Vol 13 No 5, May 2004
1009-1963/2004/13(05)/0765-07

Chinese Physics

© 2004 Chin. Phys. Soc.
and IOP Publishing Ltd

Dynamics of aggregation–annihilation process with cluster removals*

Wang Xiang-Hong(王向红)†, Ke Jian-Hong(柯见洪), and Lin Zhen-Quan(林振权)

School of Physics and Electronic Information, Wenzhou Normal College, Wenzhou 325027, China

(Received 27 October 2003; revised manuscript received 20 December 2003)

We have studied the kinetic behaviours of irreversible aggregation–annihilation models with cluster removals. In the models, an irreversible aggregation reaction occurs between any two clusters of the same species and an irreversible annihilation reaction occurs simultaneously between two different species; meanwhile, the clusters of large size are gradually removed from the system. In a mean-field limit, we obtain the general solutions of the cluster-mass distributions for the cases with an arbitrary removal probability. We found that the cluster-mass distribution of either species satisfies a generalized or modified scaling form. The results also indicate that the evolution behaviours of the systems depend strongly on the details of the reaction events.

Keywords: scaling behaviour, aggregation–annihilation process, cluster removal
PACC: 8220, 2460, 0570L

1. Introduction

In the past few decades, much interest has been focused on the irreversible aggregation and annihilation system, which may unfold nonclassical kinetic behaviours.[1−7] Recently, many works have been devoted to the understanding of the competition between aggregation and annihilation processes.[8−16] Krapivsky studied the kinetics of an irreversible two-species aggregation model with partial annihilation.[8−9] Zhang and Yang[3,14] generalized the two-species system to the multi-species case and analysed the evolution behaviour of the cluster-mass distribution in some symmetrical cases. Ben-Naim and Krapivsky proposed an irreversible aggregation model with complete annihilation and found the scaling description of the cluster-mass distribution for some special cases.[11] We then probed into the kinetic behaviour of the irreversible aggregation system with complete annihilation in general cases.[15,16] All these works show that the cluster-mass distribution in the irreversible aggregation–annihilation system may also satisfy the conventional scaling form at large times. Moreover, there are also a few works concerning the aggregation process with input and/or removal term. Practically, the input and removal processes are of practical significance in real systems. For example, in the stirred tank reactors for aerosols, the large particles are removed by outflow from the chemical chamber. On the other hand, these processes are also of theoretical interest. It was found that in the presence of sinks, the coagulation system usually evolves to a steady state and never reaches the gelation transition.[17,18] Racz[9] and Vicsek *et al*[20] investigated the aggregation process in the presence of sources and sinks and developed a scaling theory for the evolution of cluster densities. In our previous work, we investigated the kinetic behaviour of the aggregation–annihilation model with input term and found that the system possesses nonuniversal scaling properties.[21] We also found that the scaling description of the cluster-mass distribution may break down in an irreversible aggregation system with a removal term.[22] However, as far as we know the aggregation–annihilation processes with sinks remain unexplored.

In this work, we investigate the kinetics of irreversible aggregation–annihilation processes in the presence of cluster removals. There are two types of species in the system, A and B. Irreversible aggregations occur only between two clusters of the same

*Project supported by the National Natural Science Foundation of China (Grant Nos 10305009, 10275048 and 10175008) and by the Natural Science Foundation of Zhejiang Province, China (Grant Nos 102067 and 101002).
†E-mail: wangxh@wznc.zj.cn

species, $A_i + A_j \xrightarrow{I_A(i,j)} A_{i+j}$ and $B_i + B_j \xrightarrow{I_B(i,j)} B_{i+j}$; meanwhile, irreversible annihilation reactions occur between two clusters of different species, $A_i + B_j \xrightarrow{J(i,j)}$ inert product (a complete annihilation scheme) or $A_i + B_j \xrightarrow{J(i,j)} A_{i-j}(i \geq j)/B_{j-i}(j > i)$ (a partial annihilation scheme). Here A_i denotes a cluster consisting of i monomers of species A, and similarly for B_i. The rate of the aggregation between the A_i and A_j clusters is equal to $I_A(i,j)$ and that between the B_i and B_j clusters is $I_B(i,j)$. The rate of the annihilation between the A_i and B_j clusters is equal to $J(i,j)$. Meanwhile, the removal probabilities of the A_k and B_k clusters are denoted by $P_A(k;t)$ and $P_B(k;t)$, respectively.

