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Depinning dynamics of driven colloids with random pinning: a numerical study*

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Using Langevin simulations, we have investigated numerically the depinning dynamics of driven two-dimensional colloids subject to the randomly distributed point-like pinning centres. With increasing strength of pinning, we find a crossover from elastic to plastic depinnings, accompanied by an order to disorder transition of state and a substantial increase in the depinning force. In the elastic regime, no peaks are found in the differential curves of the velocity–force dependence (VFD) and the transverse motion is almost none. In addition, the scaling relationship between velocity and force is found to be valid above depinning. However, when one enters the plastic regime, a peak appears in the differential curves of VFD and transverse diffusion occurs above depinning. Furthermore, history dependence is found in the plastic regime.

Keywords: colloid, depinning, crossover
PACC: 8270D, 6460H

1. Introduction

Much attention has been focused on colloidal crystals over the past two decades, because they provide ideal model systems to investigate various problems in materials science,[1] physical chemistry [2] and condensed matter physics.[3] In particular, the general problems of ordering and dynamics in two-dimensional (2D) colloids have been addressed extensively.[4–8] There exist numerous studies about 2D and 1D melting on homogeneous substrates [9–11] and corrugated surfaces.[12] Recently, there has been considerable interest in the dynamics of driven media in the presence of quenched disorder and an applied drive. More recently, it has been demonstrated [13] that point defects can be created in a 2D colloidal crystal by manipulating individual particles with optical tweezers. Thereafter, many authors have studied the colloidal crystallization and melting in 2D systems on disordered substrates.[14,15]

Although the equilibrium thermodynamics of a 2D colloidal crystal has been studied extensively, the non-equilibrium dynamics of driven 2D colloids on disordered substrates remains less explored. When brought into non-equilibrium, colloidal systems may exhibit many different kinds of pattern formation,[16,17] which are much richer than the traditional phase transitions in equilibrium systems. Such exotic systems also include sliding charge density waves (CDWs),[18,19] the vortex lattices in disordered superconductors,[20] and Wigner crystals,[21] etc.

One of the most studied systems in this class is a vortex lattice driven by an applied current through disordered superconductors. For the vortex lattice in type-II superconductors, crossover from elastic to plastic depinnings has been studied extensively.[22–26] It has been established that for all but very weak pinning, the vortex lattice is deformed plastically and the onset of the vortex motion takes place through channels. This result goes beyond the applicability of the collective pinning theory for current–voltage (I–V) characteristics.[27] In the vortex glass picture the transport is dominated by vortex creep, leading to exponential I–V characteristics.[22] Increasing tempera-

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ture transforms the vortex glass phase into a vortex liquid.[24] It was recently pointed out that the critical current shows a peak during the vortex glass to vortex liquid transition and the peak is accompanied by a crossing of the I–V curves.[26]

Most recently, Reichardt and Olson [28] have examined numerically the dynamics of driven colloidal systems interacting with quenched disorder. However, the depinning properties of colloid systems are important and still an open problem, needing further investigation. Motivated by the recent advance in colloidal experiment and simulation, we numerically investigate the depinning behaviour of colloidal systems. We will see that the colloid systems may exhibit a series of intriguing phenomena.

2. Model

Numerical simulations offer a unique window through which one may view the qualitative behaviour of the particles. In this paper, we investigate numerically the dynamics of 2D colloids on a disordered substrate in which the point-like pinning centres are randomly distributed. The colloids are simulated by Langevin dynamics, and the colloid motion is described by

$$\frac{\mathrm{d}R_i}{\mathrm{d}t} = -\sum_{i\neq j}\nabla_i U_{\text{c-c}}(R_i - R_j) - \sum_{j'}\nabla_i U_{\text{c-p}}(R_i - r_{j'}) + F_i^{\text{L}}(t) + F_{\text{d}} \quad (1)$$

with $R_{i,j}$ and $r_{j'}$ being the colloid and pin coordinates respectively. $U_{\text{c-c}}(R_i - R_j)$, $U_{\text{c-p}}(R_i - r_{j'})$, F_{d} are the colloid–colloid interaction potential, the colloid–pin interaction potential, and the external driving force, respectively. The Langevin force $F_i^{\text{L}}(t)$ describes the coupling with a heat bath and can be expressed by the correlation function:[25]

$$< F_{i\alpha}^{\text{L}}(t)F_{j\beta}^{\text{L}}(t') >= 2\eta T\delta_{ij}\delta_{\alpha\beta}\delta(t-t'). \quad (2)$$

