Vol 13 No 5, May 2004
1009-1963/2004/13(05)/0704-08

Chinese Physics

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and IOP Publishing Ltd

Coupling and stability of bright holographic soliton pairs*

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(Received 28 July 2003; revised manuscript received 24 September 2003)

The coupling effect and stability property of symmetric bright holographic soliton pairs have been investigated numerically. Results show that when any one of the two solitary beams from a pair is perturbed in amplitude or width, both beams will be affected by such a perturbation via the coupling effect between the beams, thus resulting in both beams propagating in the medium without a constant shape; however, these two solitary beams are still stable against small perturbations. When both solitary beams from a pair are perturbed simultaneously in amplitude, for some given absolute values of the perturbations, the two beams are stable against these perturbations if both beams are perturbed with the same sign, whereas are unstable with the different signs. When the two beams are simultaneously perturbed in width, both beams exhibit their stability property similar to that when only one beam is perturbed no matter whether both perturbations have the same or different signs.

Keywords: spatial optical solitons, solitons pair, holographic grating, coupling effects
PACC: 4265J, 4265S

1. Introduction

Optical spatial solitons have been extensively studied during the last three decades. To date, many topics have been researched in the area including Kerr solitons, photorefractive (PR) solitons, incoherent solitons, quadratic solitons, Bragg solitons, discrete solitons, cavity solitons and magneto-optic solitons and so on.[1] PR spatial solitons have become a convenient playground to study soliton phenomena, since they are observed at low light powers and exhibit robust trapping in both transverse dimensions.[2−14] So far, the mechanism to support the steady-state PR solitons is the self-phase-modulation self-focusing mechanism that originates from the spatially nonuniform screening of the applied field to form screening solitons,[2,3] or from the photovoltaic effects to form photovoltaic solitons,[4,5] or from both simultaneously to form screening-photovoltaic solitons.[6−13] Through this mechanism, an optical beam modifies the refractive index (that is, the refractive index in the centre of the beam is larger than at the beam's margins) and induces an effective positive lens. The refractive-index structure of the medium then resembles that of a graded-index waveguide. When the optical beam that has induced the waveguide is also a guide mode of the waveguide that it has induced, the beam's propagation becomes stationary. Recently, Cohen *et al*[15] proposed a new kind of steady-state spatial soliton, namely holographic solitons, supported by biased PR crystals with a process of PR two-wave mixing (TWM) in a special case in which there is not any energy exchange between the beams and the absorption of the crystal is not taken into account. The mechanism to support the holographic solitons is the holographic focusing mechanism instead of the self-phase-modulation self-focusing mechanism. The holographic focusing mechanism arises from a grating in the refractive index induced by interference between the signal and pump beams via TWM. Through the grating (i.e. the induced periodic modulation of the refractive index) the beams are coupled through Bragg reflections. Each beam is Bragg reflected and coherently added into the other beam. This can lead to focusing of narrow beams when the reflected beams are $\pi/2$ phase retarded with respect to the primary beams.

*Project supported by the National Natural Science Foundation of China (Grant No 10174025) and the Key Project Foundation of the Education Ministry of China (Grant No 011118).
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The holographic focusing mechanism was most probably presented first in the pioneering experimental demonstration of spatial Kerr solitons,[16] which utilized interference between two beams to arrest transverse instability. An effect related to holographic focusing in PR materials has been reported,[17] in which the mutual focusing/defocusing of two beams with slightly different frequencies was demonstrated with a biased SBN crystal. Another new kind of spatial soliton with both the holographic focusing mechanism and the self-phase modulation self-focusing mechanism (for example, photovoltaic or screening mechanism) was proposed by Liu very recently for PR dissipative systems that consist of an open-circuit photovoltaic PR crystal[18] or a biased PR crystal[19] and a strong pump beam with a uniform spatial distribution in both transverse dimensions. Such solitons were termed rigid PR solitons, including rigid photovoltaic solitons[18] and rigid screening solitons.[19]

PR soliton pairs, such as incoherently coupled PR soliton pairs[20−22] and separate PR soliton pairs[23−29] have been investigated extensively, in which there are three types of pairs: bright–bright, dark–dark and bright–dark. A holographic soliton consists of two mutually coherent field components. As a result, holographic solitons are existent in the fashion of a pair. However, there are only two types for holographic soliton pairs: bright–bright and dark–dark. So in this paper, we use the terms bright and dark soliton pairs to denote the bright–bright and dark–dark holographic soliton pairs, respectively. The symmetric solutions of the holographic soliton pairs are obtained in Ref.[15] in which both components have the same amplitude and width. In other words, the two solitons in a pair are just the same in steady state.

Although the existence of both bright and dark holographic soliton pairs are proved in Ref.[15], the stability property and the coupling effect between the two solitons in a pair have not been investigated. In this paper, we take a numerical approach to study the stability property and coupling effect. Our results show that when one of the two solitary beams from a soliton pair is perturbed in amplitude or width, the perturbation can affect the propagation characters of both beams via the coupling effect between the two solitons. Whether one or two solitary beams from a bright soliton pair are perturbed, both beams are stable against small perturbation. For dark soliton pairs, our results indicate that they cannot overcome the numerical noises that come from the actual calculation process, which means that dark holographic solitons are unstable relative to small perturbation. In other words, we cannot find a stable numerical solution for the dark soliton case.

