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Experimental realization of information transmission between not-directly-coupled spins on NMR quantum computers*

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This paper presents a simple scheme for information transmission between two non-directly interactive qubits in an n-qubit system. An example has been realized on a three-qubit nuclear magnetic resonance (NMR) spectrometer quantum computer. The experimental result successfully demonstrates that the feasible measure can also be extended to other quantum logical gates, or other quantum algorithms, where some qubits have no direct interactions in a multi-qubit system.

Keywords: information transmission, quantum superdense coding, nuclear magnetic resonance spectrometer

PACC: 0365, 7660, 7510J

1. Introduction

Quantum entanglement, a non-local correlation inherent in entangled states, plays an important role in quantum information processing.[1−5] Such examples include quantum computation,[2] quantum teleportation,[3] quantum superdense coding,[4] and quantum cryptography[5] etc. These uses rely on the successful generation and manipulation of highly entangled states. Recently, there has been intense interest in extending quantum computation to multi-qubit quantum systems.[6,7] Several authors have proposed the schemes for creating entanglement in arrays of interacting qubits with an Ising-type interaction.[8−10] Experimentally, important progress in creating maximally entangled states has been made.[11,12] It is important to note that a universal quantum computation can be based on using teleportation and single-qubit operations,[13] and that non-local quantum gates can rely on local implementation and classical communication.[14] These involve the use of quantum entanglement and classical communication. It is well-known that the fundamental resource to generate entanglement and transmit information is the interaction between two quantum systems.[15] However, with the increase in the number of qubits, it is not easy to selectively control the coherent evolution between two distant qubits because no available interactions exist between these distant qubits. Mádi *et al*[16] used spin swap operations to solve the difficulty of controlling the coherent evolution of the spin systems where some interactions between distant spins are too weak. This idea was widely applied in the creation of Greenbareer–Horne–Zeilinger (GHZ) states, quantum erasers,[17,18] entanglement transfers,[19] and an exchange of information.[20] It is interesting to note that the quantum-eraser scheme can be used for the transfer of information among weakly interacting qubits,[18] and that swap gates can transfer entanglement between two non-directly interacting spins.[19] These applications indicate that the swap operation can be used to transfer quantum information, and

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allows an extension of quantum computing to a partially coupled spin system.[16−21] One goal of this paper is to experimentally perform controlled-NOT gates between two non-direct-coupled qubits and generate entangled states of two 'distant' qubits by using quantum swap gates.

Quantum superdense coding (QSDC), theoretically proposed by Bennett and Wiesner,[4] provides a means to transmit and encode two classical bits through the manipulation of only one of two entangled qubits. Currently, experimental realizations of QSDC are limited to two-qubit systems.[22,23] Recently, there have been several schemes for superdense coding among multiparticles.[24−26] In a nuclear magnetic resonance (NMR) realization,[23] two nuclear spins are coupled. In a multi-qubit spin system, direct interactions between two distant qubits are negligible so that there is no resource to transmit information between them. Therefore, it becomes more significative to implement quantum communication between non-directly interacting qubits. The realization of the QSDC between two distant qubits could demonstrate the striking feature of entangled states. One of the problems of interest in this paper is the experimental implementation of quantum communication between 'distant' qubits in a molecule.

In this paper, we first analyse a simple scheme to transmit information between non-directly interacting (or distant) spins on the basis of swap operations and QSDC. Then we realize this scheme experimentally and also extend the measure to transmit information in such systems as a chain of nuclear spins in molecules. In terms of our theoretical and experimental results, our scheme may be suitable for the implementation of universal quantum operations in a distributed quantum computer,[14] instead of the need for classical communication.

2. Information transmission

For simplicity, we only consider a three-qubit system, in which the qubits I_1 and I_3 have no direct interactions. In order to carry through the QSDC between the qubits I_1 and I_3, the first step is to create the entangled state of the spins I_1 and I_3 in which the controlled-NOT gate is a key factor. Without the J-coupling J_{13}, the controlled-NOT gate between I_1 and I_3 can be performed by first applying a swap operation on I_1 and I_2, which can exchange the roles of the two spins, followed by a controlled-NOT operation on I_2 and I_3, and finally by another swap gate between I_1 and I_2.[16] This process only needs two interactions: namely, the one between spins I_1 and I_2, and the other between spins I_2 and I_3. Secondly, according to the principle of QSDC, the information between spins I_1 and I_3 can be transmitted by using three single-qubit operations and two controlled-NOT gates. One great difference is that the controlled-NOT gate can only be realized indirectly.

