Vol 13 No 6, June 2004
1009-1963/2004/13(06)/0877-05

Chinese Physics

© 2004 Chin. Phys. Soc.
and IOP Publishing Ltd

Evolution of sum-chirp in polarization multiplexed communication system

Wang Jing(王 晶)† and Wang Zhen-Li(王珍丽)

Department of Physics, Ocean University of China, Qingdao 266071, *China*

(Received 21 July 2003; revised manuscript received 26 January 2004)

The evolution of sum-chirp for an initially chirped Gaussian pulse is studied in the polarization multiplexed communication system, with fibre attenuation considered. The sum-chirp is found to have the character of saturation. Its value appears different along the two different polarization axes, determined by the incidence polarization angle. We also find that sum-chirp is dominated by the initial chirp at a short distance, and by the cross-phase modulation effect at long distance. And it is influenced apparently by a wavevector mismatch parameter below 10 ps/km. Further, its saturation results from the effective distance determined by fibre attenuation.

Keywords: sum-chirp, initial chirp, incidence polarization angle, fibre attenuation, wavevector mismatch

PACC: 4265, 4280

1. Introduction

Following the 10 Gb/s fibre communication system that dominated the communication network, a new system of 40 Gb/s communication speed has attracted much attention. It shows an inevitable trend for fibre communication systems: increase of capacity and rate. However, new problems related to the polarization mode dispersion (PMD) occur simultaneously.

Obviously, PMD will do harm to both analogue and numerical communication systems. So far, several approaches have been put forward to eliminate or weaken the negative effect of PMD.[1−5] Since PMD results from the birefringence in the fibre, we wonder whether it can be well offset by the cross-phase modulation (XPM) effect in the polarization multiplexed system.[6,7]

As is well known, the soliton's ability of distortion-free propagation results from the interaction of group velocity dispersion (GVD) and self-phase modulation (SPM), when zero sum-chirp is realized by the resistance between GVD-caused chirp and the SPM-caused chirp.[8] Similarly, PMD may be resisted by XPM, which results from the resistance between the relative chirps. Hence, it is possible for distortion-free communication to come to be truly realized in a new high-speed system. In this light, the sum-chirp is studied.

2. Basic theory

When propagating in the strong birefringence fibre, the incidence polarization pulse is found to satisfy the equations [8]

$$\begin{aligned}&\frac{\partial A_x}{\partial z}+\beta_{1x}\frac{\partial A_x}{\partial t}+\frac{\mathrm{i}}{2}\beta_{2x}\frac{\partial^2 A_x}{\partial t^2}+\frac{\alpha}{2}A_x\\&=\mathrm{i}\gamma[|A_x|^2+\frac{2}{3}|A_y|^2]A_x,\\&\frac{\partial A_y}{\partial z}+\beta_{1y}\frac{\partial A_y}{\partial t}+\frac{\mathrm{i}}{2}\beta_{2y}\frac{\partial^2 A_y}{\partial t^2}+\frac{\alpha}{2}A_y\\&=\mathrm{i}\gamma[|A_y|^2+\frac{2}{3}|A_x|^2]A_y,\end{aligned}\tag{1}$$

where A_x and A_y are the amplitudes of the pulse along the x and y axes respectively, β_{1x} and β_{1y} are the reciprocals of the group rates, β_{2x} and β_{2y} are the GVD parameters, α is the fibre attenuation, γ is the nonlinear parameter, z is the propagation distance, and t is time.

2.1. SPM and XPM-caused chirps

To consider the nonlinear effects only, we choose β_{2x}=β_{2y}=0. And by making the transformation $T=$

†E-mail: wjing@mail.ouc.edu.cn

$t - z\beta_{1x}$, we obtain

$$\frac{\partial A_x}{\partial z} = \mathrm{i}\gamma\left[|A_x|^2 + \frac{2}{3}|A_y|^2\right]A_x - \frac{\alpha}{2}A_x,$$
$$\frac{\partial A_y}{\partial z} + d\frac{\partial A_y}{\partial T} = \mathrm{i}\gamma\left[|A_y|^2 + \frac{2}{3}|A_x|\right]A_y - \frac{\alpha}{2}A_y, \quad (2)$$

where $d = \beta_{1y}$–β_{1x} is the wavevector mismatch.