This paper is organized as follows. In Sec.2, the formulae to be used in the paper are listed. In Sec.3, the effects of the perturbations in one beam on the stability property of both beams are analysed numerically. In Sec.4, the effects of the perturbations in both beams on the stability property of both beams are analysed numerically. Finally, we draw some conclusions in Sec.5.

2. Formulae

Figure 1 shows the configuration to support holographic solitons, which is taken from Ref.[15] (see Fig.1 and the text describing the arrangement in Ref.[15]). We start with the nonlinear coupled-wave equations that have been given in Ref.[15] and they read as

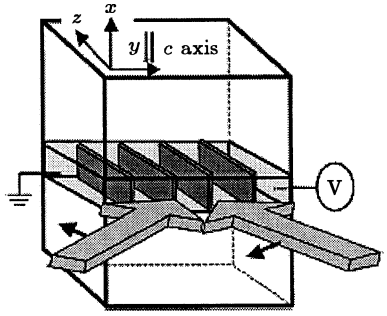


Fig.1. The schematic of a holographic soliton in a PR medium. Taken from Ref.[15].

$$\frac{\partial^2 A}{\partial x^2} + 2\mathrm{i}k\cos(\theta)\frac{\partial A}{\partial z} + \frac{2k^2\Delta n_0\cos(2\theta)}{n_0} \times \frac{|B|^2}{|A|^2+|B|^2+I_B}A = 0, \tag{1a}$$

$$\frac{\partial^2 B}{\partial x^2} + 2\mathrm{i}k\cos(\theta)\frac{\partial B}{\partial z} + \frac{2k^2\Delta n_0\cos(2\theta)}{n_0} \times \frac{|A|^2}{|A|^2+|B|^2+I_B}B = 0, \tag{1b}$$

where A and B are the complex envelope of the amplitude of the beams, $k = n_0 k_0$, $k_0 = 2\pi/\lambda_0$ and λ_0 is the free-space wavelength of the lightwave employed, n_0 is the ordinary index of refraction, θ is the angle between the vector k of each beam and the z-axis, Δn_0 is, in general, a complex factor that depends on the specific material parameters and the applied voltage, and I_B is the uniformly illuminated background intensity. Holographic solitons can exist only when Δn_0 is

real, which can be approximately satisfied by selecting a very small angle between the two beams, such as 0.1^0 and a strong bias field, such as 1000 V/cm, as proposed in Ref.[15].

Let $s = x/x_0$, $\xi = z/z_0$, $z_0 = kx_0^2\cos\theta$, $\sigma = (x_0k)^2\Delta n_0\cos(2\theta)/n_0$, $A = I_B^{1/2}U$, $B = I_B^{1/2}V$, where x_0 is an arbitrary spatial width. Equations (1) can then be written as two coupled equations in dimensionless variables:

$$\mathrm{i}\frac{\partial U}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U}{\partial s^2} + \sigma\frac{|V|^2}{|U|^2+|V|^2+1}U = 0, \tag{2a}$$

$$\mathrm{i}\frac{\partial V}{\partial \xi} + \frac{1}{2}\frac{\partial^2 V}{\partial s^2} + \sigma\frac{|U|^2}{|U|^2+|V|^2+1}V = 0. \tag{2b}$$

To seek a symmetric soliton solution, we can express the normalized envelops U and V as

$$U = V = (r/2)^{1/2}y(s)\exp(\mathrm{i}\nu\xi), \tag{3}$$

where ν represents a nonlinear shift to the propagation constant, r is the total peak input intensity of the two light beams normalized to I_B, namely $r = I_0/I_B$, and $I_0 = I(s=0,\xi=0)$ for the bright soliton and $I_0 = I(s=\pm\infty,\xi=0)$ for the dark soliton, and $I(s,\xi) = I_B[|U(s,\xi)|^2 + |V(s,\xi)|^2]$ is the sum of the average intensity of the two beams.

Substituting of Eqs.(3) into Eqs.(2), we have

$$\frac{\mathrm{d}^2y}{\mathrm{d}s^2} - 2\nu y + \sigma\frac{ry^3}{1+ry^2} = 0. \tag{4}$$

Both bright and dark soliton solutions of Eq.(4) have been discussed in great detail [7–9] and this equation is known to allow bright and dark solitons when $\sigma > 0$ and $\sigma < 0$, respectively.

