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Dynamic scaling of migration-driven aggregate growth*

Ke Jian-Hong(柯见洪)†, Wang Xiang-Hong(王向红),
Lin Zhen-Quan(林振权), and Zhuang You-Yi(庄友谊)

School of Physics and Electronic Information, Wenzhou Normal College, Wenzhou 325027, China

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We study the kinetic behaviour of the growth of aggregates driven by reversible migration between any two aggregates. For a simple model with the migration rate $K(i;j) = K'(i;j) \propto i^u j^v$ at which the monomers migrate from the aggregates of size i to those of size j, we find that the aggregate size distribution in the system with $u + v \leq 3$ and $u < 2$ approaches a conventional scaling form, which reduces to the Smoluchovski form in the $u = 1$ case. On the other hand, for the system with $u < 2$, the average aggregate size $S(t)$ grows exponentially in the $u + v = 3$ case and as $(t \ln t)^{1/(5-2u)}$ in another special case of $v = u - 2$. Moreover, this typical size $S(t)$ grows as $t^{1/(3-u-v)}$ in the general $u - 2 < v < 3 - u$ case; while it always grows as $t^{1/(5-2u)}$ in the $v < u - 2$ case.

Keywords: kinetic behaviour, migration, aggregate growth, scaling law
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1. Introduction

Irreversible growth of aggregates is an important phenomenon in natural science.[1−6] The general mechanism for aggregate growth is a binary coalescence scheme, $A_i + A_j \to A_{i+j}$. Here, A_i denotes an aggregate consisting of i monomers, and two aggregates A_i and A_j can bond spontaneously to form a larger aggregate A_{i+j}. In the last few decades, many works have been devoted to the investigation of the kinetics of these aggregation processes and considerable understanding has been achieved.[7−11] On the other hand, there also exist some other classes of aggregate growth mechanisms in nature and in social science.[12−15] In order to study universal aggregation mechanisms, Ispolatov *et al*[16] introduced several asset exchange models for the evolution of the wealth distribution in economical interactions, and Leyvraz and Redner proposed a migration-driven aggregation model for the evolution of city populations.[17] In these models, the aggregates evolve according to an irreversible monomer migration scheme, $A_k + A_l \longrightarrow A_{k-1} + A_{l+1}$ $(k \leq l)$, and the results exhibit that the kinetics of such a system obeys a scaling law different from that for conventional aggregation processes. More generally, the migration direction may not depend on the relative sizes of the two aggregates. In our previous work, we proposed a migration-driven aggregation model with a special rate kernel, in which migration goes from the larger to the smaller aggregates as well as from the smaller to the larger, and it was found that this system evolves according to a scaling law different from the above-mentioned models with complete migration bias.[18]

In this work, we investigate the migration-driven aggregate growth model with a general size-dependent rate kernel. The system evolves according to the migration mechanism defined in Ref.[18], which is somewhat different from that proposed by Refs.[16,17]. The aggregate A_k of size k losing a monomer with a size-dependent rate $K(k;j)$ is described by the emigration scheme

$$A_k + A_j \stackrel{K(k;l)}{\longrightarrow} A_{k-1} + A_j^{+1},$$

while the aggregate A_l of size l gains a monomer with a rate $K'(l;j')$ according to the immigration scheme

$$A_l + A_{j'} \stackrel{K'(l;j')}{\longrightarrow} A_{l+1} + A_{j'}^{-1}.$$

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†E-mail: kejianhong@yahoo.com.cn

Here A_j^{+1} represents the A_j aggregate containing a 'positively charged' monomer and $A_{j'}^{-1}$ denotes the $A_{j'}$ aggregate with a 'negatively charged' monomer. Meanwhile, we assume that a couple of charged aggregates react spontaneously in the scheme $A_j^{+1} + A_{j'}^{-1} \xrightarrow{J(j;j')} A_j + A_{j'}$, with the rate $J(j;j') \to \infty$. The migration scheme of our model is thus equivalent to the catalysis-driven migration reaction

$$A_k + A_l \xrightarrow{A_j;A_{j'}} A_{k-1} + A_{l+1},$$

where the aggregates A_j and $A_{j'}$ play the roles analogous to the monomer sink and source, respectively.