One can introduce a normalized amplitude U by using the definition

$$A_x(z,T) = \sqrt{P_x}\exp\left(-\frac{\alpha z}{2}\right)U_x(z,T),$$
$$A_y(z,T) = \sqrt{P_y}\exp\left(-\frac{\alpha z}{2}\right)U_y(z,T), \quad (3)$$

where $P_x = P_0\cos^2\theta$, $P_y = P_0\sin^2\theta$, P_0 and θ stand for the incidence peak power and polarization angle respectively. Then Eqs.(2) take the form

$$\mathrm{i}\frac{\partial U_x}{\partial z} = -\gamma\left[P_x\exp(-\alpha z)|U_x(z,T)|^2\right]U_x(z,T) + 2\gamma/3\left[P_y\exp(-\alpha z)|U_y(z,T)|^2\right]U_x(z,T),$$
$$\mathrm{i}\frac{\partial U_y}{\partial z} = -\gamma\left[P_y\exp(-\alpha z)|U_y(z,T)|^2\right]U_y(z,T) + 2\gamma/3\left[P_x\exp(-\alpha z)|U_x(z,T)|^2\right]U_y(z,T), \quad (4)$$

whose general solution is given by

$$U_x(z,T) = U_x(0,T)\exp(\mathrm{i}\phi_x),$$
$$U_y(z,T) = U_y(0,T-zd)\exp(\mathrm{i}\phi_y), \quad (5)$$

where

$$\phi_x = \gamma P_x\frac{1-\exp(-\alpha z)}{\alpha}|U_x(0,T)|^2 + \frac{2}{3}\gamma P_y\int_0^z\exp(-\alpha z)|U_y(0,T-zd)|^2\,\mathrm{d}z,$$
$$\phi_y = \gamma P_y\frac{1-\exp(-\alpha z)}{\alpha}|U_y(0,T)|^2 + \frac{2}{3}\gamma P_x\int_0^z\exp(-\alpha z)|U_x(0,T+zd)|^2\,\mathrm{d}z. \quad (6)$$

Equation (6) shows that the nonlinear phase is modulated due to SPM and XPM, and the initial normalized amplitude also contributes to the modulation. Here, the initial normalized amplitudes are given by

$$U_x(0,T) = U_y(0,T) = \exp\left(-\frac{1+\mathrm{i}C}{2}\frac{T^2}{T_0^2}\right), \quad (7)$$

where T_0 is the pulse width, and C is the initial chirp parameter. By defining

$$\tau = \frac{T}{T_0},$$
$$L_{\mathrm{NL}} = \frac{1}{\gamma P_0},$$
$$L_{\mathrm{eff}} = \frac{1-\exp(-\alpha z)}{\alpha},$$
$$\varepsilon = \frac{\alpha T_0}{2d} = \frac{\alpha}{2}L_{\mathrm{w}},$$
$$\delta = \frac{zd}{T_0} = \frac{z}{L_{\mathrm{w}}},$$

we have

$$f(x) = \mathrm{erf}(\tau + x) - \mathrm{erf}(\tau + x - \delta),$$
$$\kappa(\alpha) = \exp\left(\frac{\alpha^2T_0^2 + 4\alpha Td}{4d^2}\right),$$

where the function erf stands for error function. Then Eq.(6) changes into the form

$$\phi_x = \left[\frac{L_{\mathrm{eff}}}{L_{\mathrm{NL}}}\exp\left(-\tau^2\right)\right]\cos^2\theta + \left[\frac{\sqrt{\pi}T_0}{3dL_{\mathrm{NL}}}\kappa(-\alpha)f(-\varepsilon)\right]\sin^2\theta,$$
$$\phi_y = \left[\frac{L_{\mathrm{eff}}}{L_{\mathrm{NL}}}\exp\left(-\tau^2\right)\right]\sin^2\theta + \left[\frac{\sqrt{\pi}T_0}{3dL_{\mathrm{NL}}}\kappa(\alpha)f(\varepsilon+\delta)\right]\cos^2\theta. \quad (8)$$