Our investigation is based on the mean-field theory, which assumes that the reaction proceeds at a rate proportional to the reactant concentrations. Thus the mean-field approximation neglects the spatial fluctuations of the reactant densities and, therefore, applies to the case of the spatial dimension d of the system greater than or equal to a critical dimension d_c. For the $d < d_c$ system, the fluctuations in the densities of the reactants may lead to a dimension-dependent kinetic behaviour in the long-time limit. It is found that for the irreversible two-species aggregation–annihilation system, $d_c = 2$.[10,11] For simplicity, we assume that the spatial dimension d of our system is greater than 2. By solving the mean-field rate equations, we have derived the general asymptotic solutions of the cluster-mass distributions for arbitrary removal probabilities. The results show that the evolution behaviour of either species may satisfy a generalized or modified scaling law.

This paper is organized as follows. In Sec.2, we consider an irreversible two-species aggregation and complete annihilation model with cluster removals. We derive the asymptotic solutions of cluster-mass distributions for different cases and then discuss the evolution properties of the system. In Sec.3, we study the kinetics of the irreversible aggregation and partial annihilation process with removal term. A brief summary is given in Sec.4.

2. Model of irreversible aggregation and complete annihilation process with cluster removals

In our investigation, the theoretical approach to this model is based on the mean-field rate equations. We first investigate the system with complete annihilation. The concentrations of A_k and B_k clusters of k-mers are denoted as a_k and b_k, respectively. We consider a simple model with a constant reaction rate kernel. The annihilation rates $J(i,j)$ are equal to a constant J, and the aggregation rates $I_A(i,j)$ of species A are equal to I_1, and those of $I_B(i,j)$ of species B equal to I_2. Without any loss of generality, we assume $I_1 \geq I_2$. The removal probabilities $P_A(k;t)$ and $P_B(k;t)$ are the same size-independent function $P(t)$ $(P(t) > 0)$, which may be defined by the practical process. By generalizing the rate equations of the aggregation–annihilation process given by Ben-Naim and Krapivsky [11] the corresponding rate equations for our system read as

$$\begin{aligned}\frac{\mathrm{d}a_k}{\mathrm{d}t} =& I_1 \left(\frac{1}{2} \sum_{i+j=k} a_i a_j - a_k \sum_{j=1}^{\infty} a_j \right) \\ & - J a_k \sum_{j=1}^{\infty} b_j - P a_k, \\ \frac{\mathrm{d}b_k}{\mathrm{d}t} =& I_2 \left(\frac{1}{2} \sum_{i+j=k} b_i b_j - b_k \sum_{j=1}^{\infty} b_j \right) \\ & - J b_k \sum_{j=1}^{\infty} a_j - P b_k. \end{aligned} \tag{1}$$

Consider the monodisperse initial case, i.e. $a_k(0) = A_0 \delta_{k1}$ and $b_k(0) = B_0 \delta_{k1}$. With the help of ansatz [8]

$$a_k(t) = A(t)[a(t)]^{k-1}, \quad b_k(t) = B(t)[b(t)]^{k-1}, \tag{2}$$

we transform the rate equations (1) into the following differential equations:

$$\begin{aligned}\frac{\mathrm{d}a}{\mathrm{d}t} &= \frac{I_1 A}{2}, \quad \frac{\mathrm{d}A}{\mathrm{d}t} = -\frac{I_1 A^2}{1-a} - J\frac{AB}{1-b} - PA, \\ \frac{\mathrm{d}b}{\mathrm{d}t} &= \frac{I_2 B}{2}, \quad \frac{\mathrm{d}B}{\mathrm{d}t} = -\frac{I_2 B^2}{1-b} - J\frac{AB}{1-a} - PB, \end{aligned} \tag{3}$$

with the initial values

$$\begin{aligned} & a = 0, \quad b = 0, \\ & A = A_0, \quad B = B_0, \quad P = P_0 \quad \text{at} \quad t = 0. \end{aligned} \tag{4}$$

Introducing two new variables,

$$\alpha = (1-a)^{-1}, \quad \beta = (1-b)^{-1}, \tag{5}$$

we recast the differential equations (3) to the following equations:

$$\begin{aligned}\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} &= -\frac{2J}{I_2}\frac{1}{\beta}\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\mathrm{d}\beta}{\mathrm{d}t} - P\frac{\mathrm{d}\alpha}{\mathrm{d}t}, \\ \frac{\mathrm{d}^2\beta}{\mathrm{d}t^2} &= -\frac{2J}{I_1}\frac{1}{\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\mathrm{d}\beta}{\mathrm{d}t} - P\frac{\mathrm{d}\beta}{\mathrm{d}t}, \end{aligned} \tag{6}$$

with the corresponding initial conditions

$$\alpha = 1, \quad \beta = 1,$$
$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{I_1 A_0}{2}, \quad \frac{\mathrm{d}\beta}{\mathrm{d}t} = \frac{I_2 B_0}{2} \quad \text{at} \quad t = 0. \tag{7}$$