The colloids interact through a screened Coulomb potential:[7,8,27]

$$U_{\text{c-c}}(R_i - R_j) = \frac{Q^2}{|R_i - R_j|}\exp(-\kappa|R_i - R_j|), \quad (3)$$

with Q the charge of colloidal particles, and $1/\kappa$ the screening length.

Different from Ref.[28], where the quenched disorder is modelled as randomly placed parabolic traps, we choose the colloid–pin interaction as a conventional attracting Gaussian potential:[25,26]

$$U_{\text{c-p}}(R_i - r_{j'}) = -A_p\exp\frac{|R_i - r_{j'}|^2}{r_p^2}, \quad (4)$$

with A_{p} being a constant.

All the lengths are measured with respect to the lattice constant a_0 of the ideal triangular lattice. We take $\kappa = 2/a_0$ and the scale for temperature is chosen as the 'bare' Kosterlitz–Thouless melting temperature T_{m0}.[27] The size of pinning centres r_{p} is chosen to be r_{p}=$0.2a_0$. The colloids are initially placed in a perfect triangular lattice subject to periodic triangular boundary conditions, and the point-like pinning centres are randomly distributed. The driving force F_{d} is increased from zero by small increments along the horizontal symmetry axis (x-axis) and the average colloid velocity $V_x = \frac{1}{N_{\text{c}}}\sum_{i=1}^{N_{\text{c}}} V_i\cdot\widehat{x}$ is measured at each increase. A time integration step of Δt=0.001 is used and averages are taken over 2×10^5 steps after 8×10^5 for equilibrium.

3. Numerical results and discussion

In Fig.1, we present a series of velocity force dependence (VFD) curves for different values of the strength of pinning centres (through changing the dimensionless prefactor A_{p}) at a fixed temperature (T/T_{m0}=0.1). One can see that there exists a critical driving force F_{c} in each VFD curve, below which the colloids are pinned. In addition, we find a remarkable change in the gross features of the VFD curves as A_{p} is varied. For a weak disorder with small values of A_{p} (A_{p} $\leq$0.18), the VFD response is basically linear above F_{c}. In this case, the colloid systems depin elastically and the motion of the colloid is homogeneous above F_{c}. Each colloid keeps the same neighbours as it moves. The trajectories are similar to that in the inset (a) of Fig.1 where we find a set of periodic elastic channels parallel to the driving force, consistent with recent simulations.[28] For a strong disorder with large values of A_{p} (A_{p} $\geq$0.20), plastic flow takes places above F_{c} and the VFD curves show a pronounced concave upward curvature, which is induced by defects in the flowing colloids. This is one of the typical characteristics of plastic depinning, like the vortex lattice in superconductors.[25] The flow pattern is shown in the inset (b) of Fig.1, where a fraction of colloids move in an intricate network of channels. In contrast to the elastic case, these channels are not

static but changeable over time. The flow consists of two components: unpinned individual colloids in the flow state and pinned colloids. The latter contributes to the flow through intermittent motion. In this case, the colloid medium is broken up by an external force and the transport is highly inhomogeneous.