We believe that our model may mimic some social and economical processes. For example, one person leaves his own company and temporarily joins another one, so the latter company contains a 'positively charged' staff member, while a company that is deprived of a staff member by another one leaves an unoccupied position. On the other hand, if possible, the temporary person in the 'positively charged' company may easily take over the vacant position in the 'negatively charged' one. This phenomenon may occur frequently in a commercially developed area. It is also believed that the migration-driven aggregation processes are of interest in studying the scaling properties of their evolution behaviours. According to this work, it was found that the evolution of the system falls in a conventional scaling regime and the aggregate size distribution approaches a scaling form quite different from those found in Refs. [16,17]. The results also exhibited that in some special cases, the conventional scaling description of the aggregate size distribution may reduce to the well-known Smoluchovski scaling form.

This paper is organized as follows. In Sec.2, we study a migration-driven aggregate growth model with a general rate kernel and obtain the asymptotic solution of the aggregate size distribution by means of mean-field rate equation. We also propose a generalized scaling form to describe the evolution characteristic of the aggregates. Finally, a brief summary is given in Sec.3.

2. The model of migration-driven aggregate growth

We assume that the system has a spatial homogeneity, so that the fluctuations in densities of the reactants are ignored and the aggregates are considered to be homogeneously distributed in the space throughout the process. Thus, the theoretical approach to investigate the kinetics of aggregate growth can be based on the mean-field rate equation, which assumes that the reaction proceeds at a rate proportional to the reactant concentrations. Let $a_k(t)$ be the concentration of the A_k aggregates of size k at time t. By generalizing the rate equation of the migration-driven aggregation model given by Leyvraz *et al*,[17] we write the corresponding rate equation for our system as follows:

$$\begin{aligned}\frac{\mathrm{d}a_k}{\mathrm{d}t} =& \sum_{j=1}^{\infty} K(k+1;j)a_{k+1}a_j \\ &+ \sum_{j=1}^{\infty} K'(k-1;j)a_{k-1}a_j \\ &- \sum_{j=1}^{\infty} [K(k;j) + K'(k;j)]a_k a_j, \end{aligned} \tag{1}$$

where we impose the boundary condition as $a_0(t) \equiv 0$.

For simplicity, we consider a model with a symmetrical reaction rate kernel. The rate of the A_k aggregate gaining or losing one monomer is directly proportional to k^u of itself and to j^v of the opposite aggregate A_j, i.e. $K(k;j) = K'(k;j) = Ik^u j^v$ (I is a constant). Here, u and v are the migration rate indices which may interpret the degree of 'richness' in the migration of an aggregate. When the values of u and v increase, the aggregate becomes much more generous in emigration and much greedier in immigration. This restriction of the rate kernel can follow the conservation law of the total size of the aggregates. It is also natural that the overall migration rate varies as a power law in some scale factor.[17] On the other hand, the resulting model can be explicitly solved and this provides a helpful understanding of the reversible migration mechanism. Then Eq.(1) reduces to

$$\frac{\mathrm{d}a_k}{\mathrm{d}t} = IM_v[(k+1)^u a_{k+1} + (k-1)^u a_{k-1} - 2k^u a_k], \tag{2}$$

where $M_v(t) = \sum_{j=1}^{\infty} j^v a_j(t)$. By summing up Eq.(2) we find $\dot{M}_1(t) = \sum_{k=1}^{\infty} k\dot{a}_k(t) = 0$, which indicates that the total size of the aggregates obeys the conservation law. This is a natural characteristic for our migration-driven aggregation system without any annihilation of the aggregates.