A quantum circuit for implementing the information transmission on a three-qubit NMR quantum computer is shown in Fig.1. The circuit can be decomposed into three parts: the first part is to produce the maximally entangled state of the qubits I_1 and I_3, including one Hadamard transformation ($R_{\rm H}$),[27] two swap operations ($U_{\rm S}$) and one controlled-NOT ($U_{\rm CN}$) operation. The second one is to encode on qubit I_3, which is denoted by $U_{ij}(U_{00}, U_{01}, U_{10}$ and $U_{11})$. The last part, including two swap operations ($U_{\rm S}$), one controlled-NOT ($U_{\rm CN}$) operation and one Hadamard transformation ($R_{\rm H}$), is to decode the encoded value.

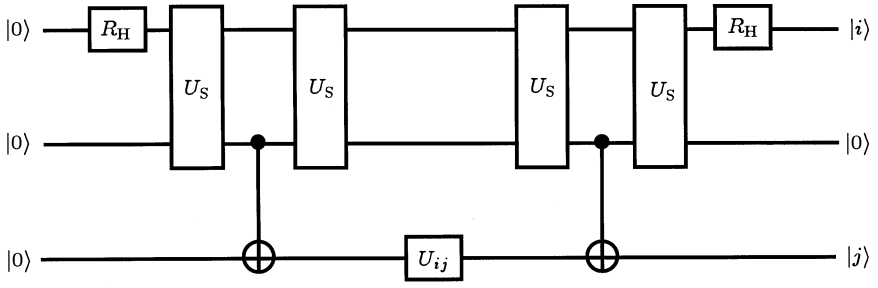


Fig.1. The quantum circuit for implementing the information transmission between two non-direct-coupled qubits in a three-qubit system, where $U_{\rm S}$ denotes the swap gate, U_{ij} the encoding operation.

Starting from a pseudopure state,[21] the first part can be expressed as follows:

$$U_{\rm Bell} = (U_{\rm S}\otimes E)(E\otimes U_{\rm CN})(U_{\rm S}\otimes E)(R_{\rm H_1}\otimes E\otimes E), \quad (1)$$

where the time order is from right to left, and E is a unit operation. Note that the swap operator ($U_{\rm S}$) is a unitary transformation with $U_{\rm S}^2 = 1$, where $U_{\rm S}^{+} = U_{\rm S}$. The whole transformation ($U_{\rm QSDC}^{ij}$) of the three parts can be denoted as

$$\begin{aligned} U_{\rm QSDC}^{ij} =& (R_{\rm H}\otimes E\otimes E)(U_{\rm S}\otimes E)(E\otimes U_{\rm CN}) \\ & \times (U_{\rm S}\otimes E)(E\otimes E\otimes U_{ij}) \\ & \times (U_{\rm S}\otimes E)(E\otimes U_{\rm CN}) \\ & \times (U_{\rm S}\otimes E)(R_{\rm H}\otimes E\otimes E) \\ =& (R_{\rm H}\otimes E\otimes E)(U_{\rm S}\otimes E) \\ & \times (E\otimes U_{\rm CN})(E\otimes E\otimes U_{ij}) \\ & \times (E\otimes U_{\rm CN})(U_{\rm S}\otimes E)(R_{\rm H}\otimes E\otimes E), \quad (2) \end{aligned}$$

where

$$R_{\rm H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$

$$U_{\rm S} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$U_{\rm CN} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$U_{00} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$U_{01} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

$$U_{10} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$U_{11} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

The execution of a series of NMR pulse sequences for the transformation $U_{\rm QSDC}^{ij}$ shown in Table 1, where $\frac{\pi}{2}$ indicates the $\frac{\pi}{2}$ flip degree of the radio-frequency pulse, the superscripts denote the operated spins, the subscripts stand for the phases of the radio-frequency pulse, and $\frac{1}{2J_{kl}}$ refers to the evolution time dominated by the J-coupling between spins k and l, produces the resulting state

$$\psi_{i0j} = U_{\rm QSDC}^{ij}|000\rangle = |i0j\rangle. \quad (3)$$

Table 1. The operations and their corresponding pulse sequences for information transmission in a three-qubit system.

