_l}{\mathrm{d}t} + \frac{\mathrm{d}\varphi_l}{\mathrm{d}t}\sin\lambda_l\right)$$
$$+ \frac{1}{2}\hat{K}_-^l \mathrm{e}^{\mathrm{i}\varphi_l(t)}\left(-\mathrm{i}\frac{\mathrm{d}\lambda_l}{\mathrm{d}t} + \frac{\mathrm{d}\varphi_l}{\mathrm{d}t}\sin\lambda_l\right), \quad (8)$$

and noticing the adiabatic condition that $\frac{\mathrm{d}\lambda_l}{\mathrm{d}t} \simeq 0$ and $\frac{\mathrm{d}\varphi_l}{\mathrm{d}t} \simeq 0$, we obtain the Hamilton operator

$$\hat{H}' = \sum_{l=1}^{4} \sqrt{\varepsilon_l^2 + 4\Omega_l^2}\hat{K}_0^l, \tag{9}$$

with auxiliary parameter λ_l chosen as

$$\sin\lambda_l = -\frac{2\Omega_l}{\sqrt{\varepsilon_l^2 + 4\Omega_l^2}},$$
$$\cos\lambda_l = \frac{\varepsilon_l}{\sqrt{\varepsilon_l^2 + 4\Omega_l^2}},$$
$$\varphi_{1,2} = -\omega t, \qquad \varphi_{3,4} = 0.$$

It is easy to obtain the exact general solution of the original Schrödinger equation

$$\begin{aligned}|\psi(t)\rangle =& \prod_{l=1}^{4}\sum_{n_l} c_{n_l}\exp[-\mathrm{i}\alpha_{n_l}(t)]\,|n_l, t\rangle \\ =& \prod_{l=1}^{4}\sum_{n_l} c_{n_l}\exp[-\mathrm{i}\alpha_{n_l}(t)]\hat{R}_l(t)\,|n_l\rangle\,, \end{aligned}\tag{10}$$

with $|n_l\rangle$ being the usual eigenstate of $\hat{K}_0^l$ which satisfies $\hat{K}_0^l\,|n_l\rangle = K_n^l\,|n_l\rangle$, where

$$\begin{aligned}\alpha_{n_l}(t) =& \int_0^t \mathrm{d}t'\,\langle n_l, t'|\,\mathrm{i}\frac{\partial}{\partial t'} - \hat{H}(t')\,|n_l, t'\rangle \\ =& \int_0^t \mathrm{d}t'\left(\sqrt{\varepsilon_l^2 + 4\Omega_l^2}\right)K_n^l. \end{aligned}\tag{11}$$

To study the dynamics of the spin, it is useful to derive the explicit time-evolution operator such that

$$|\psi(t)\rangle = \hat{U}(t)\,|\psi(0)\rangle\,,$$

where the time-evolution operator is found from the exact solution of the time-dependent Schrödinger equation $|\psi(t)\rangle$ in Eq.(10) as

$$\begin{aligned}\hat{U}(t) =& \hat{\mathcal{R}}(t)\exp\left[-\mathrm{i}\sum_l \chi_l(t)\,\hat{K}_0^l\right]\hat{\mathcal{R}}^\dagger(0) \\ =& \prod_l \hat{R}_l(t)\,\mathrm{e}^{-\mathrm{i}\chi_l(t)\hat{K}_0^l}\hat{R}_l^\dagger(0)\,, \end{aligned}\tag{12}$$

where $\chi_l(t) = \sqrt{\varepsilon_l^2 + 4\Omega_l^2}t$. For a given initial state $|\psi(0)\rangle$ of the system the spin-up current can be calculated from the time evolution of the occupation number

$$N_\uparrow(t) = \langle\psi(0)|\,\hat{N}_\uparrow(t)\,|\psi(0)\rangle\,,$$

where the operator $\hat{N}_\uparrow = \hat{n}_{1,\uparrow} - \hat{n}_{2,\downarrow} = \hat{K}_0^1 + \hat{K}_0^2 + \hat{K}_0^3 - \hat{K}_0^4$ in the Heisenberg picture which is obtained with the help of Eqs.(5)–(7) as

$$\hat{N}_\uparrow(t) = \hat{U}^\dagger(t)\,\hat{N}_\uparrow\hat{U}(t)\,. \tag{13}$$

3. Conclusion

There are six kinds of two-electron distribution in the double QD, which are shown in Table 1. The time-evolution of population of $N_\uparrow(t)$ is evaluated for various initial states as follows. For the sake of simplicity, we set the initial phase to zero, i.e. $\varphi_l(0) = 0$.

