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Superposition of Fock states via adiabatic passage in a coupled four-level atom–cavity system*

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We have investigated the behaviour of an atom–cavity system via a stimulated Raman adiabatic passage technique in a four-level system, in which two dark states are present. We find, because of the coherent control field, that a superposition of Fock states can be prepared, even when the cavity is initially not in its vacuum state. This method provides a way to generate arbitrary quantum states of a cavity field.

Keywords: superposition of Fock states, adiabatic passage, cavity QED
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1. Introduction

Recently there has been considerable interest in investigation of the generation of the Fock state and its superposition state, because of their important potential application in quantum information processing. A particularly interesting and topical avenue of research into the generation of nonclassical light fields is that associated with cavity quantum electrodynamics (cavity QED). In the context of cavity QED, the generation of nonclassical light has been performed both in the microwave domain [1–4] and in the optical domain.[5–9] In the microwave domain, the preparation of Fock states and their superposition is based on the vacuum Rabi oscillation in a two-level atom–cavity system. The experiments in the microwave domain are achieved by using Rydberg atoms and very high-Q superconducting cavities, so that spontaneous emission and cavity damping are negligible in the time scale of the atom–cavity interaction.

In the optical domain, the generation of Fock states and their superposition is based on the coherent population transfer in a coupled three-level Λ atom–cavity system.[5] An important feature of this scheme is that the adiabatic transformation is applied to the dark eigenstate of the system, i.e. the relevant eigenstate contains no contribution from the excited atomic state. Therefore this technique is immune to the detrimental consequences of atomic spontaneous emission. In Ref.[5], Parkins *et al* considered one of the light (Stokes) fields to be the field mode of a high-finesse cavity and showed how to prepare a Fock state in the cavity. They found that, if the pump and Stokes frequencies maintain two-photon resonance, and if the Stokes laser pulse precedes the pump laser pulse (counter-intuitively ordered pulses), the adiabatic passage of an atom through the interaction region will create a single-photon Fock state and its superposition 'out of vacuum'. However, the intuitively ordered pulses do not allow the preparation in the three-level atom–cavity system. The first experimental results to generate single photons were obtained by Hennrich *et al*[7] based on the single-photon generation scheme discussed in Ref.[8].

Recently, using the stimulated Raman adiabatic passage (STIRAP) technique in a four-level system, we studied the properties of the atom–cavity states in an optical cavity. We found[10] that a cavity-mode Fock state can always be generated by both the in-

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tuitively and counter-intuitively ordered pulses due to the interaction of the two dark states.

On the basis of our previous paper,[10] we propose a method to create a more general superposition of Fock states. We find that a superposition of Fock states can be prepared due to the coherent control field, even if the cavity-mode is initially not in its vacuum state. This method provides a way to generate arbitrary quantum states of a cavity field.

This paper is organized as follows. In Section 2, we present the basic model for a four-level atom–cavity system. In Section 3, we propose a method to implement a superposition of Fock states by using the adiabatic passage scheme. Finally, in Section 4, we offer some conclusions.

2. The basic model

We consider a four-state atomic system with excited state $|e\rangle$, and degenerate ground states $|g_1\rangle$, $|g_2\rangle$ and $|g_3\rangle$, as illustrated in Fig.1. The pump pulse $P(t)$ couples the state $|g_1\rangle$ to the excited state $|e\rangle$. The control pulse $Q(t)$ couples the states $|g_2\rangle$ and $|e\rangle$. Here $P(t)$ and $Q(t)$ are classical laser fields. The Stokes pulse $g_{\rm S}(t)$ is provided by a cavity field mode, it couples the states $|g_3\rangle$ and $|e\rangle$. The three fields interact with the atom sequentially, each of the fields is in one-photon resonance with the respective transition. Thus the interaction Hamiltonian in the rotating-wave approximation for this system can be written as follows:

$$\begin{aligned} H(t) =& \hbar\omega a^+a + \hbar\omega_{eg}|e\rangle\langle e| - {\rm i}\hbar g_{\rm S}(t) \\ & \times [{\rm e}^{-{\rm i}\varphi}|e\rangle\langle g_3|a - {\rm e}^{{\rm i}\varphi}a^+|g_3\rangle\langle e|] \\ & + {\rm i}\hbar P(t)[|e\rangle\langle g_1|{\rm e}^{-{\rm i}\omega t} - {\rm e}^{{\rm i}\omega t}|g_1\rangle\langle e|] \\ & + {\rm i}\hbar Q(t)[|e\rangle\langle g_2|{\rm e}^{-{\rm i}\omega t} - {\rm e}^{{\rm i}\omega t}|g_2\rangle\langle e|], \quad (1) \end{aligned}$$