From Eqs.(6) one can easily obtain

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{I_1 A_0}{2} E\beta^{-2J/I_2},$$
$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = \frac{I_2 B_0}{2} E\alpha^{-2J/I_1}, \tag{8}$$

where the shorthand notation

$$E(t) \equiv \exp\left[-\int_0^t P(t')\mathrm{d}t'\right].$$

Introducing a new time variable

$$\tau(t) = \int_0^t \mathrm{d}t' \exp\left[-\int_0^{t'} P(t'')\mathrm{d}t''\right],$$

one can recast Eqs.(8) to the following equations:

$$\left(\frac{\mathrm{d}\alpha}{\mathrm{d}\tau}\right)^{-I_2/2J} \frac{\mathrm{d}^2\alpha}{\mathrm{d}\tau^2} = -JB_0 \left(\frac{I_1 A_0}{2}\right)^{-I_2/2J} \alpha^{-2J/I_1} \frac{\mathrm{d}\alpha}{\mathrm{d}\tau},$$
$$\left(\frac{\mathrm{d}\beta}{\mathrm{d}\tau}\right)^{-I_1/2J} \frac{\mathrm{d}^2\beta}{\mathrm{d}\tau^2} = -JA_0 \left(\frac{I_2 B_0}{2}\right)^{-I_1/2J} \beta^{-2J/I_2} \frac{\mathrm{d}\beta}{\mathrm{d}\tau}. \tag{9}$$

It is reasonable to assume that the system reaches its steady state as $t \to \infty$ and the steady state conditions are given as follows:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{1}{\alpha^2}\frac{\mathrm{d}\alpha}{\mathrm{d}t} = 0,$$
$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{2}{I_1\alpha^2}\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} - \frac{4}{I_1\alpha^3}\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 = 0,$$
$$\frac{\mathrm{d}b}{\mathrm{d}t} = \frac{1}{\beta^2}\frac{\mathrm{d}\beta}{\mathrm{d}t} = 0,$$
$$\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{2}{I_2\beta^2}\frac{\mathrm{d}^2\beta}{\mathrm{d}t^2} - \frac{4}{I_2\beta^3}\left(\frac{\mathrm{d}\beta}{\mathrm{d}t}\right)^2 = 0. \tag{10}$$

It can be concluded from Eqs.(10) that we must have either α $(\beta) \to \infty$ or $\mathrm{d}\alpha/\mathrm{d}t$, $(\mathrm{d}\beta/\mathrm{d}t) \to 0$ as $\to \infty$.

When $\tau(t) \to \tau_0$ (τ_0 is a finite constant) as $t \to \infty$, from Eqs.(9) and (10) one can find that $\alpha(\infty)$ and $\beta(\infty)$ are two finite constants. This case also implies that the function $E(t)$ decreases with time and its decay rate is faster than that of t^{-1}. In the long-time limit, from Eqs.(9) we derive the asymptotic solutions

$$\alpha(t) \simeq C_1\tau(t), \quad \beta(t) \simeq C_2\tau(t), \tag{11}$$

where

$$C_1 = (I_1 A_0/2)\beta_1^{-2J/I_2}$$

and

$$C_2 = (I_2 B_0/2)\alpha_1^{-2J/I_1}.$$

The asymptotic solutions of $a_k(t)$ and $b_k(t)$ are then obtained as follows:

$$a_k(t) \sim [1 - (C_1\tau_0)^{-1}]^k E(t)\exp(-x),$$
$$b_k(t) \sim [1 - (C_2\tau_0)^{-1}]^k E(t)\exp(-x), \tag{12}$$

where $x \propto k\int_t^\infty E(t)\mathrm{d}t$. Equations (12) indicate that for the monodisperse initial case the cluster-mass distribution of either species falls in a scaling regime. In general irreversible aggregation processes, the average mass of the clusters increases indefinitely with time. Thus one may assume that this typical mass plays a role analogous to that of the correlation length in ordinary critical phenomena. Introducing such a characteristic mass $S(t)$, one can write the cluster-mass distribution in the following conventional scaling form:[8]

$$a_k(t) \simeq t^{-w}\Phi[k/S(t)], \quad S(t) \propto t^z. \tag{13}$$

It is found that for this case, both species do not scale according to the conventional form of Eq.(13) but their cluster-mass distributions satisfy the modified scaling form $a_k(t) \simeq h^k F(t)\,\Phi[k/S(t)]$ (here h denotes a constant and $0 < h < 1$). The modified scaling form indicates that two different mass scales, a growing scale and a time-independent scale, are associated with either species. Both species have the same growing scale $S(t) \propto \left(\int_t^\infty E(t')\mathrm{d}t'\right)^{-1}$, which is forced by the removal process. The time-independent scale for species A is $S_A = C_1\tau_0$ and that for species B is $S_B = C_2\tau_0$, which will respectively dominate the asymptotic behaviours of species A and B in the long-time limit. It is also instructive to determine the total mass densities. By summing up Eqs.(12) we find that in the long-time limit,

$$M_A(t) = \sum_{k=1}^\infty k a_k(t) \propto E(t),$$
$$M_B(t) = \sum_{k=1}^\infty k b_k(t) \propto E(t). \tag{14}$$

Equations (14) imply that both species cannot survive at the end.