Table 1. The two-electron distribution in double quantum dots.

	States of the system					
	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$	$\|4\rangle$	$\|5\rangle$	$\|6\rangle$
dot 1	$\uparrow\downarrow$		$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
dot 2		$\uparrow\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$

We consider, first of all, the initial state $|\psi(0)\rangle_1 = |1\rangle$: we find from Eq.(13) that

$$\begin{aligned}N_\uparrow^1(t) =& \langle 1|\prod_l \hat{R}_l(0)\mathrm{e}^{\mathrm{i}\chi_l(t)\hat{K}_0^l}\hat{R}_l^\dagger(t)\hat{N}_\uparrow\prod_l \hat{R}_l(t)\mathrm{e}^{-\mathrm{i}\chi_l(t)\hat{K}_0^l}\hat{R}_l^\dagger(0)\,|1\rangle \\ =& \langle 1|\prod_l \hat{R}_l(0)\mathrm{e}^{\mathrm{i}\chi_l(t)\hat{K}_0^l}\hat{R}_l^\dagger(t)(\hat{K}_0^1 + \hat{K}_0^2 + \hat{K}_0^3 - \hat{K}_0^4)\prod_l \hat{R}_l(t)\mathrm{e}^{-\mathrm{i}\chi_l(t)\hat{K}_0^l}\hat{R}_l^\dagger(0)\,|1\rangle \\ =& \frac{1}{2}\sum_{3,4}\Big[\cos\lambda_l(t)\cos\lambda_l(0) - \frac{1}{2}\mathrm{e}^{\mathrm{i}[\chi_l(t)-\varphi_l(t)]}\sin\lambda_l(t)\sin\lambda_l(0) - \frac{1}{2}\mathrm{e}^{-\mathrm{i}[\chi_l(t)-\varphi_l(t)]}\sin\lambda_l(t)\sin\lambda_l(0)\Big] \\ =& \frac{1}{2}\sum_{3,4}\Big\{\cos^2\lambda_l(t) + \sin^2\lambda_l(t)\cos\Big[\chi_l(t) - \varphi_l(t)\Big]\Big\} \\ =& \frac{1}{2}\sum_{3,4}\left\{\frac{\varepsilon_l^2}{\varepsilon_l^2 + 4\Omega_l^2} + \frac{4\Omega_l^2}{\varepsilon_l^2 + 4\Omega_l^2}\cos\left[\left(\omega - \sqrt{\varepsilon_l^2 + 4\Omega_l^2}\right)t\right]\right\} = \cos(2\Omega_t t); \end{aligned}$$