where a and a^+ are the annihilation and creation operators for the cavity mode, respectively, the atom–cavity-mode coupling strength is $g_{\rm S}(t)$, and φ is the initial phase of the cavity field (for simplicity, we have chosen the initial phases of the classical fields to be zero). The time dependence of $P(t)$, $Q(t)$ and $g_{\rm S}(t)$ is provided by the motion of the atoms across the laser and cavity-field profiles.

The interaction Hamiltonian for this system has the property that it couples only states within the manifold of dressed states $|g_1,n\rangle$, $|g_2,n\rangle$, $|g_3,n+1\rangle$ and $|e,n\rangle$, where $|g,n\rangle \equiv |g\rangle|n\rangle$, $|e,n\rangle \equiv |e\rangle|n\rangle$, and $|n\rangle$ represents an n-photon Fock state of the cavity mode. Such a family is shown in Fig.1. It is easy to derive that $H(t)$ has four eigenstates; the two degenerate dark states corresponding to zero eigenvalue are

$$|\Phi_n^1(t)\rangle = \cos\theta(t){\rm e}^{-{\rm i}\varphi}|g_1,n\rangle + \sin\theta(t)|g_3,n+1\rangle, \quad (2)$$

$$\begin{aligned} |\Phi_n^2(t)\rangle =& -\sin\theta(t)\sin\phi(t){\rm e}^{-{\rm i}\varphi}|g_1,n\rangle \\ & + \cos\theta(t)\sin\phi(t)|g_3,n+1\rangle \\ & + \cos\phi(t)|g_2,n\rangle, \quad (3) \end{aligned}$$

and

$$\tan\theta(t) = \frac{P(t)}{g_{\rm S}(t)\sqrt{n+1}}, \quad (4)$$

$$\tan\phi(t) = \frac{Q(t)}{\sqrt{P^2(t)+g_{\rm S}^2(t)(n+1)}}, \quad (5)$$

where $\theta(t)$ is the mixing angle used in standard STIRAP,[11] whereas $\phi(t)$ is an additional mixing angle related to the control pulse $Q(t)$.[12,13] If the control pulse $Q(t)$ is absent, the system reduces to a three-state atom–cavity system. Indeed, Eq.(4) recovers the result from Parkins *et al.*[5] For $Q \neq 0$, the two dark states are degenerate, and nonadiabatic coupling between them will occur. The evolution of the four-level atom–cavity system will be significantly influenced by that coupling of the two dark states. In the following, we set the control pulse $Q(t)$=const (which implies that the diameter of the Q-laser beam is large compared to the waist of the cavity mode and the P-beam) and require $QT \gg 1$. This is realizable for our system. For example, if using the values from Refs.[5,9], the adiabatic condition is met even if Q is as small as $Q = \Gamma^{-1}$ (here Γ^{-1} is the population lifetime); in this case $QT \approx 10$.

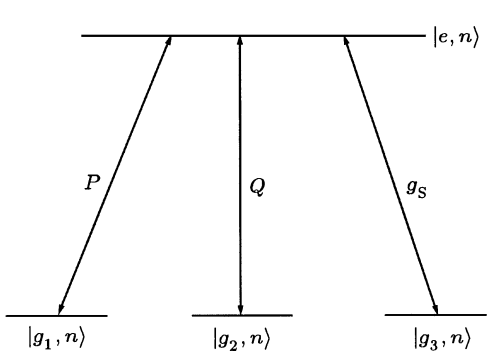


Fig.1. A four-level atom with excited state $|e\rangle$, and degenerate ground states $|g_1\rangle$, $|g_2\rangle$ and $|g_3\rangle$. The pump pulse $P(t)$ couples state $|g_1\rangle$ to excited state $|e\rangle$. The control pulse $Q(t)$ couples state $|g_2\rangle$ to $|e\rangle$. The Stokes pulse is a cavity field mode, it couples state $|g_3\rangle$ to $|e\rangle$. $g_{\rm S}(t)$ gives the atom–cavity-mode coupling strength.