If we define

$$g(x) = \exp(-(\tau+x)^2) - \exp(-(\tau+x-\delta)^2),$$
$$\alpha_1 = -\alpha, \qquad \varepsilon_1 = -\varepsilon, \qquad \varepsilon_2 = \varepsilon + \delta,$$

then the sum-chirps are given by

$$\Delta\omega_x = \frac{L_{\mathrm{eff}}}{L_{\mathrm{NL}}}\left[\frac{2\tau}{T_0}\exp\left(-\tau^2\right)\right]\cos^2\theta + \frac{\kappa(\alpha_1)}{3L_{\mathrm{NL}}}\left[\frac{\alpha\sqrt{\pi}T_0}{d^2}f(\varepsilon_1) - \frac{2}{d}g(\varepsilon_1)\right]\sin^2\theta,$$
$$\Delta\omega_y = \frac{L_{\mathrm{eff}}}{L_{\mathrm{NL}}}\left[\frac{2\tau}{T_0}\exp\left(-\tau^2\right)\right]\sin^2\theta + \frac{\kappa(\alpha)}{3L_{\mathrm{NL}}}\left[\frac{\alpha\sqrt{\pi}T_0}{d^2}f(\varepsilon_2) - \frac{2}{d}g(\varepsilon_2)\right]\cos^2\theta. \quad (9)$$

2.2. GVD-caused chirp

As a matter of fact, Eq.(9) only shows the sum of SPM-caused chirp and XPM-caused chirp. When GVD is considered only and the negative β_2 is chosen, then Eqs.(1) change into the form

$$\frac{\partial A_x}{\partial z} + \frac{\mathrm{i}}{2}\beta_{2x}\frac{\partial^2 A_x}{\partial T^2} = 0,$$
$$\frac{\partial A_y}{\partial z} + \frac{\mathrm{i}}{2}\beta_{2y}\frac{\partial^2 A_y}{\partial T^2} = 0. \quad (10)$$

As a result, the GVD-caused chirp is given as [8]

$$\Delta\omega_{\mathrm{D}} = \frac{C-(1+C^2)z/L_{\mathrm{D}}}{(1-Cz/L_{\mathrm{D}})^2+(z/L_{\mathrm{D}})^2}\frac{\tau}{T_0}. \qquad (11)$$

When $\beta_2 < 0$, C=0, the GVD-caused chirp will resist the SPM-caused chirp, as long as $L_{\mathrm{NL}} = L_{\mathrm{D}}$, while the fibre attenuation will diminish the pulse amplitude along the fibre length, which in turn will affect the SPM-caused chirp. Hence, with the initial chirp considered, the sum-chirp under this circumstance will surely take on new characters.

3. Analysis and discussion

According to Eqs.(9) and (11), the sum of SPM-caused and XPM-caused chirps will increase proportionally to the nonlinear effect parameter γ and the incidence pulse peak power P_0, while the incidence polarization angle θ, initial chirp C, fibre attenuation α and wavevector mismatch parameter d affect the sum-chirp in a complicated manner.

We use the method of numerical simulation in MATLAB to demonstrate the evolution of chirp for an incidence pulse with a width of 10 ps inside a fibre with the specification as $L_{\mathrm{NL}} = L_{\mathrm{D}}$=5 km.

3.1. Influence of incidence polarization angle on the chirp derived from SPM and XPM

By numerical simulation, the incidence polarization angle θ is found to make the chirp have different values along the x and y axes, with the exception of $\theta = \pi/4$. The evolutions of the chirp along the x-axis are shown in Fig.1. We observe that the value of the chirp decreases as the angle increases. The reason for this is that the incidence polarization angle determines the distribution of the pulse power along the two polarization axes. When $\theta = \pi/4$, the power is distributed equally. But there is a common trend for the chirps to be saturated at corresponding levels as the distance increases beyond 20 km.