for other initial states we have

$$
\begin{aligned}
N_{\uparrow}^{2}(t) =& \langle 2| \prod_{l} \hat{R}_{l}(0) \mathrm{e}^{\mathrm{i}\chi_{l}(t)\hat{K}_{0}^{l}} \hat{R}_{l}^{\dagger}(t) \hat{N}_{\uparrow}(t) \prod_{l} \hat{R}_{l}(t) \mathrm{e}^{-\mathrm{i}\chi_{l}(t)\hat{K}_{0}^{l}} \hat{R}_{l}^{\dagger}(0) |2\rangle ,\\
=& -\cos(2\Omega_{t}t),\\
N_{\uparrow}^{3}(t) =& \langle 3| \prod_{l} \hat{R}_{l}(0) \mathrm{e}^{\mathrm{i}\chi_{l}(t)\hat{K}_{0}^{l}} \hat{R}_{l}^{\dagger}(t) \hat{N}_{\uparrow}(t) \prod_{l} \hat{R}_{l}(t) \mathrm{e}^{-\mathrm{i}\chi_{l}(t)\hat{K}_{0}^{l}} \hat{R}_{l}^{\dagger}(0) |3\rangle ,\\
=& \frac{1}{2} \sum_{1,2} \left\{ \frac{\varepsilon_{l}^{2}}{\varepsilon_{l}^{2}+4\Omega^{2}} + \frac{4\Omega^{2}}{\varepsilon_{l}^{2}+4\Omega^{2}} \cos\left[\left(\omega - \sqrt{\varepsilon_{l}^{2}+4\Omega^{2}} \right) t \right] \right\},\\
N_{\uparrow}^{4}(t) =& \langle 4| \prod_{l} \hat{R}_{l}(0) \mathrm{e}^{\mathrm{i}\chi_{l}(t)\hat{K}_{0}^{l}} \hat{R}_{l}^{\dagger}(t) \hat{N}_{\uparrow} \prod_{l} \hat{R}_{l}(t) \mathrm{e}^{-\mathrm{i}\chi_{l}(t)\hat{K}_{0}^{l}} \hat{R}_{l}^{\dagger}(0) |4\rangle ,\\
=& -\frac{1}{2} \sum_{1,2} \left\{ \frac{\varepsilon_{l}^{2}}{\varepsilon_{l}^{2}+4\Omega^{2}} + \frac{4\Omega^{2}}{\varepsilon_{l}^{2}+4\Omega^{2}} \cos\left[\left(\omega - \sqrt{\varepsilon_{l}^{2}+4\Omega^{2}} \right) t \right] \right\},\\
N_{\uparrow}^{5}(t) =& \langle 5| \prod_{l} \hat{R}_{l}(0) \mathrm{e}^{\mathrm{i}\chi_{l}(t)\hat{K}_{0}^{l}} \hat{R}_{l}^{\dagger}(t) \hat{N}_{\uparrow} \prod_{l} \hat{R}_{l}(t) \mathrm{e}^{-\mathrm{i}\chi_{l}(t)\hat{K}_{0}^{l}} \hat{R}_{l}^{\dagger}(0) |5\rangle\\
=& -\frac{1}{2} \sum_{1,3} \left\{ \frac{\varepsilon_{l}^{2}}{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} + \frac{4\Omega_{l}^{2}}{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} \cos\left[\left(\omega - \sqrt{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} \right) t \right] \right\}\\
&+ \frac{1}{2} \sum_{2,4} \left\{ \frac{\varepsilon_{l}^{2}}{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} + \frac{4\Omega_{l}^{2}}{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} \cos\left[\left(\omega - \sqrt{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} \right) t \right] \right\}\\
=& 0,\\
N_{\uparrow}^{6}(t) =& \langle 6| \prod_{l} \hat{R}_{l}(0) \mathrm{e}^{\mathrm{i}\chi_{l}(t)\hat{K}_{0}^{l}} \hat{R}_{l}^{\dagger}(t) \hat{N}_{\uparrow} \prod_{l} \hat{R}_{l}(t) \mathrm{e}^{-\mathrm{i}\chi_{l}(t)\hat{K}_{0}^{l}} \hat{R}_{l}^{\dagger}(0) |6\rangle\\
=& \frac{1}{2} \sum_{1,3} \left\{ \frac{\varepsilon_{l}^{2}}{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} + \frac{4\Omega_{l}^{2}}{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} \cos\left[\left(\omega - \sqrt{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} \right) t \right] \right\}\\
&- \frac{1}{2} \sum_{2,4} \left\{ \frac{\varepsilon_{l}^{2}}{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} + \frac{4\Omega_{l}^{2}}{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} \cos\left[\left(\omega - \sqrt{\varepsilon_{l}^{2}+4\Omega_{l}^{2}} \right) t \right] \right\}\\
=& 0.
\end{aligned}
$$

From the above results, we can see that when the initial state is prepared at $|1\rangle$ or $|2\rangle$, i.e. both electrons are put in a dot initially; the spin current varies according to the cosine function, with the frequency equal to the tunnelling coefficient Ω_t. When the electrons are prepared in two dots with the same spin in initial state $|3\rangle$ or $|4\rangle$, it is found that one can obtain a steady spin current $N_{\uparrow}^{3,4}(t) = \pm 1$, which means that the initial state is preserved all the time only if $\omega_l = \sqrt{\varepsilon_l^2 + 4\Omega_l^2}$ (l=1, 2) is satisfied. Otherwise, when $|\psi(0)\rangle = |5\rangle$ or $|6\rangle$, the electrons in different dots with different spins, we have $N_{\uparrow}^{5,6}(t) = 0$ and the initial state is kept at any time for any condition.

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