In Ref.[10], we have obtained the atom–cavity states based on the non-adiabatic interaction of the two degenerate dark states (2) and (3). In the adiabatic limit, without loss of generality, we consider the case that initially $|\Psi(t)\rangle$ coincides with $|\Phi_n^1(t)\rangle$,

$$|\Psi_n(-\infty)\rangle = |\Phi_n^1(-\infty)\rangle. \tag{6}$$

At a later time, due to the coupling of the two degenerate eigenstates, $|\Psi_n(t)\rangle$ acquires a component along $|\Phi_n^2(t)\rangle$. The state vector then takes the form (see Ref.[13]) written as

$$|\Psi_n(t)\rangle = \sum_j B_{ij}(t)|\Phi_n^j(t)\rangle \quad (i,j=1,2), \tag{7}$$

where the matrix B, after the interaction, is given by

$$B(\infty) = \begin{pmatrix} \cos\gamma_f & \sin\gamma_f \\ -\sin\gamma_f & \cos\gamma_f \end{pmatrix}, \tag{8}$$

$$\gamma_f = \oint_C \frac{Q\sqrt{n+1}}{\sqrt{P^2+g_S^2(n+1)}\sqrt{P^2+Q^2+g_S^2(n+1)}} \times (g_S \mathrm{d}P - P\mathrm{d}g_S), \tag{9}$$

and C is the closed path in the parameter space. Here γ_f is called the geometric phase.[14] It is independent of pulse areas, depending only on the ratios τ/T, P_0/Q_0 and G_0/Q_0, where T and τ are the pulse lengths and the delay between the Stokes and pump pulses, respectively, and P_0, Q_0 and G_0 are the peak values of P, Q and g_S.

For the intuitively ordered pulses (Fig.2(a)), if the atom is initially in state $|g_1\rangle$, after the interaction, according to Eqs.(2), (3) and (6)–(8), we have

$$|\Psi_n^{(2)}(\infty)\rangle_{\mathrm{in}} = \sin\gamma_f(n)|g_1,n\rangle - \cos\gamma_f(n)\mathrm{e}^{\mathrm{i}\varphi}|g_3,n+1\rangle. \tag{10}$$

If the atom is initially in $|g_3\rangle$, then the state vector emerges into

$$|\Psi_n^{(1)}(\infty)\rangle_{\mathrm{in}} = \cos\gamma_f(n)\mathrm{e}^{-\mathrm{i}\varphi}|g_1,n\rangle + \sin\gamma_f(n)|g_3,n+1\rangle. \tag{11}$$

Here and in what follows the subscript 'in' ('cin') denotes a quantity related to the intuitively (counter-intuitively) ordered pulses, while the superscript '(2)' ('(1)') means that the state vector is initially identical to $|\Phi_n^2\rangle$ ($|\Phi_n^1\rangle$).

For the counter-intuitively ordered pulses (Fig.2(b)), we obtained the atom–cavity state with the atoms initially in state $|g_1\rangle$,

$$|\Psi_n^{(1)}(\infty)\rangle_{\mathrm{cin}} = \cos\gamma_f(n)\mathrm{e}^{\mathrm{i}\varphi}|g_3,n+1\rangle - \sin\gamma_f(n)|g_1,n\rangle, \tag{12}$$

or for the reverse process, when the atom is initially in $|g_3\rangle$,

$$|\Psi_n^{(2)}(\infty)\rangle_{\mathrm{cin}} = -\sin\gamma_f(n)|g_3,n+1\rangle - \cos\gamma_f(n)\mathrm{e}^{-\mathrm{i}\varphi}|g_1,n\rangle. \tag{13}$$

Obviously, the atom–cavity entangled states can be prepared by both the intuitive and counter-intuitive ordering of pulses, because of the coherent control field Q, unlike the case discussed in the three-level Λ system,[5] where only counter-intuitively ordered pulses can be used to produce such states.

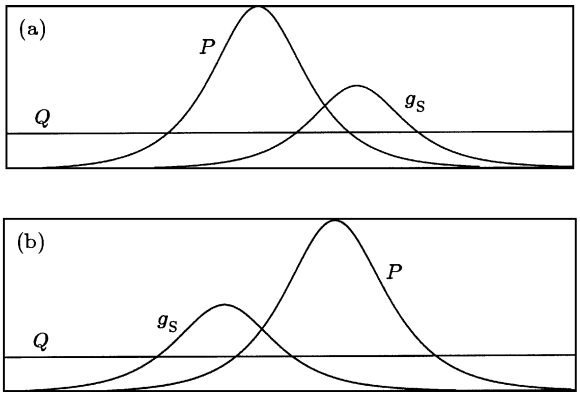


Fig.2. The intuitively ordered pulses (a) and the counter-intuitively ordered pulses (b).