Operations	Pulse sequences
$R_{\rm H}$	$\left(\frac{\pi}{2}\right)_{y}^{I_1} - (\pi)_{x}^{I_1}$
$U_{\rm S}$	$\left(\frac{\pi}{2}\right)_{y}^{I_1,I_2} - \frac{1}{2J_{12}} - \left(\frac{\pi}{2}\right)_{x}^{I_1,I_2} - \frac{1}{2J_{12}} - \left(\frac{\pi}{2}\right)_{-y}^{I_1,I_2}$
$U_{\rm CN}$	$\left(\frac{\pi}{2}\right)_{-y}^{I_3} - \frac{1}{2J_{23}} - \left(\frac{\pi}{2}\right)_{y}^{I_2,I_3} - \left(\frac{\pi}{2}\right)_{x}^{I_2,I_3} - \left(\frac{\pi}{2}\right)_{-y}^{I_2,I_3} - \left(\frac{\pi}{2}\right)_{y}^{I_3}$
U_{00}	do nothing operation
U_{01}	$(\pi)_{y}^{I_3}(\pi)_{-x}^{I_3}$
U_{10}	$(\pi)_{-x}^{I_3}$
U_{11}	$(\pi)_{y}^{I_3}$

In order to simplify the quantum circuit, the pulse sequences in Table 2 can also lead to the same results as those in Table 1, which rely on the fact that in NMR quantum computers any absolute phase is meaningless.[21] Under this condition, the first operation $R_{\rm H}$ in Fig.1 was replaced by $R_{\rm H_1}$, the second $R_{\rm H}$ was replaced by $R_{\rm H_2}$, and the matrix of a controlled-NOT gate becomes

$$U_{\mathrm{CN}} = \begin{bmatrix} 1+\mathrm{i} & 0 & 0 & 0 \\ 0 & 1-\mathrm{i} & 0 & 0 \\ 0 & 0 & 0 & -1-\mathrm{i} \\ 0 & 0 & 1-\mathrm{i} & 0 \end{bmatrix}. \quad (4)$$

With the modified pulse sequences, the states can be written as

$$\begin{aligned} \psi_{00} &= U^{00}_{\mathrm{QSDC}}|000\rangle = \mathrm{e}^{\mathrm{i}\frac{\pi}{2}}|000\rangle, \\ \psi_{01} &= U^{01}_{\mathrm{QSDC}}|000\rangle = \mathrm{e}^{\mathrm{i}\pi}|001\rangle, \\ \psi_{10} &= U^{10}_{\mathrm{QSDC}}|000\rangle = \mathrm{e}^{\mathrm{i}\pi}|100\rangle, \\ \psi_{11} &= U^{11}_{\mathrm{QSDC}}|000\rangle = \mathrm{e}^{\mathrm{i}\frac{\pi}{2}}|101\rangle. \end{aligned} \quad (5)$$

Compared with Eq.(3), Eq.(5) have the same final states except for the phase differences. By measuring the final states of all qubits, we can conclude which encoding operation has been applied.

From the above analyses, the information transmission project between non-directly interacting spins (I_1 and I_3) is theoretically successful. Furthermore, with increase in the number of spins, the above scheme of information transmission can also be extended to n-qubit systems containing just interactions between two neighbouring qubits. The quantum network in Fig.2, which is simplified, is used to achieve the quantum communication between qubits I_1 and I_{10} in a ten-qubit system on the basis of the principle of the QSDC and quantum swap gates.[28]

Table 2. The modified operations for information transmission and their pulse sequences.