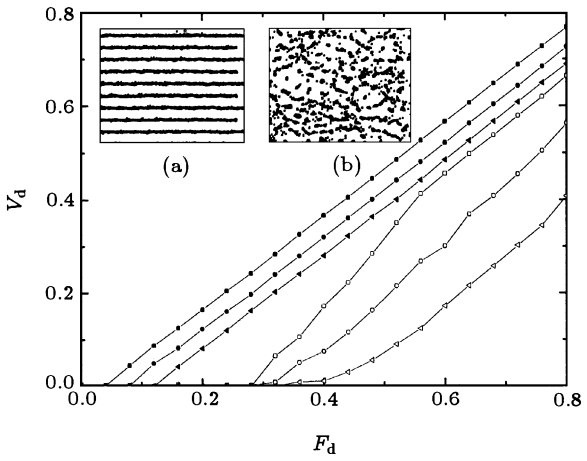


Fig.1. Series of VFD curves for different values of A_p (from left to right)=0.1, 0.15, 0.18, 0.20, 0.25, 0.30). Inset: colloid trajectories above depinning (F_d/F_c=1.2) for (a) A_p=0.15 and (b) A_p=0.5.

To further identify the elastic–plastic crossover, we now study the structure factor

$$S(k) = \left\langle \left| \frac{1}{N_c} \sum_i \exp[ik \cdot r_i(t)] \right|^2 \right\rangle$$

at depinning. Figure 2 shows $S(k)$ at different values of A_p. For a weak disorder with small values of A_p, the interaction between colloids dominates. Thus, the lattice is stiff and the hexagonal order is robust. In this case, the tearing and defect formation of the moving lattice may be ignored. One can see this point from the sixfold coordinated Bragg peaks of $S(k)$ in Fig.2 (a). With increasing strength of pinning centres, disorder increases and plays a dominant role when A_p is above 0.2. The lattice is torn up and the sixfold coordinated Bragg peaks in $S(k)$ disappear, as shown in Fig.2(b), where only a peak is found at the centre ($k_x = k_y$=0).

To characterize the order–disorder transition, we plot the averaged intensity of the Bragg peaks $S(G) = [S(G_1) + S(G_2) + S(G_3)]$ with different A_p, where $G_1 = (\pm 2\pi/a_0, 0)$, $G_2 = \pm 2\pi/a_0(1/2, \sqrt{3}/2)$ and $G_3 = \pm 2\pi/a_0(\sqrt{3}/2, 1/2)$ in Fig.3(a). The value of $S(G)$ can resemble the degree of order of the lattice.[29] We notice that the crossover from elastic to plastic depinning coincides with an abrupt decreasing of $S(G)$ at certain A_p ($A_p \approx$0.18), indicating the occurrence of a simultaneous transition from the order–disorder state. The order–disorder transition as a function of pinning strength has never been observed numerically in other systems, such as vortex matter.

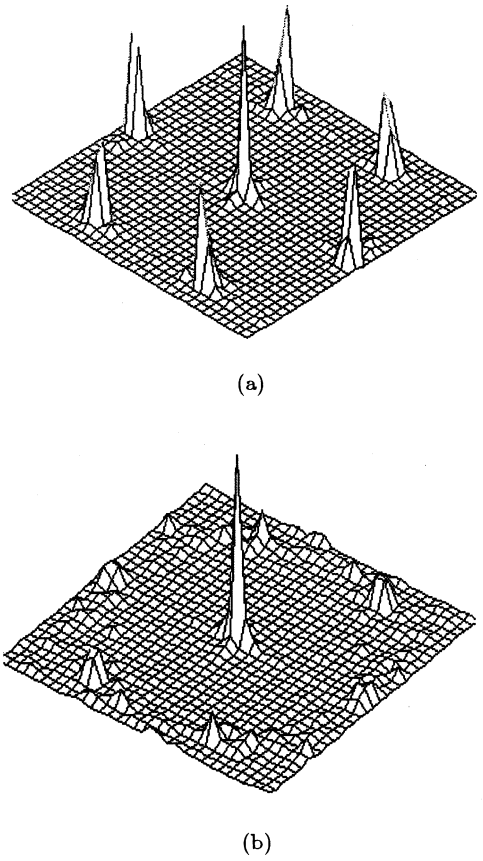


Fig.2. The static structure factor $S(k)$. (a) A_p=0.15 and (b) A_p=0.5.