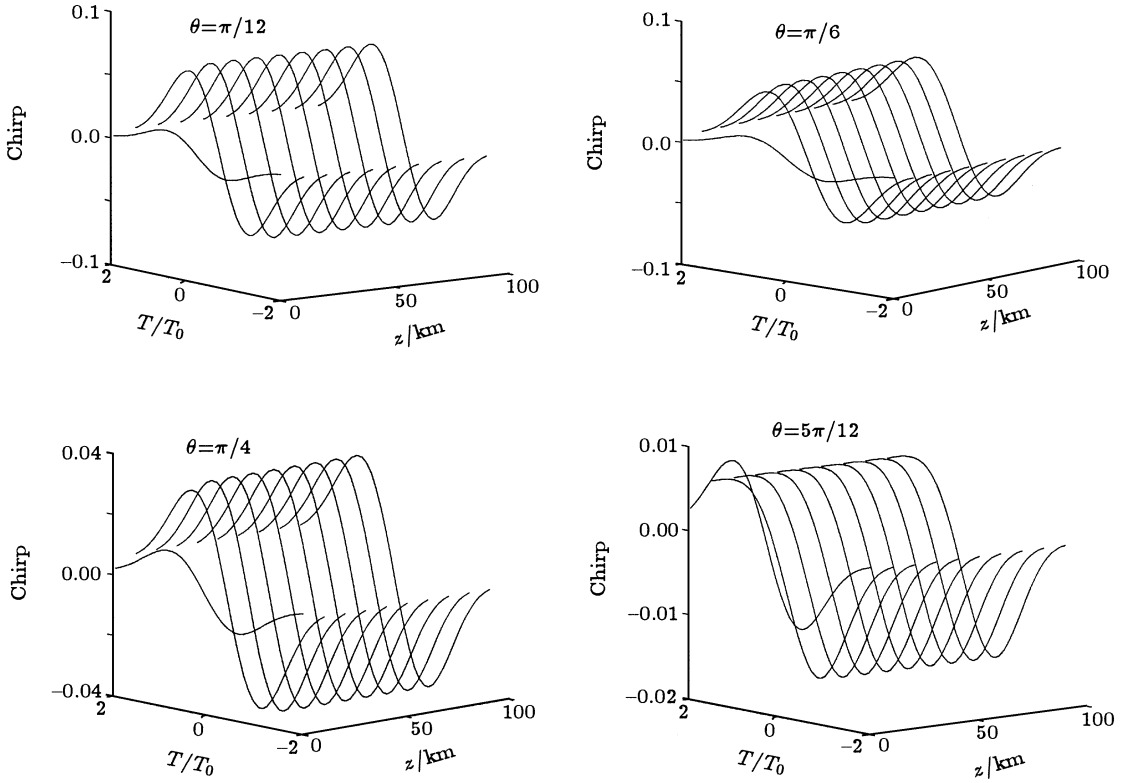


Fig.1. Evolution of the chirp for a pulse with certain incidence polarization angles inside a fibre of d=7 ps/km, α=0.25 dB/km.

3.2. Influence of initial chirp on the sum-chirp

According to Fig.2, when the distance z is below 20 km, we find that whether the sum-chirp is positive or negative depends on the sign of the initial chirp parameter. The instantaneous sum-chirp increases from the leading to the trailing edge (up-chirp) for $C > 0$, while the opposite occurs (down-chirp) for $C < 0$, and the sum-chirp will look like a sine profile for C=0. When the distance is longer than 20 km, GVD will be offset by SPM, while XPM will be dominant; as a result, the sum-chirp is mainly determined by the degree of mode coupling effect.