When $\tau \to \infty$ as $t \to \infty$, Eqs.(9) are similar to the governing differential equations (11) in Ref.[23], except for some trivial difference in the algebraic denotations. Employing the technique introduced in Ref.[23], we obtain the scaling solutions of the cluster-mass distributions in the following different cases.

2.1. $I_1 > 2J$ and $I_2 > 2J$ case

From Eqs.(9) one can easily obtain the following equations:

$$\begin{aligned} \frac{\mathrm{d}\alpha}{\mathrm{d}\tau} &= (C_2\alpha^{1-2J/I_1} + C_3)^{-2J/(I_2-2J)}, \\ \frac{\mathrm{d}\beta}{\mathrm{d}\tau} &= (C_4\beta^{1-2J/I_2} + C_5)^{-2J/(I_1-2J)}, \end{aligned} \tag{15}$$

where

$$\begin{aligned} C_2 =& [(I_2-2J)I_1B_0/2(I_1-2J)](I_1A_0/2)^{-I_2/2J}, \\ C_3 =& (I_1/2)(I_1A_0/2)^{-I_2/2J} \\ & \times [A_0 - B_0(I_2-2J)/(I_1-2J)], \\ C_4 =& [(I_1-2J)I_2A_0/2(I_2-2J)](I_2B_0/2)^{-I_1/2J} \end{aligned}$$

and

$$\begin{aligned} C_5 =& (I_2/2)(I_2B_0/2)^{-I_1/2J} \\ & \times [B_0 - A_0(I_1-2J)/(I_2-2J)]. \end{aligned}$$

From Eqs.(15) we know that both $\alpha \to \infty$ and $\beta \to \infty$ as $t \to \infty$. Hence, $\alpha \gg 1$ and $\beta \gg 1$ at $t \gg 1$. Thus one can obtain the following asymptotic solutions of $\alpha(t)$ and $\beta(t)$ in the long-time limit:

$$\alpha(t) \simeq C_6[\tau(t)]^{\gamma_1}, \quad \beta(t) \simeq C_7[\tau(t)]^{\gamma_2}, \tag{16}$$

where

$$\begin{aligned} C_6 &= C_2^{-2I_1J/(I_1I_2-4J^2)}\gamma_1^{-\gamma_1}, \\ C_7 &= C_4^{-2I_2J/(I_1I_2-4J^2)}\gamma_2^{-\gamma_2}, \\ \gamma_1 &= (I_1I_2 - 2I_1J)/(I_1I_2 - 4J^2) \end{aligned}$$

and

$$\gamma_2 = (I_1I_2 - 2I_2J)/(I_1I_2 - 4J^2).$$

We then obtain the asymptotic scaling solutions for the cluster-mass distributions in the long-time limit,

$$\begin{aligned} a_k(t) &\sim E(t)[\tau(t)]^{-\gamma_1-1}\exp(-x), \quad x \propto k\tau^{-\gamma_1}, \\ b_k(t) &\sim E(t)[\tau(t)]^{-\gamma_2-1}\exp(-y), \quad y \propto k\tau^{-\gamma_2}. \end{aligned} \tag{17}$$

These indicate that the cluster-mass distribution of either species can be written in the following generalized scaling form:[15,16]

$$\begin{aligned} c_k(t) &\simeq [g(t)]^{-w_1}[f(t)]^{-w_2}\,\Phi[k/S(t)], \\ S(t) &\propto [g(t)]^{z_1}[f(t)]^{z_2}, \end{aligned} \tag{18}$$

where $f(t)$ and $g(t)$ are two unusual functions of time, and $g'(t)$ and $f'(t) > 0$ (such as e^t, $\ln t$, 2^t, and so on). The results also imply that the governing scaling exponents may depend crucially on the aggregation–annihilation reaction rates (I_1, I_2, J) and the removal probability function $P(t)$. Moreover, we obtain the total mass densities, $M_A(t) \propto E(t)[\tau(t)]^{\gamma_1-1}$ and $M_B(t) \propto E(t)[\tau(t)]^{\gamma_2-1}$. It is not difficult to find that for an arbitrary removal probability $P(t)$, the total mass of either species always decreases with time. Hence, neither species can be conserved finally.