By defining a modified time variable $T(t)$,

$$T(t) = I \int_0^t \mathrm{d}t' M_v(t'), \tag{3}$$

we recast Eq.(2) to

$$\frac{\mathrm{d}a_k(T)}{\mathrm{d}T} = (k+1)^u a_{k+1}(T) + (k-1)^u a_{k-1}(T) - 2k^u a_k(T), \quad (4)$$

with the boundary condition $a_0(T) \equiv 0$. From Eq.(4) we find that $M_1(T)$ is conserved, namely, $\mathrm{d}M_1(T)/\mathrm{d}T = 0$. Now, the problem is reduced to determining the solution of the aggregate size distribution $a_k(T)$ from Eq.(4). We first investigate several simple cases in which the integral index u can be explicitly solved.

When $u = 0$, Eq.(4) reduces to the simple discrete diffusion equation

$$\frac{\mathrm{d}a_k(T)}{\mathrm{d}T} = a_{k+1}(T) + a_{k-1}(T) - 2a_k(T). \quad (5)$$

Equation (5) can be readily solved under arbitrary initial conditions.[19] Here, we consider a simple but important case in which there only exist monomer aggregates at $t = 0$ and their concentration is equal to unity. Then the initial condition is $a_k(0) = \delta_{k1}$, $k = 1, 2, 3, \ldots$ Under the monodisperse initial condition, from Eq.(5) one can obtain the solution

$$a_k(T) = \mathrm{e}^{-2T}[I_{k-1}(2T) - I_{k+1}(2T)], \quad (6)$$

where I_n denotes the modified Bessel function of order n.[20] In the long-time limit, Eq.(6) can be asymptotically rewritten as

$$a_k(T) \simeq \frac{k}{\sqrt{4\pi T^3}} \exp\left(-\frac{k^2}{4T}\right). \quad (7)$$

This result is similar to that for the random asset exchange model in Ref.[15]. Equation (7) shows that the expression of the aggregate size distribution $a_k(T)$ may be written in the conventional scaling form [7]

$$a_k(T) \simeq k^{-\tau} T^{-w} \Phi[k/S(T)], \quad S(T) \propto T^z, \quad (8)$$

where $S(T)$ denotes the system's typical aggregate size which plays a role analogous to that of the correlation length in critical phenomena and the scaling function $\Phi(x) \simeq 1$ for $x \ll 1$ and $\Phi(x) \simeq 0$ for $x \gg 1$. In this case, the scaling function takes a stretched exponential expression, $\Phi(x) = \exp(-x^2)$, and the typical aggregate size grows as $T^{1/2}$. Moreover, the governing exponents for this case are $\tau = -1$, $w = 3/2$ and $z = 1/2$. On the other hand, each $a_k(T)$ continuously decreases to zero at the end.

We then discuss the $u = 1$ case. Equation (4) reduces to

$$\frac{\mathrm{d}a_k(T)}{\mathrm{d}T} = (k+1)a_{k+1}(T) + (k-1)a_{k-1}(T) - 2ka_k(T). \quad (9)$$

This system has been investigated by Ref.[18]. Here, we simply recover the results by using another mathematical method. Under the unitary monodisperse initial condition, Eq.(9) can be solved with the help of ansatz [21]

$$a_k(T) = A(T)[a(T)]^{k-1}, \quad (10)$$

under the initial condition $a(0) = 0$ and $A(0) = 1$. Substituting Eq.(10) into Eq.(9), we obtain

$$\frac{\mathrm{d}a(T)}{\mathrm{d}T} = [1 - a(T)]^2,$$
$$\frac{\mathrm{d}A(T)}{\mathrm{d}T} = -2A(T)[1 - a(T)]. \quad (11)$$

Equations (11) can be easily solved to yield the exact solutions

$$a(T) = 1 - (T+1)^{-1}, \quad A(T) = (T+1)^{-2}. \quad (12)$$

Thus we obtain the exact expression of $a_k(T)$ as follows:

$$a_k(T) = (T+1)^{-2}[1 - (T+1)^{-1}]^{k-1}. \quad (13)$$

In the region of $k \gg 1$ and $T \gg 1$, the aggregate size distribution $a_k(T)$ approaches the well-known Smoluchovski scaling form (see, e.g., Refs.[1,21–23])

$$a_k(T) \simeq T^{-2}\Phi[k/S(T)], \quad S(T) \propto T, \quad (14)$$

with the scaling function $\Phi(x) = \exp(-x)$. It is obvious that Eq.(14) is a reduced form of Eq.(8).