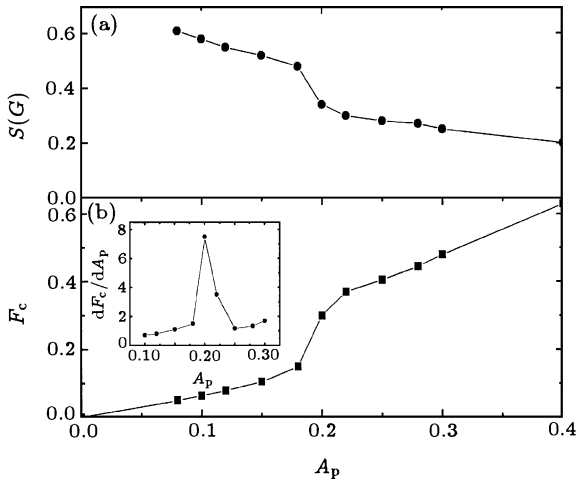


Fig.3. (a) Average intensity of Bragg peaks $S(G)$. (b) Depinning force vs pinning strength A_p. Inset: corresponding dF_c/dA_p.

An obvious feature in Fig.1 is that F_c increases with increasing A_p, and a substantial increase is found

in F_c within the crossover from elastic to plastic depinnings, as shown in Fig.3(b), where we present the F_c versus A_p curve. The corresponding differential curve is plotted in the inset of Fig.3(b) where we find a peak. Crossing over the peak, plastic flow takes place above F_c. These behaviours are similar to the vortex systems in superconductors.[13]

The systematic evolution of these VFD curves with increasing A_p is better illustrated by the differential curves dV_x/dF_d (proportional to the differential resistance, $R_d = dV/dI$) in Fig.4. Dramatic changes are observed in the curves. In the elastic regime, the VFD curve of colloids is linear and thus no peaks are found in dV_x/dF_d above F_c. However, once the system enters the plastic regime, the VFD is highly nonlinear and a large peak appears in the dV_x/dF_d slightly above F_c. The position of the peak moves to high driving forces as A_p is increased.

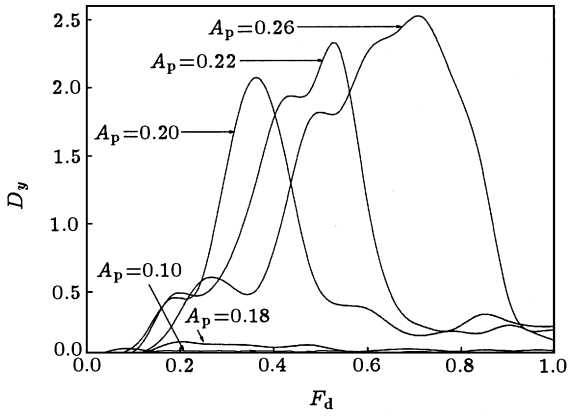


Fig.4. Evolution of the differential curve dV_x/dF_d with increasing the values of A_p (from left to right)=0.15, 0.18, 0.20, 0.22..

To understand the influence of disorder on the dynamic properties further, it is helpful to analyse the average quadratic displacement of colloids in the direction vertical to the driving force from their centre-of-mass position Y_{cm} as a function of time. We define $W_y = \frac{1}{N_c}\sum[\tilde{Y}_i(t) - \tilde{Y}_i(0)]^2$, where $\tilde{Y}_i(t) = Y_i(t) - Y_{cm}(t)$. In the plastic regime, the colloid motion is observed to be diffusive in the transverse direction and $W_y(t) \sim D_y t$, where D_y is the coefficient of diffusion. Figure 5 shows the curves of D_y versus F_d. For a weak disorder, the transverse diffusion is so small that it can be neglected (see the curves of A_p=0.1, 0.18), implying that the colloid directly enters into the moving Bragg phase above depinning (which can be demonstrated by structure factors in Fig.2, too). For a strong disorder, plastic flow occurs above F_c. Thus the transverse diffusion increases rapidly with increasing F_d. With further increasing of F_d, D_y decreases, indicating the occurrence of an order state.