2.2. $I_1 > 2J$ and $I_2 = 2J$ case

In this case, by employing the above-mentioned technique, the asymptotic solutions of the long-time mass distributions of A and B clusters are then obtained:

$$\begin{aligned} a_k(t) &\sim E(t)[\tau(t)]^{-1}[\ln\tau(t)]^{-1-\gamma_3}\exp(-x), \\ x &\propto k[\ln\tau(t)]^{-\gamma_3}, \\ b_k(t) &\sim E(t)[\tau(t)]^{-2}[\ln\tau(t)]^{\gamma_4}\exp(-y), \\ y &\propto k[\tau(t)]^{-1}[\ln\tau(t)]^{\gamma_4}, \end{aligned} \tag{19}$$

where $\gamma_3 = I_1/(I_1-2J)$ and $\gamma_4 = 2J/(I_1-2J)$. The results indicate that the cluster-mass distribution of either species also satisfies the generalized scaling law (18). Comparison between the total mass of species A and that of species B, $M_A(t) \propto E(t)[\tau(t)]^{-1}[\ln\tau(t)]^{\gamma_4}$ and $M_B(t) \propto E(t)[\ln\tau(t)]^{-\gamma_4}$ at $t \gg 1$, shows that the large B clusters dominate over the corresponding A clusters in the long-time limit. This is independent of the initial concentrations A_0 and B_0. Moreover, we again find that for this case both species cannot survive at the end.

2.3. $I_1 > 2J$ and $I_2 < 2J$ case

In this case, by solving Eqs.(9) we can also obtain the scaling solutions of the cluster-mass distributions as follows:

$$\begin{aligned} a_k(t) &\sim \left(\frac{C_8-1}{C_8}\right)^k E(t)[\tau(t)]^{-\gamma_5-1}\exp(-x), \\ x &\propto k[\tau(t)]^{-\gamma_5}, \\ b_k(t) &\sim E(t)[\tau(t)]^{-2}\exp(-y), \\ y &\propto k[\tau(t)]^{-1}, \end{aligned} \tag{20}$$

where $C_8 = (I_2B_0/2)^{I_1/2J}C_5^{I_1/(I_1-2J)}$ and $\gamma_5 = (2J - I_2)/I_2$. These show that in this case, the evolution of species B obeys the scaling law (18) while

that of species A obeys a modified form $c_k(t) \simeq h^k g(t)^{-w_1}[f(t)]^{-w_2}\Phi[k/S(t)]$. The typical mass of species B is $S(t) \propto \tau(t)$. As for species A, the growing scale is $S(t) \propto [\tau(t)]^{\gamma_5}$ and the time-independent scale is $S_1 = C_8$. We also determine the total mass of clusters A and of clusters B, $M_A(t) \propto E(t)[\tau(t)]^{-\gamma_5-1}$ and $M_B(t) \propto E(t)$, which imply that the large clusters B dominate over the corresponding clusters A in the long-time limit. In this case, only species B may survive finally, which is independent of the initial concentrations A_0 and B_0.

2.4. $I_1 = I_2 = 2J$ **case**

For this special case, the scaling solutions of $a_k(t)$ and $b_k(t)$ are obtained as

$$a_k(t) \sim E(t)[\tau(t)]^{-\gamma_6-1}\exp(-x), \quad x \propto k[\tau(t)]^{-\gamma_6},$$
$$b_k(t) \sim E(t)[\tau(t)]^{-\gamma_7-1}\exp(-y), \quad y \propto k[\tau(t)]^{-\gamma_7}, \tag{21}$$

where $\gamma_6 = A_0/(A_0+B_0)$ and $\gamma_7 = B_0/(A_0+B_0)$. These scaling solutions indicate that the mass distributions of both species can be written in the scaling form (18). We also find that in the long-time limit the total masses of the two species are $M_A(t) \propto E(t)[\tau(t)]^{\gamma_6-1}$ and $M_B(t) \propto E(t)[\tau(t)]^{\gamma_7-1}$, respectively. Since $\gamma_6/\gamma_7 = A_0/B_0$, the heavy species with the larger initial concentration will dominate over the light species throughout the process. However, both species cannot be conserved at the end.

2.5. $I_1 = 2J$ **and** $I_2 < 2J$ **case**

For this case, the solutions of Eqs.(9) are similar to those for the third case of this section and therefore the cluster-mass distributions also take the forms of Eqs.(20) except for the constant C_8 substituted by another constant $\exp[I_1A_0/(I_1-I_2)B_0]$. Thus we obtain the same results for this case as those in Case 2.3.

2.6. $I_1 < 2J$ **and** $I_2 < 2J$ **case**

We also obtain the differential equations (15) for this case. It is obvious that $C_2 > 0$ and $C_4 > 0$ in this case. The solutions of $\alpha(t)$ and $\beta(t)$ are dependent on the values of C_2 and C_4.