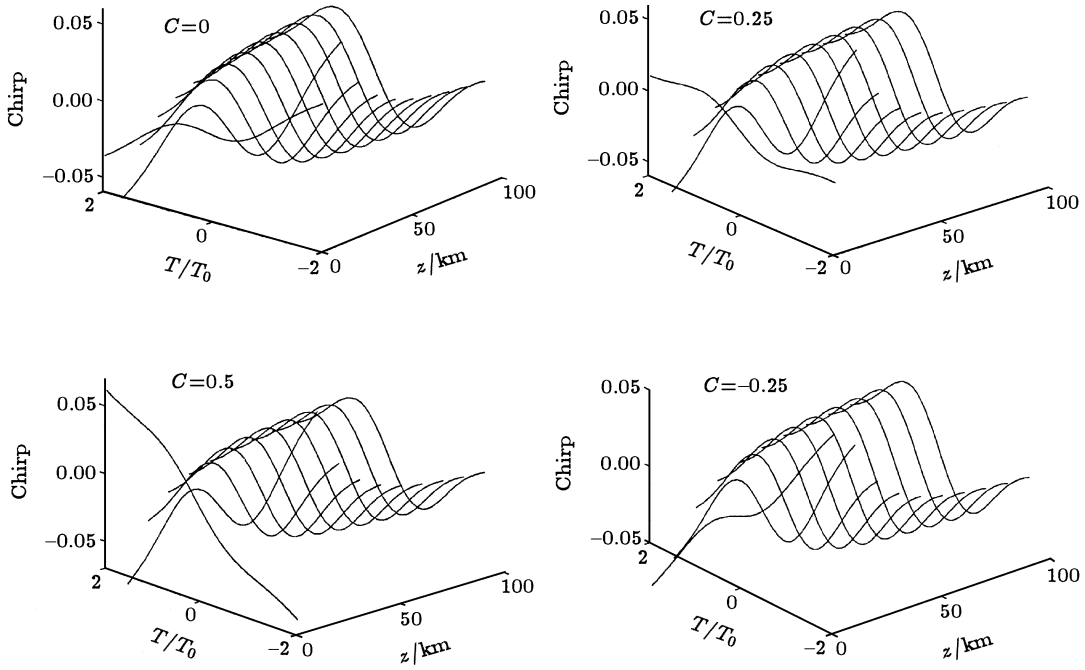


Fig.2. Evolutions of sum-chirp for a pulse with certain initial chirp parameters inside a fibre of d=7 ps/km, α=0.25 dB/km at $\theta = \pi/4$.

3.3. Influence of fibre attenuation on the chirp derived from SPM and XPM

We find that the fibre attenuation mainly affects the saturation and value of the chirp. How it works is demonstrated in Fig.3. It shows that the chirp increases continuously over the distance of 100 km, when α=0. However, when α=0.25 dB/km, the chirp becomes saturated as the distance reaches 20 km. And at z=100 km, the value of the sum-chirp for α=0.25 dB/km is only 5% that of the sum-chirp for α=0.

The variation of fibre attenuation and sum-chirp

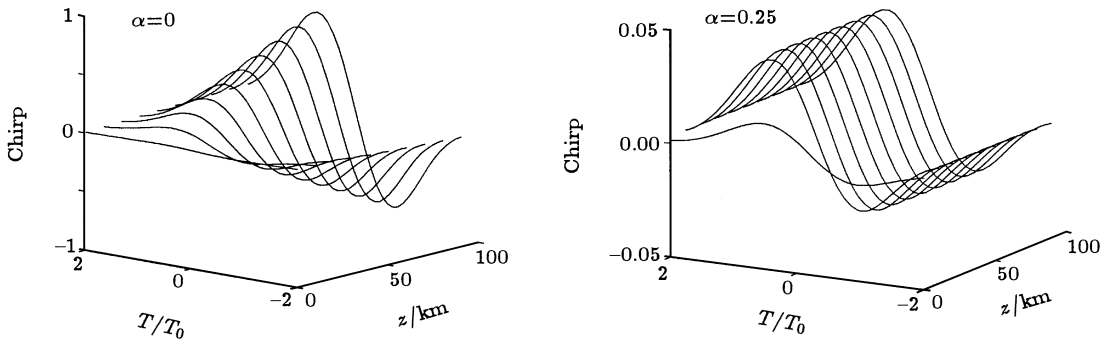


Fig.3. Evolution of the chirp for a pulse inside a fibre of d=7 ps/km, with certain attenuations, at $\theta = \pi/4$.

is shown in Fig.4. In the figure, at distance z=50 km, for T=0.25T_0, the sum-chirp will decrease quickly as α increases gradually when α lies in the range of 0–3 dB/km; and when α is in the range of 3–5 dB/km, the sum-chirp becomes constant.