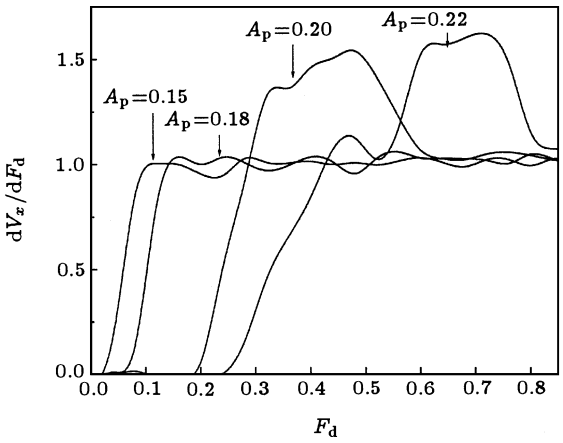


Fig.5. Diffusion coefficient of transverse motion D_y vs F_d.

Although we have given much information about colloidal dynamics, it is beyond the scope of this paper to present the dynamic phase diagram for a driven colloid with varying disorder strength. It also needs further study to clarify and identify the quality of the continuous phase transition of the creeping Bragg state to moving Bragg state in 2D colloid systems.

Strong metastability of the vortex matter, such as history-dependent vortex correlations in the pinned state and memory effects, illustrates the dominance of disorder [30] and is strong evidence of plastic flow. Due to the metastability of channels and the defects in moving colloids, the system becomes noisy and history-dependent, as shown in Fig.6. In contrast, no history-dependence is found in the elastic flow regime. For a weak disorder with small values of A_p, the VFD curves are linear down to F_c. The scaling fit can be obtained above F_c, which is a general feature of an elastic medium. For A_p=0.15, the apparent critical exponent is found to be ζ=1.10±0.01, as shown in the inset of Fig.6. This is close to that measured in 2D CDWs and vortex systems.[30] No scaling fit is found for a strong disorder, where plastic flow takes place.

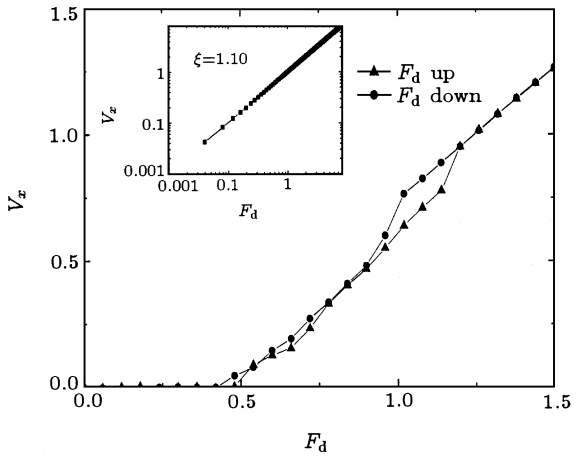


Fig.6. History dependence of the depinning processes in the plastic flow regime (A_p=0.28). Inset: scaling behaviour in the elastic regime. The line shows a scaling fit with the apparent critical exponent $\zeta = 1.1$.

4. Conclusion

In conclusion, we have investigated numerically the depinning dynamics of the driven 2D colloids subject to the randomly distributed point-like pining centres. With increasing strength of pinning, we find a crossover from elastic to plastic depinnings, accompanied by an order–disorder transition and a substantial increase in the depinning force. In the elastic regime, no peaks are found in the VFD differential curves and the transverse motion is almost none. In addition, the scaling relationship between velocity and force is found to be valid above depinning. However, when the system enters the plastic regime, a peak appears in the differential curves of VFD and transverse diffusion occurs above depinning. Furthermore, history dependence is found in the plastic regime. Our results are in good agreement with recent experiments.

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