3. Superposition of Fock states via the STIRAP technique

As we know,[5] in the three-level Λ atom–cavity QED system, if an atom is prepared in a coherent superposition of ground states and the cavity is initially in its vacuum state, a superposition of Fock states can be prepared by the adiabatic passage technique in the case of counter-intuitively ordered pulses. But it cannot be realized if the cavity is initially in the Fock state of $|n \neq 0\rangle$, because the coherent population transfer cannot be realized by the intuitively ordered pulses. However, in the four-level atom–cavity QED system, we will show that due to the coherent control field, the superposition of Fock states can always be produced, even if the cavity is initially not in its vacuum state. Here we assume that the population in $|g_1\rangle$ is transferred by the counter-intuitively ordered pulses, the population in $|g_3\rangle$ is, correspondingly, transferred

by the intuitively ordered pulses (similar results can be obtained by using the reverse-ordered pulses).

In order to prepare the superposition of Fock states, we assume that an atom initially prepared in a superposition of atomic ground states $\cos\alpha|g_1\rangle + \sin\alpha|g_3\rangle$ (which can be easily prepared, for example, by the extended STIRAP technique in the four-level system [12]). First, we consider the case in which the cavity is initially in its vacuum state, so we have the initial atom–cavity state $(\cos\alpha|g_1\rangle + \sin\alpha|g_3\rangle)\otimes|0\rangle$. After the end of the interaction, $|g_3\rangle|0\rangle$ remains unaltered, while

$$|g_1\rangle|0\rangle \Longrightarrow -\sin\gamma_f(0)|g_1\rangle|0\rangle + \cos\gamma_f(0)\mathrm{e}^{\mathrm{i}\varphi}|g_3\rangle|1\rangle$$

according to Eq.(12). So we have the atom–cavity state

$$\begin{aligned}&-\cos\alpha\sin\gamma_f(0)|g_1\rangle|0\rangle\\&+\cos\alpha\cos\gamma_f(0)\mathrm{e}^{\mathrm{i}\varphi}|g_3\rangle|1\rangle+\sin\alpha|g_3\rangle|0\rangle.\end{aligned}\quad(14)$$

As for the relative phase between $|g_1\rangle$ and $|g_3\rangle$, we know that they have zero relative phase at the beginning. They should also have zero relative phase (except for the phase via evolution of the atom–cavity system) after the end of interaction, so we have $\varphi = 0$. We can see, from Eq.(14), that if the atom is detected in its ground state $|g_3\rangle$, the atom–cavity is projected into

$$\sin\alpha|0\rangle + \cos\alpha\cos\gamma_f(0)|1\rangle. \quad(15)$$

For $Q \ll P_0, G_0$, $\gamma_f(0) \simeq 0$,[10] thus we can realize the quantum-state mapping between the atom and the cavity field, which is similar to the case of the three-level atom–cavity QED system.

Now we consider the situation in which the cavity is initially in its Fock state $|n \neq 0\rangle$. In this case, the superposition of Fock states cannot be prepared in a three-level atom–cavity system. However, in the four-level atom–cavity coupled system, the coherent population transfer can always be realized because of the coherent control field Q. We can prepare the superposition of Fock states in terms of this behaviour.

When the cavity is initially in its Fock states $|n \neq 0\rangle$, we have the initial atom–cavity state $(\cos\alpha|g_1\rangle + \sin\alpha|g_3\rangle)\otimes|n\rangle$. After the end of the interaction,

$$\begin{aligned}|g_1\rangle|n\rangle \Longrightarrow &-\sin\gamma_f(n)|g_1\rangle|n\rangle\\&+\cos\gamma_f(n)\mathrm{e}^{\mathrm{i}\varphi}|g_3\rangle|n+1\rangle\end{aligned}$$

according to Eq.(12), and

$$\begin{aligned}|g_3\rangle|n\rangle \Longrightarrow &-\cos\gamma_f(n)|g_1\rangle|n-1\rangle\\&+\sin\gamma_f(n)\mathrm{e}^{\mathrm{i}\varphi}|g_3\rangle|n\rangle\end{aligned}$$

according to Eq.(11). So the atom–cavity state turns into

$$\begin{aligned}&-\cos\alpha\sin\gamma_f(n)|g_1\rangle|n\rangle\\&+\cos\alpha\cos\gamma_f(n)\mathrm{e}^{\mathrm{i}\varphi}|g_3\rangle|n+1\rangle\\&+\sin\alpha\cos\gamma_f(n-1)\mathrm{e}^{-\mathrm{i}\varphi}|g_1\rangle|n-1\rangle\\&+\sin\alpha\sin\gamma_f(n-1)|g_3\rangle|n\rangle.\end{aligned}\quad(16)$$