Now we investigate the general cases. Under the monodisperse initial condition, we generalize the asymptotic solutions (7) and (14) of $a_k(T)$ in the above-mentioned special cases and, therefore, make the conventional scaling ansatz (8) for the large-time behaviour of $a_k(T)$ in general cases. Since our system obeys the mass conservation law, one can easily find the following exponent relation in the scaling form (8):

$$w = (2 - \tau)z. \quad (15)$$

Equation (15) implies that for the system in which the total mass is conserved, the scaling ansatz (8) is valid only in the case of $\tau < 2$. On the other hand, if $\tau \geq 2$, it follows from the mass conservation that the exponent w in Eq.(8) is $w \equiv 0$; however, this can never be the case unless the system gels because the total density of aggregates in the nongelling system always

decreases with time. Here we only consider the non-gelling case, i.e. $\tau < 2$. Inserting Eq.(15) into Eq.(8), we then rewrite the scaling ansatz as follows:

$$a_k(T) \simeq k^{-\tau}[S(T)]^{\tau-2}\Phi[k/S(T)], \quad S(T) \propto T^z. \tag{16}$$

Then what we shall do is to find the index τ, the typical aggregate size $S(T)$ and the scaling function $\Phi(x)$.

We first determine the typical aggregate size $S(T)$. By making use of Eq.(16) one can find $M_2(T) \simeq S(T)$. Multiplying Eq.(4) by k^2 and summing over k, we obtain

$$\frac{\mathrm{d}M_2(T)}{\mathrm{d}T} = 2M_u(T). \tag{17}$$

It is not difficult to find that for the scaling function $\Phi(x)$ defined in Eq.(8), $\sum_{j=1}^{\infty} j^{\nu}\Phi[j/S(t)]$ is approximately equal to $C_{1\nu}[S(t)]^{\nu+1}$ for $\nu > -1$, $C_{2\nu}\ln[S(t)]$ for $\nu = -1$ or $C_{3\nu}$ for $\nu < -1$ (here $C_{1\nu}$, $C_{2\nu}$ and $C_{3\nu}$ are integration constants dependent on the value of the index ν). By substituting Eq.(16) into Eq.(17) we find

$$\frac{\mathrm{d}S(T)}{\mathrm{d}T} \simeq \begin{cases} [S(T)]^{u-1} & \text{if } \tau < u+1, \\ [S(T)]^{\tau-2}\ln S(T) & \text{if } \tau = u+1, \\ [S(T)]^{\tau-2} & \text{if } \tau > u+1. \end{cases} \tag{18}$$

On the other hand, we sum up Eq.(4) and then obtain the evolution behaviour of $M_0(t)$ as follows:

$$\frac{\mathrm{d}M_0(T)}{\mathrm{d}T} = -a_1(T). \tag{19}$$

Inserting Eq.(16) into Eq.(19) yields another set of evolution equations of $\dot{S}(T)$ in different cases,

$$\frac{\mathrm{d}S(T)}{\mathrm{d}T} \simeq \begin{cases} [S(T)]^{\tau} & \text{if } \tau < 1, \\ S(T)/\ln S(T) & \text{if } \tau = 1, \\ S(T) & \text{if } \tau > 1. \end{cases} \tag{20}$$

We then make a trivial comparison between Eqs.(18) and (20). It is found that the asymptotic behaviour (18) (or equivalently, Eq.(20)) of $\dot{S}(T)$ is only valid in the first case of $\tau < u+1$ (or $\tau < 1$) and $\tau = u-1$.