Operations	Pulse sequences
R_{H1}	$\left(\frac{\pi}{2}\right)^{I_1}_{-x}$
U_{S}	no modification
U_{CN}	$\left(\frac{\pi}{2}\right)^{I_3}_{x} - \frac{1}{2J_{23}} - \left(\frac{\pi}{2}\right)^{I_3}_{y}$
U_{00}	do nothing operation
U_{01}	$(\pi)^{I_3}_{y}$
U_{10}	$(\pi)^{I_3}_{y}(\pi)^{I_3}_{-x}$
U_{11}	$(\pi)^{I_3}_{-x}$
R_{H2}	$\left(\frac{\pi}{2}\right)^{I_1}_{y}$

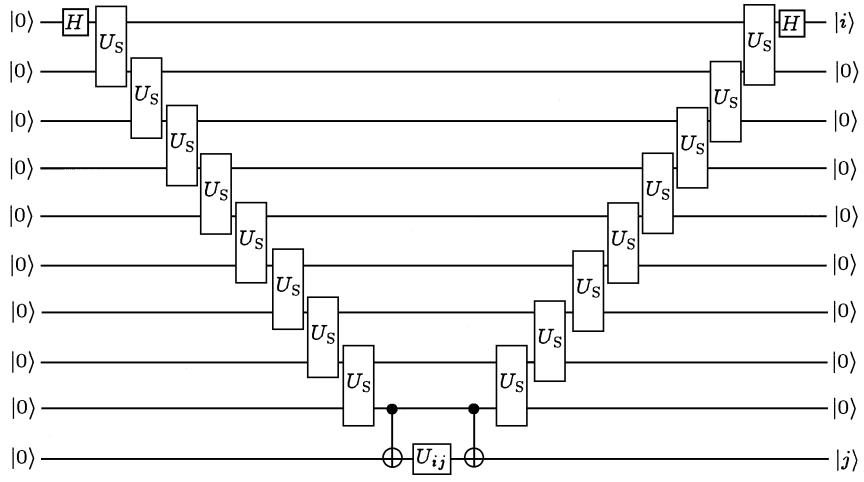


Fig.2. The simplified quantum circuit for implementing the information transmission between two non-direct-coupled qubits (I_1 and I_{10}) in a ten-qubit system.

3. Experiment and results

We carried out the experiment with a Bruker ARX 500 MHz spectrometer. The sample was ^{13}C-labelled alanine ($^{13}_{\mathrm{I}}\mathrm{CH_3}$—$^{13}_{\mathrm{II}}\mathrm{CH(NH_2)}$—$^{13}_{\mathrm{III}}\mathrm{COOH}$) in deuterated water,[29] in which the three carbons denote the qubits I_1, I_2 and I_3. The experimental temperature was maintained at 25°C. The molecular structure and coupling constants (J_{12}=35.1 Hz, J_{23}=54.1 Hz, J_{13}=−1.2 Hz) of alanine can be found in Ref.[29]. The relaxation times T_1 for the three carbons are about 1.6, 2.5, and 21 s, while the times T_2 are about 800, 420 and 550 ms, respectively.[30] The total time of each experiment is approximately 100 ms. Compared with the other two coupling constants, J_{13} coupling is relatively small and can be regarded as zero, or is always refocused. This condition means that the qubits I_1 and I_3 have no interactions which

agrees with our demand.

In terms of the temporal averaging method proposed by Knill *et al*,[31] the pseudopure state [11] $|000\rangle$ was assembled as a mixture of five terms ($I_z^1+I_z^2+I_z^3$, $2I_z^1I_z^2$, $2I_z^1I_z^3$, $2I_z^2I_z^3$, $4I_z^1I_z^2I_z^3$) which were prepared from the thermal equilibrium state as shown in Table 3. This involved five experiments and a sum of these five experimental results.

Table 3. The five terms and the corresponding pulse sequences for the preparation of the pseudopure state. PFG denotes the pulse field of gradient.