When $A_0(2J-I_1) > B_0(2J-I_2)$, we obtain the asymptotic scaling solutions of the cluster-mass distributions

$$\begin{aligned} &a_k(t) \sim E(t)[\tau(t)]^{-2}\exp(-x),\\ &x \propto k\tau(t),\\ &b_k(t) \sim \left(\frac{C_9-1}{C_9}\right)^k E(t)[\tau(t)]^{-\gamma_8-1}\exp(-y),\\ &y \propto k[\tau(t)]^{-\gamma_8}, \end{aligned} \tag{22}$$

where $C_9 = (I_1A_0/2C_{17})^{I_2/2J}$ and $\gamma_8 = (2J-I_1)/I_1$. The results indicate that for this case the evolution of species A obeys the scaling law (18) while the mass distribution of clusters B satisfies a modified scaling form $c_k(t) \simeq h^k g(t)^{-w_1}[f(t)]^{-w_2}\Phi[k/S(t)]$. Comparison between the total mass of clusters A and that of clusters B shows $M_A(t) \propto E(t)$ and $M_B(t) \propto E(t)[\tau(t)]^{-\gamma_8-1}$. This implies that clusters A dominate over clusters B in the long-time limit. Moreover, only species A might survive in the system at the end. The results of this case are contrary to those in the third case of this section.

When $A_0(2J-I_1) < B_0(2J-I_2)$, we find that $\beta \to \infty$ as $t \to \infty$. By employing the above method, one can obtain the asymptotic equations (20) for $a_k(t)$ and $b_k(t)$. So, the evolution behaviour of this system is similar to that in Case 2.3.

Finally, we turn to the borderline case, $A_0(2J-I_1) = B_0(2J-I_2)$. It is found that for this case, the results of this case are identical to those in Case 2.1.

3. Model of irreversible aggregation and partial annihilation process with cluster removals

In this section, we turn to investigate the kinetics of the irreversible aggregation and partial annihilation process with cluster removals. We also consider a constant-reaction-rate model with the generic conditions similar to those in Sec.2. Then the governing rate equations for this system read as

$$\begin{aligned} \frac{\mathrm{d}a_k}{\mathrm{d}t} =& I_1\left(\frac{1}{2}\sum_{i+j=k}a_ia_j - a_k\sum_{j=1}^{\infty}a_j\right)\\ &+ J\left(\sum_{p-q=k}a_pb_q - a_k\sum_{j=1}^{\infty}b_j\right) - Pa_k,\\ \frac{\mathrm{d}b_k}{\mathrm{d}t} =& I_2\left(\frac{1}{2}\sum_{i+j=k}b_ib_j - b_k\sum_{j=1}^{\infty}b_j\right)\\ &+ J\left(\sum_{p-q=k}a_pb_q - a_k\sum_{j=1}^{\infty}b_j\right) - Pb_k. \end{aligned} \tag{23}$$

Under the monodisperse initial conditions, with the help of ansatz (2) we again introduce the variables $\alpha = (1-a)^{-1}$ and $\beta = (1-b)^{-1}$ to transform the rate equations (23) into the following differential equations:

$$\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} = -\frac{2J}{I_2}\frac{1}{\alpha+\beta-1}\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\mathrm{d}\beta}{\mathrm{d}t} - P\frac{\mathrm{d}\alpha}{\mathrm{d}t},$$
$$\frac{\mathrm{d}^2\beta}{\mathrm{d}t^2} = -\frac{2J}{I_1}\frac{1}{\alpha+\beta-1}\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\mathrm{d}\beta}{\mathrm{d}t} - P\frac{\mathrm{d}\beta}{\mathrm{d}t}, \qquad (24)$$

with the initial values (7). We then find two integrals of Eqs.(24) as follows:

$$I_2\frac{\mathrm{d}\alpha}{\mathrm{d}\tau} - I_1\frac{\mathrm{d}\beta}{\mathrm{d}\tau} = \frac{I_1I_2}{2}(A_0-B_0),$$
$$\left(\frac{\mathrm{d}\alpha}{\mathrm{d}\tau}\right)^{I_2}\left(\frac{\mathrm{d}\beta}{\mathrm{d}\tau}\right)^{I_1} = \left(\frac{I_1A_0}{2}\right)^{I_2}\left(\frac{I_2B_0}{2}\right)^{I_1} \times(\alpha+\beta-1)^{-2J}. \qquad (25)$$

We then discuss the solutions of Eqs.(25) in two different cases.

3.1. Symmetric initial case

For the symmetric initial conditions $A_0 = B_0$, Eqs. (25) can be immediately solved to yield

$$\alpha(t) = \frac{I_1}{I_1+I_2}[C_{10}\tau(t)+1]^{\gamma_9} + \frac{I_2}{I_1+I_2},$$
$$\beta(t) = \frac{I_2}{I_1+I_2}[C_{10}\tau(t)+1]^{\gamma_9} + \frac{I_1}{I_1+I_2}, \qquad (26)$$

where $C_{10} = (I_1+I_2+2J)A_0/2$ and $\gamma_9 = (I_1+I_2)/(I_1+I_2+2J)$.