For the same reason as Eq.(14), we have $\varphi = 0$. We obtain the result that, if the atom is detected in the ground state $|g_1\rangle$, the cavity state is projected into

$$\begin{aligned}|\Psi_{\mathrm{c}1}\rangle = &-\cos\alpha\sin\gamma_f(n)|n\rangle\\&+\sin\alpha\cos\gamma_f(n-1)|n-1\rangle;\end{aligned}\quad(17)$$

otherwise it is projected into

$$\begin{aligned}|\Psi_{\mathrm{c}2}\rangle = &\sin\alpha\sin\gamma_f(n-1)|n\rangle\\&+\cos\alpha\cos\gamma_f(n)|n+1\rangle,\end{aligned}\quad(18)$$

if the atom is detected in the ground state $|g_3\rangle$. So we prepared the superpositions of Fock states ($|n\rangle$, $|n-1\rangle$) and ($|n\rangle$, $|n+1\rangle$), respectively, according to the detected results.

We can see, from Eqs.(16)–(18), that the superposition of Fock states can always be produced via the STIRAP technique in the four-level atom–cavity system because of the coherent control field Q, even if the cavity mode is in its Fock state $|n \neq 0\rangle$. Using these results, we can prepare the desired superposition of Fock states beginning with any initial cavity state. For instance, if the initial cavity state is in $|0\rangle + |n \neq 0\rangle$, with the similar steps, we can produce the superposition state

$$\begin{aligned}|\Psi_{\mathrm{c}1}\rangle = &-\cos\alpha\sin\gamma_f(0)|0\rangle\\&-\cos\alpha\sin\gamma_f(n)|n\rangle\\&+\sin\alpha\cos\gamma_f(n-1)|n-1\rangle,\end{aligned}\quad(19)$$

if the atom is detected in the ground state $|g_1\rangle$, while if the atom is detected in the ground state $|g_3\rangle$, we can prepare the superposition state

$$\begin{aligned}|\Psi_{\mathrm{c}2}\rangle = &\sin\alpha|0\rangle + \cos\alpha\cos\gamma_f(0)|1\rangle\\&+\sin\alpha\sin\gamma_f(n-1)|n\rangle\\&+\cos\alpha\cos\gamma_f(n)|n+1\rangle.\end{aligned}\quad(20)$$

Hence, beginning with the initial state $|0\rangle + |n \neq 0\rangle$ of the cavity, we have produced a superposition of Fock

states $|0\rangle$, $|n-1\rangle$ and $|n\rangle$ for the atom detected in $|g_1\rangle$, and a superposition of Fock states $|0\rangle$, $|1\rangle$, $|n\rangle$ and $|n+1\rangle$ for the atom detected in $|g_3\rangle$.

We have noticed that such a superposition of Fock states can be prepared by using pulse area technique in a two-level atom–cavity system. The pulse area technique is simple to apply in theory, but it is very sensitive to actual experimental conditions. In our scheme, the STIRAP technique is robust; it does not require an accurate area of pulses. Furthermore, in the process of preparing such a superposition of Fock states, there are no excited states of atoms involved, thus this technique is immune to the detrimental consequences of atomic spontaneous emission.

4. Summary and discussion

In this paper, using the STIRAP technique in a four-level system, we have investigated the properties of the atom–cavity states in an optical cavity. We have proposed a method to prepare the superposition of Fock states. We found that, due to the interaction of the two dark states, the superposition of Fock states can be prepared, even if the cavity is initially not in its vacuum state. This method provides a way to generate arbitrary quantum states of a cavity field. In experiment, the adiabatic passage technique requires the condition [5] $\Omega_0, G_0 \gg 1/T \gg n_{\max}\kappa, \Gamma$. Here κ is the cavity linewidth and $n_{\max}$ is the maximum photon number attained by the cavity mode. The realization of parameters that satisfy the condition appears feasible with realistic improvements to the experiments reported recently.[15–18]

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