Terms	Pulse sequences
$I_z^1+I_z^2+I_z^3$	do nothing
$2I_z^1I_z^2$	$\left(\frac{\pi}{2}\right)_x^{I_2,I_3}-\mathrm{PFG}-\left(\frac{\pi}{2}\right)_x^{I_1}-\left(\frac{1}{2J_{12}}\right)-\left(\frac{\pi}{2}\right)_y^{I_1}$
$2I_z^2I_z^3$	$\left(\frac{\pi}{2}\right)_x^{I_1,I_3}-\mathrm{PFG}-\left(\frac{\pi}{2}\right)_x^{I_2}-\left(\frac{1}{2J_{23}}\right)-\left(\frac{\pi}{2}\right)_y^{I_2}$
$4I_z^1I_z^2I_z^3$	$\left(\frac{\pi}{2}\right)_x^{I_1,I_3}-\mathrm{PFG}-\left(\frac{\pi}{2}\right)_x^{I_2}-\left(\frac{1}{2J_{12}}\right)-\left(\frac{1}{2J_{23}}\right)-\left(\frac{\pi}{2}\right)_y^{I_2}$
$2I_z^1I_z^3$	$\left(\frac{\pi}{2}\right)_x^{I_1,I_3}-\mathrm{PFG}-\left(\frac{\pi}{2}\right)_x^{I_2}-\left(\frac{1}{2J_{12}}\right)-\left(\frac{1}{2J_{23}}\right)-\left(\frac{\pi}{2}\right)_y^{I_2}-\left(\frac{\pi}{2}\right)_x^{I_1}-\left(\frac{1}{2J_{12}}\right)-\left(\frac{\pi}{2}\right)_y^{I_1}$