3.4. Influence of wavevector mismatch on the chirp derived from SPM and XPM

The wavevector mismatch parameter d affects the degree of mode coupling, so it also contributes to the value of the chirp. At a distance of z=50 km, for T=0.25T_0, α=0.25 dB/km, when the wavevector mismatch parameter d <3 ps/km, the absolute value of the chirp increases as d increases until a peak value is approached, as shown in Fig.5. Then, when d > 3 ps/km the absolute value of the chirp decreases slowly and approaches a constant.

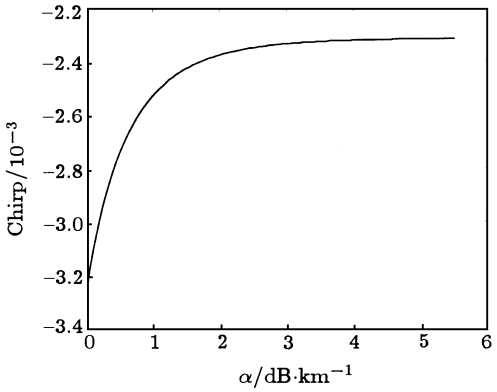


Fig.4. Variation of fibre attenuation and the chirp. The parameter values are identical to those used in Fig.3.

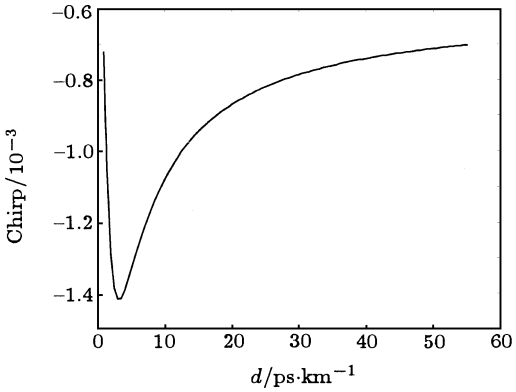


Fig.5. Variation of wavevector mismatch parameter and the sum-chirp. The parameter values are identical to those used in Fig.3.

4. Conclusions

1) The sum-chirp has the character of saturation, which is caused by the fibre attenuation.

2) The values of the sum-chirp are slightly different between the two polarization axes caused by the incidence polarization angle θ.

3) Whether the value of sum-chirp is positive or negative depends on the sign of the initial chirp parameter C during the early stage of the pulse propagation.

4) Both the saturation and the value of the sum-chirp are affected by fibre attenuation α.

5) The value of the chirp derived from SPM and XPM is related to the wavevector mismatch parameter d, but only when $0 < d < 30\,\mathrm{ps/km}$ does the wavevector mismatch effect influence the chirp evidently.

6) The study of sum-chirp confirms and explains the physical mechanism for the phenomenon of pulse propagation mentioned in the work of Schäfer *et al.*[9]

References

[1] Suzuki K, Kubota H, Sahara A and Nakazawa M 1998 *Electron. Lett.* **34** 98

[2] Lee S, Khosravani R, Feinberg J, Grubsky V, Starodubov D S, Willner A E and Feinberg J 1999 *IEEE. Photon. Lett.* **11** 1277

[3] Hinz S, Sandel D, Yoshida-Dierolf M, Mirvoda V, Noé R, Feise G, Herrmann H, Ricken R, Sohler W, Suche H, Wehrmann F and Wessel R 1999 *Electron. Lett.* **35** 1185

[4] Wai P K A, Menyuk C R and Chen H H 1991 *Opt. Lett.* **16** 1231

[5] Chen Y and Hans H A 2000 *Opt. Lett.* **25** 290

[6] Eleftherianos C A, Syvridis D, Sphicopoulos T, Caroubalos C 1998 *Opt. Commun.* **154** 14

[7] Gong J M, Fang Q, Liu J *et al* 2000 *Acta Phys. Sin.* **49** 449 (in Chinese)

[8] Agrawal G P 1989 *Nonlinear Fiber Optics* (New York: Academic) p66, 104, 172

[9] Schäfer T, Moore R O and Jones C K R T 2002 *Opt. Commun.* **214** 353