From Eq.(18) we easily derive the asymptotic solution of the typical size $S(T)$ as follows:

$$S(T) \simeq [(2-u)T]^{1/(2-u)}, \tag{21}$$

which is valid only in the case of $u < 2$. In the $u > 2$ case, from Eq.(18) we find that the typical size $S(T)$ grows only at finite time $T < T_{\mathrm{c}}$ (T_{c} is the critical time of the gel point) and the scaling form of Eq.(16) is thus invalid; hence, the system may undergo a gelation-like transition after a sufficiently long time (see, e.g., Refs.[24–26]). As for the borderline case of $u = 2$, we have the relation $\tau = u - 1$ so that we have $\tau = 1$, and thus the conventional scaling form (16) of the aggregate size distribution breaks down. One may modify the scaling ansatz (8) as $a_k(T) \simeq k^{-\tau}T^{-w}[\ln S(T)]^{-1}\Phi[k/S(T)]$, with $S(T) \propto T^z$. By employing the above-mentioned technique we find that for the $u = 2$ case, the typical size $S(T)$ grows exponentially, i.e. $S(T) \simeq \exp(T)$. In this work, we only focus on the more interesting case of $u < 2$, in which the aggregate size distribution infinitely evolves according to a conventional scaling regime.

Now we turn to determining the scaling function $\Phi(x)$. Substituting the scaling ansatz (16) into Eq.(4), we find the following differential equation of $\Phi(x)$ for the A_k aggregate with size $k \gg 1$:

$$x\Phi''(x) + (2 + px^{2-u})\Phi'(x) \simeq p(u-3)x^{1-u}\Phi(x), \tag{22}$$

where p is such a separation constant for the x and T dependence that $[S(T)]^{1-u}\dot{S}(T) = p$. Equation (22) can be solved to yield

$$\Phi(x) \simeq C_1\exp(-C_2x^{2-u}), \tag{23}$$

where C_1 is an integration constant and $C_2 = p/(2-u)$. Since our system obeys the mass conservation law, C_1 and p are related as

$$\int_0^{\infty} \mathrm{d}x\{C_1x^{2-u}\exp[-px^{2-u}/(2-u)]\} = 1.$$

For simplicity, one may choose the constant p so that $C_1 = 1$, and it is thus verified that $\Phi(x) \simeq 1$ for $x \ll 1$ and $\Phi(x) \simeq 0$ for $x \gg 1$.

In order to determine the explicit t-dependent solution of the aggregation size distribution, we devote our attention to the important case of $x \ll 1$ and $k \gg 1$. Based on the above analyses, one can find that $a_k(T)$ approaches the following scaling form:

$$a_k(T) \simeq k^{1-u}[(2-u)T)]^{-(3-u)/(2-u)}\exp(-C_2x^{2-u}), \quad x \propto kT^{-1/(2-u)}. \tag{24}$$

We can then use the scaling form (24) to approximately estimate $M_v(T)$ in the $u < 2$ case,

$$M_v(T)=\sum_{k=1}^{\infty}k^v a_k(T)\simeq\begin{cases}T^{(v-1)/(2-u)} & \text{if } v>u-2,\\ T^{(u-3)/(2-u)}\ln T & \text{if } v=u-2,\\ T^{(u-3)/(2-u)} & \text{if } v<u-2.\end{cases}\tag{25}$$

Inserting Eq.(25) into Eq.(3), we then obtain the asymptotic expression of the modified time variable $T(t)$ as follows:

$$T(t)\sim\begin{cases}\exp(C_3t) & \text{if } v=3-u,\\ t^{(2-u)/(3-u-v)} & \text{if } u-2<v<3-u,\\ (t\ln t)^{(2-u)/(5-2u)} & \text{if } v=u-2,\\ t^{(2-u)/(5-2u)} & \text{if } v<u-2,\end{cases}\tag{26}$$

where C_3 is a constant dependent on the details of the migration kernel. On the other hand, from Eqs.(3) and (25) we cannot deduce the infinite t-dependent solution of $T(t)$ if $u+v>3$. So we conclude that for our model the system may undergo the gelation-like transition in both the $u+v>3$ and $u>2$ cases.