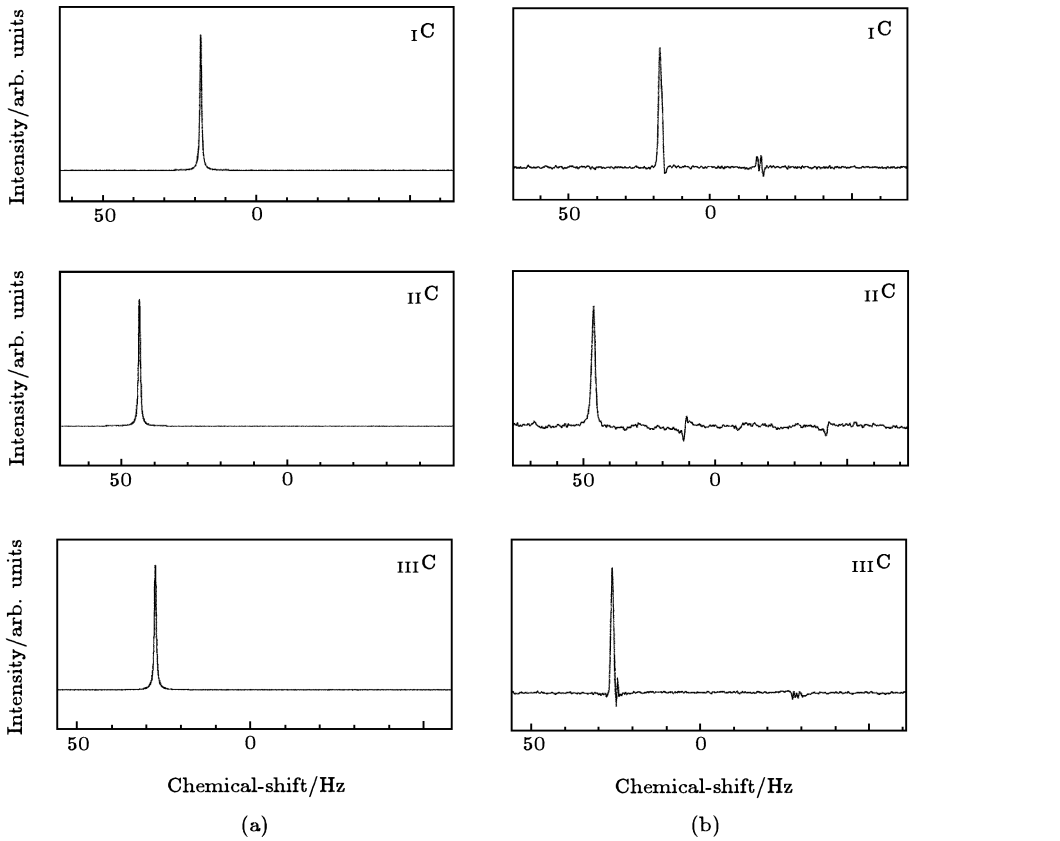


Fig.3. Simulated and experimental spectra of the spins ${}^{13}_{\mathrm{I}}\mathrm{C}$, ${}^{13}_{\mathrm{II}}\mathrm{C}$, ${}^{13}_{\mathrm{III}}\mathrm{C}$, corresponding to the operation U^{00}_{QSDC}. (a) Simulated spectra, (b) experimental spectra.

The shape of the soft pulse is Gaussian. We firstly give one of the experimental spectra corresponding to the operation U^{00}_{QSDC} shown in Fig.3. In order to show the experimental result more obviously, using state tomography,[32] we reconstruct the experimental results (as described in Fig.4) in which the matrices ρ_1, ρ_2, ρ_3, and ρ_4 express the output results after performing the operations U^{00}_{QSDC}, U^{01}_{QSDC}, U^{10}_{QSDC} and U^{11}_{QSDC}, respectively. Observing the above matrices, we can conclude that the final states are $|000\rangle$, $|001\rangle$,