When $\tau \to \tau_0$ as $t \to \infty$, we obtain the asymptotic scaling solutions

$$a_k(t) \sim (1-C_{11})^k E(t)\exp(-x),$$
$$x \propto k\int_t^\infty E(t')\mathrm{d}t',$$
$$b_k(t) \sim (1-C_{12})^k E(t)\exp(-y),$$
$$y \propto k\int_t^\infty E(t')\mathrm{d}t', \qquad (27)$$

where

$$C_{11} = (I_1+I_2)/[I_2+I_1(C_{10}\tau_0+1)^{\gamma_9}]$$

and

$$C_{12} = (I_1+I_2)/[I_1+I_2(C_{10}\tau_0+1)^{\gamma_9}].$$

Equations (27) are similar to Eqs.(12) and, thus, the results of this case are identical with those for the corresponding case in Sec.2. This implies that for this condition the evolution of the system may depend only on the removal term.

When $\tau \to \infty$ as $t \to \infty$, we again obtain the revised scaling solutions (17) for the cluster-mass distributions, where both γ_1 and γ_2 in the origin forms (17) are now replaced by γ_9. The results indicate that the cluster-mass distributions of both species take the scaling form (18). It is also observed that the total mass of either species decays as $E(t)[\tau(t)]^{\gamma_9-1}$. We can conclude that for arbitrary removal conditions both species cannot survive finally.

3.2. Asymmetric initial case

Now we turn to the asymmetric initial case $A_0 > B_0$. We discuss the general solutions of the cluster-mass distributions in two different subcases.

When $\tau \to \tau_0$ as $t \to \infty$, the solutions $\alpha(t)$ and $\beta(t)$ of Eqs.(25) are similar to Eqs.(11). So, this case is also equivalent to the corresponding $\tau \to \tau_0$ case in Sec.2.

When $\tau(t) \to \infty$ as $t \to \infty$, from Eqs.(25) we derive the asymptotic solutions at large times,

$$\alpha(t) \simeq C_{13}\tau(t), \quad \frac{\mathrm{d}\beta}{\mathrm{d}\tau} \simeq C_{14}\tau^{-2J/I_1}, \qquad (28)$$

where

$$C_{13} = I_1(A_0-B_0)/2$$

and

$$C_{14} = I_2B_0C_{13}^{-2J/I_1}[A_0/(A_0-B_0)]^{I_2/I_1}/2.$$

Equations (29) imply that the evolution of species A obeys the scaling law (18) while the evolution of species B depends crucially on the ratio between the aggregation rate I_1 and the annihilation rate J. Moreover, it is not difficult to find that the total mass of species A is $M_A(t) \propto E(t)$. When $E(t) \to$ const. as $t \to \infty$, clusters A can survive at the end. When $E(t)$ decreases with time, species A cannot be conserved finally. We then investigate the evolution behaviour of species B in several subcases as follows.

When $I_1 > 2J$, from Eq.(28) we have $\beta \simeq I_1C_{14}\tau^{1-2J/I_1}/(I_1-2J)$. Thus we obtain the asymptotic scaling solution

$$b_k(t) \sim E(t)[\tau(t)]^{-2+2J/I_1}\exp(-y),$$
$$y \propto k[\tau(t)]^{-1+2J/I_1}. \qquad (29)$$

This indicates that for $I_1 > 2J$ the evolution of species B also obeys the scaling law (18). In this case, the asymptotic behaviour of the total mass of species B is $M_B(t) \propto E(t)[\tau(t)]^{-2J/I_1}$, which implies that species B cannot survive at the end. When $E(\infty) \to$ const.,

the exhaustion of species B is forced by the annihilation process. When $E(\infty) \to 0$, that species B cannot survive is crucially due to the removal process.

When $I_1 < 2J$, from Eq.(28) we again have $\beta \simeq \beta_\infty - I_1 C_{14} \tau^{1-2J/I_1}/(2J - I_1)$, where β_∞ is a finite value and $\beta \to \beta_\infty$ as $t \to \infty$. Unfortunately, we cannot determine the explicit expression of β_∞. The asymptotic solution of $b_k(t)$ is then obtained:

$$b_k(t) \sim (1 - \beta_\infty^{-1})^k E(t)[\tau(t)]^{-2J/I_1} \exp(-y),$$
$$y \propto k[\tau(t)]^{-1+2J/I_1}. \tag{30}$$

This indicates that for $I_1 < 2J$ the evolution of species B obeys the modified scaling law $b_k(t) \simeq h^k g(t)^{-w_1}[f(t)]^{-w_2} \Phi[k/S(t)]$. In this case, we observe that the total mass of species B always decreases with time and clusters B are exhausted by the annihilation reaction as well as the removal event.