Thus we find that in the system with $u+v\leq 3$ and $u<2$, the size distribution of the aggregates $a_k(t)$ with $k\gg 1$ approaches the conventional scaling form

$$a_k(t)\sim\begin{cases}k^{1-u}e^{-(3-u)C_3t/(2-u)}\exp(-C_2x_1^{2-u}) & \text{if } v=3-u,\\ k^{1-u}t^{-(3-u)/(3-u-v)}\exp(-C_2x_2^{2-u}) & \text{if } u-2<v<3-u,\\ k^{1-u}(t\ln t)^{-(3-u)/(5-2u)}\exp(-C_2x_3^{2-u}) & \text{if } v=u-2,\\ k^{1-u}t^{-(3-u)/(5-2u)}\exp(-C_2x_4^{2-u}) & \text{if } v<u-2,\end{cases}\tag{27}$$

with the different scaling variables $x_1\propto ke^{-C_3t/(2-u)}$, $x_2\propto kt^{-1/(3-u-v)}$, $x_3\propto k(t\ln t)^{-1/(5-2u)}$ and $x_4\propto kt^{-1/(5-2u)}$. It is also found that the typical size $S(t)$ grows as $t^{1/(3-u-v)}$ in the general $u-2<v<3-u$ case; while it grows as $\exp[C_3t/(2-u)]$ in the $u+v=3$ case and as $(t\ln t)^{1/(5-2u)}$ in the $v=u-2$ case. The most interesting results in this model is that the typical size $S(t)$ grows as $t^{1/(5-2u)}$ in the $v<u-2$ cases, which is independent of the index v. Moreover, it is worth noting that the index u of the priority aggregate (which gains or loses a monomer on its own initiative) plays a more important role in the scaling behaviour of the system than the index v. On the other hand, Eq.(27) also shows that in the general case, the evolution behaviour of our migration-driven aggregation system obeys a scaling law quite different from the generalized Smoluchovski form $a_k(t)\simeq t^{-w}\exp(-k/t^z)$ abided by the irreversible self-coagulation models (see, e.g., Refs.[1,21,23]).

3. Summary

We have proposed a general migration-driven aggregation model with a rate kernel $K(k;j)=K'(k;j)=Ik^uj^v$. Based on the mean-field theory, we analysed the evolution behaviour of the aggregate size distribution in several cases with varying indices u and v. The results showed that in most cases with $u<2$ and $u+v\leq 3$, the aggregate size distribution always approaches a conventional scaling law, while the system may undergo the gelation-like transition both in the $u>2$ case and in the $u+v>3$ case. In particular, the aggregate size distribution satisfies the exponential-correction conventional scaling form in the case of $u+v=3$ and $u<2$, while the evolution behaviour of the system evolves according to the logarithm-correction scaling law in the case of $v=u-2$ and $u<2$. Moreover, the conventional scaling description of the aggregate size distribution may reduce to the Smoluchovski form in the $u=1$ case.

On the other hand, for the system with $u<2$, the typical average size $S(t)$ grows exponentially in the $u+v=3$ case and as $(t\ln t)^{1/(5-2u)}$ in the $v=u-2$ case; while $S(t)$ grows as $t^{1/(3-u-v)}$ in the general case of $u-2<v<3-u$. The most interesting result is that if $v<u-2$ and $u<2$, the typical size $S(t)$ always grows as $t^{1/(5-2u)}$ and its increasing rate is independent of the index v. This model is expected to provide some predictions for the distribution of city populations as well as the evolution of the wealth distribution in economical activities.

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