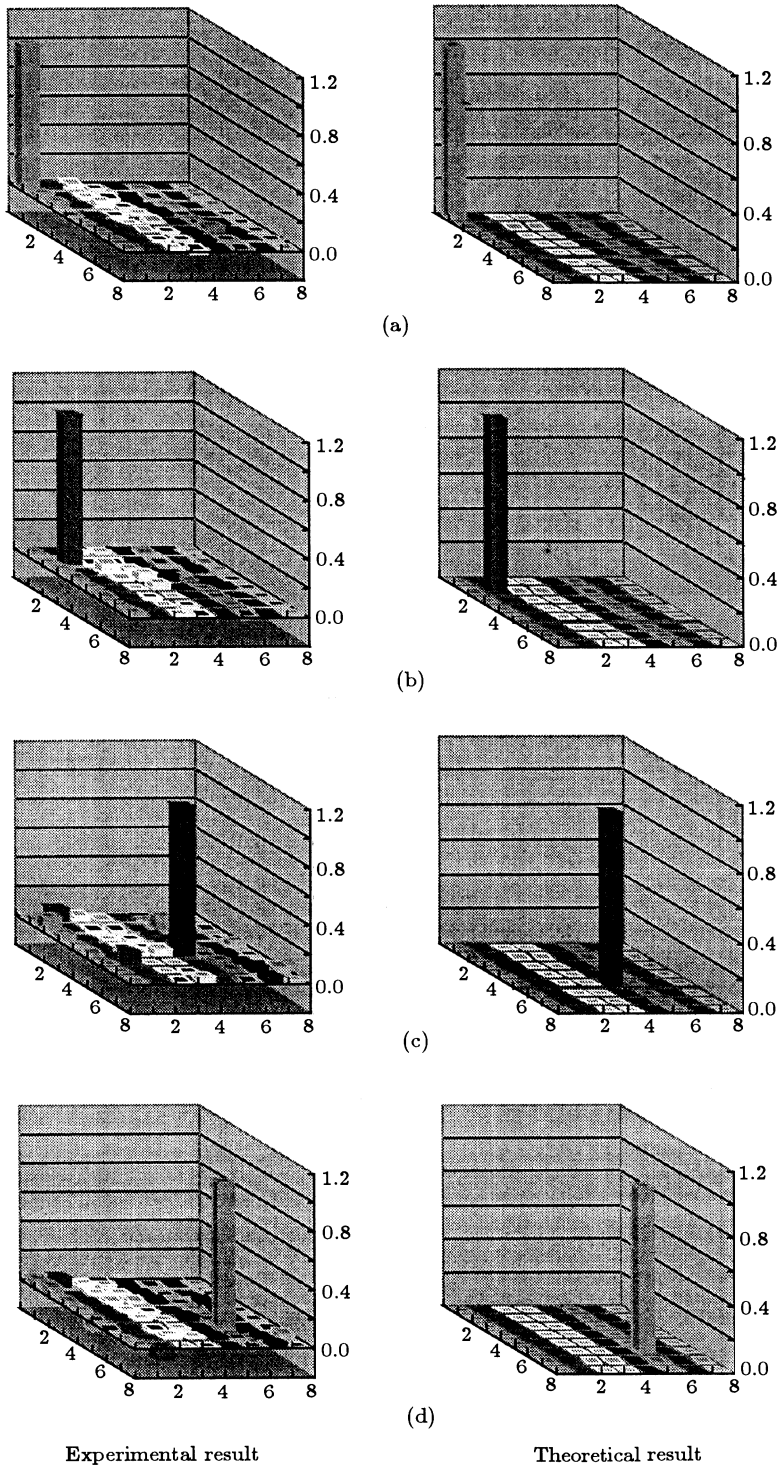


Fig.4. The deviation matrices (in normalized units) for the final states: (a) $|000\rangle$, (b) $|001\rangle$, (c) $|100\rangle$ and (d) $|101\rangle$. The real parts of the experimental results (left), and the real parts of the theoretical results (right).

$|100\rangle$, and $|101\rangle$, respectively, which indicate that the experiments have finished the information transmission between the spins I_1 and I_3.

An extension of Schumacher's fidelity [33] measures the efficacy with quantum information transmission, which calculates not only systematic errors, but also the net loss of magnetization.[18] This evaluation, which is called the attenuated correlation,[18] is given by

$$c(\hat{\rho}^{\text{expt}}) = \frac{\text{Tr}(\hat{\rho}^{\text{the}}\hat{\rho}^{\text{expt}})}{\text{Tr}(\hat{\rho}^{\text{the}}\hat{\rho}^{\text{the}})}. \quad (6)$$

Here $\hat{\rho}^{\text{the}}$ is the theoretical output density matrix,

obtained by the ideal transformation on a computer for an input density matrix $\hat{\rho}_{\rm ini}$; $\hat{\rho}^{\rm expt}$ is the experimental result, realized by a series of pulse sequences according to the same transformation on the spectrometer. Note that the above equation implies that $-1 \leq c(\hat{\rho}^{\rm expt}) \leq 1$. With the definition of $c(\hat{\rho}^{\rm expt})=1$, the values of the correlation obtained for the four tomographic results are, respectively, $c(\hat{\rho}^{\rm expt}_{000})=0.98$, $c(\hat{\rho}^{\rm expt}_{001})=0.96$, $c(\hat{\rho}^{\rm expt}_{100})=0.96$, and $c(\hat{\rho}^{\rm expt}_{101})=0.93$.

It should be noted that the imaginary parts of the resulting deviation density matrices result from experimental errors. The experimental errors mainly come from (1) the inhomogeneity of the static and radio-frequency magnetic fields, (2) the imperfection of the selective pulse, (3) the inaccuracy of the 90° pulse, and (4) the decaying of signals during the acquisition time.

4. Conclusion

We have analysed a feasible implementation of the controlled-NOT gate between two distant qubits which can be used to create entangled states among non-directly coupled spins, especially in catenulate molecules that only contain interactions between neighbouring spins. Relying on the swap operations and QSDC, we have also put forward a scheme, experimentally realized on our three-qubit NMR quantum computer, to transmit information between distant qubits. The experimental result shows that the information transmission between non-direct-coupled spins has been successfully accomplished with our method. Therefore, a combination of our method with the universal quantum computation scheme [13] may allow for the realization of universal quantum operations, instead of the requirements of classical communication. Quantum swap gates can be used to move quantum information among partly coupled spins.[18−21] Certainly, all of these quantum information processes have commendably demonstrated the quantum property of entangled states,[34] and shown that nonlocal operations can be implemented via a series of quantum swap operations. We have also extended this scheme to a multi-qubit system and given a simple example for a ten-qubit system. The idea to realize the two-qubit operations between two non-direct-coupled qubits also can be used to implement other quantum gates or some quantum algorithms.

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