In the last case of $I_1 = 2J$, we find that $\beta \simeq C_{14} \ln \tau$ in the long-time limit. We then obtain the scaling solution

$$b_k(t) \simeq \frac{2}{I_2 C_{14}} E(t)[\tau(t)]^{-1}[\ln \tau(t)]^{-2} \exp(-y),$$
$$y = (k/C_{14})[\ln \tau(t)]^{-1}. \tag{31}$$

This indicates that for $I_1 = 2J$ the evolution of species B also obeys the generalized scaling law (18). In this case, the total mass $M_B(t)$ of species B changes as $E(t)/\tau(t)$ at large times. Thus species B is also exhausted finally for this case.

4. Summary

We have studied an irreversible aggregation–annihilation model with cluster removals on the basis of mean-field theory. Considering the constant-reaction-rate model, we analysed the kinetic behaviours of the aggregation–annihilation processes with cluster removals.

We first considered the irreversible aggregation and complete annihilation system with the removal term. By solving the mean-field rate equation, we obtained the general solutions of the cluster-mass distributions in the cases with an arbitrary injection probability $P(t)$. When $\tau(\infty) = \int_0^\infty \mathrm{d}t' \exp\left[-\int_0^{t'} P(t'')\mathrm{d}t''\right] \to$ const., the cluster-mass distribution of either species may obey a modified scaling law and both species cannot survive in the end, which is independent of the initial monomer concentrations as well as the reaction rates. When $\tau(\infty) \to \infty$, the evolution behaviour of the system then depends strongly on the ratios of the aggregation rates to the annihilation rate J, and the cluster-mass distributions may satisfy the generalized scaling form. In short, the evolution of the total mass depends crucially on the removal probability. Meanwhile, the details of the reaction event also play an important role in the kinetic behaviour of the system.

We have also studied the irreversible aggregation and partial annihilation model with cluster removals. The results indicate that the evolution behaviour of the system depends strongly on the initial conditions. For the symmetric initial case, the two species evolve according to a same scaling law. Under the asymmetric initial conditions, the evolution of the heavy species with larger initial concentration satisfies a sole scaling law while the light species may scale according to three distinct forms in different cases. On the other hand, the kinetic behaviour of the system also depends on the details of the reaction rates and removal probability.

References

[1] Friedlander S K 1977 *Smoke, Dust and Haze: Fundamentals of Aerosol Behavior* (New York: Wiley)
[2] Ovchinnikov A A and Zeldovich Y B 1978 *Chem. Phys.* **28** 215
[3] Toussaint D and Wilczek F 1983 *J. Chem. Phys.* **78** 2642
[4] Meankin P 1992 *Rep. Prog. Phys.* **55** 157
[5] Liao B, Xue Y and Chen G Z 2002 *Acta. Phys. Sin.* **51** 215 (in Chinese)
[6] Xue Y and Chen G Z 2002 *Chin. Phys.* **11** 684
[7] Ke J H, Lin Z Q and Wang X H 2003 *Chin. Phys. Lett.* **20** 151
[8] Krapivsky P L 1993 *Physica* A **198** 135
[9] Krapivsky P L 1993 *Physica* A **198** 150
[10] Sokolov I M and Blumen A 1994 *Phys. Rev.* E **50** 2335
[11] Ben-Naim E and Krapivsky P L 1995 *Phys. Rev.* E **52** 6066
[12] Balboni D, Rey P A and Droz M 1995 *Phys. Rev.* E **52** 6220
[13] Zhang L G and Yang Z R 1997 *Physica* A **237** 441
[14] Zhang L G and Yang Z R 1997 *Phys. Rev.* E **55** 1442
[15] Ke J H and Lin Z Q 2002 *Phys. Rev.* E **65** 051107
[16] Ke J H and Lin Z Q 2003 *Phys. Rev.* E **67** 062101
[17] Crump J G and Seinfeld J H 1982 *J. Colloid Interface Sci.* **90** 469
[18] Hendriks E M 1984 *J. Phys. A: Math. Gen.* **17** 2299
[19] Racz Z 1985 *Phys. Rev.* A **32** 1129
[20] Vicsek T, Meakin P and Family F 1985 *Phys. Rev.* A **32** 1122
[21] Ke J H and Lin Z Q 2003 *Physica* A **320** 261
[22] Ke J H, Lin Z Q and Chen Y S 2003 *Commun. Theor. Phys.* **40** 123
[23] Ke J H, Lin Z Q and Wang X H 2003 *Chin. Phys.* **12** 443