For bright soliton, we require that $y(0) = 1$, $\dot{y}(0) = 0$ and $y(s\to\pm\infty) = 0$, the solution of Eq.(4) can be written as

$$\sigma^{1/2}s = \pm\int_y^1 \frac{r^{1/2}\mathrm{d}\hat{y}}{[\ln(1+r\hat{y}^2) - \hat{y}^2\ln(1+r)]^{1/2}}, \tag{5a}$$

$$\nu = \frac{\sigma}{2}\left[1 - \frac{1}{r}\ln(1+r)\right]. \tag{5b}$$

For the dark soliton, we require that $y(0) = 0$, $\dot{y}(\infty) = 0$, $\ddot{y}(\infty) = 0$ and $y(s\to\pm\infty) = 1$, the solution of the Eq.(4) can be written as

$$(-\sigma)^{1/2}s = \pm\int_y^0 \frac{(1+r)^{1/2}\mathrm{d}\hat{y}}{\left[(\hat{y}^2-1) - \dfrac{r+1}{r}\ln\left(\dfrac{1+r\hat{y}^2}{1+r}\right)\right]^{1/2}} \tag{6a}$$

$$\nu = \frac{r\sigma}{2(1+r)}. \tag{6b}$$

Equations (3) and (5), as well as Eqs.(3) and (6), are the bright and dark symmetric soliton solutions of Eqs.(2), respectively. These solutions are equivalent to the symmetric holographic soliton solutions given in Ref.[15].

When σ and r are given, from Eqs.(3) and (5), we can determine two bright soliton solutions in a pair, i.e. U and V. Taking the two solitary beams as the input beams, their dynamical evolution in the medium can be analysed by numerically solving Eqs.(2) with the same value of σ. Figure 2 gives an example for $r = 10$ and $\sigma = 4$, from which we can see the two bright solitary states can remain invariant with propagation distance. For the two bright solitary beams, their intensity full width at half maximum (FWHM) is W_0=1.75.

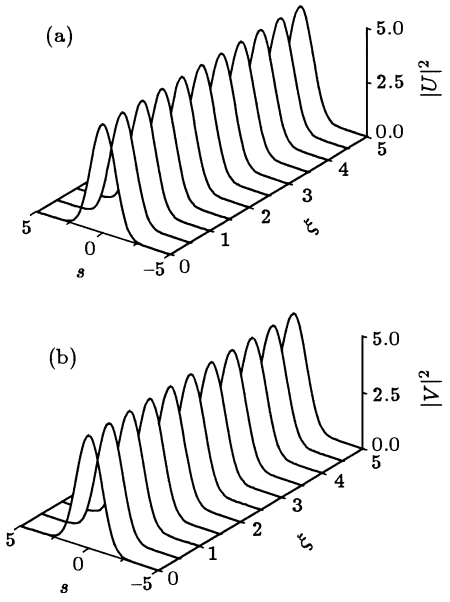


Fig.2. Simultaneous propagation of a bright holographic soliton pair (intensity profiles) with r=10 and σ=4. (a) U and (b) V. The intensity FWHM of U and V is W_0=1.75.

3. Soliton stability property for one perturbed beam

In this section, we will take a numerical method to simulate the dynamical evolution of bright holographic soliton pairs in biased PR crystals when one solitary beam from this pair is perturbed in amplitude or width. The approach used in this section is as follows. At first, we give two bright solitary solutions, U and V, from Eqs.(3) and (5) with $r = r_0$

and $\sigma = \sigma_0$, and the intensity FWHM of these beams is W_0. And then, we use a matching Gaussian beam to express any one of the two bright solitary beams. Such as, U is expressed as $U = (r/2)^{1/2}\exp(-s^2/\omega_0^2)$ with $r = r_0$ and $\omega_0 = 0.85W_0$, which has approximately the same amplitude and width as that of the holographic bright solitary solutions from Eqs.(3) and (5) with $r = r_0$ and $\sigma = \sigma_0$. The following tasks are to perturb the matching Gaussian beam by introducing two additional parameters δr and δs in the expression of U as follows.

$$U = [r(1+\delta r)/2]^{1/2}\exp\{-s^2/[\omega_0(1+\delta s)]^2\}, \quad (7)$$

where $\delta r = \Delta r/r$ and $\delta s = \Delta s/\omega_0$ denote the relative amplitude and width perturbations and, Δr and Δs denote the absolute ones, respectively. Taking U and V as the input beams, in which U is a perturbed matching Gaussian beam given by Eq.(7) and V is still an exact symmetric bright holographic-soliton solution determined by Eqs.(3) and (5) with $\sigma = \sigma_0$ and $r = r_0$, the evolutions of the two input beams in the medium can be analysed by numerically solving Eqs.(2) with $\sigma = \sigma_0$ and a given set of δr and δs, from which we can obtain the information about the effects of the amplitude or width perturbation on the stability of the two solitary beams.

Here we consider a holographic bright pair with $r = 10$ and $\sigma = 4$ that are shown in Fig.2 as the unperturbed original beams with their intensity FWHM being $W_0 = 1.75$. As a result, the amplitude and width of the matching Gaussian beam U should be taken as $r = 10$ and $\omega_0 = 0.85W_0 = 1.49$. The evolutions of U and V in the medium are shown in Fig.3 for δr =50% and $\delta s = 0$. Although V is still an exact bright holographic-soliton solution, both solitary beams from this pair cannot propagate smoothly with a constant shape because the amplitude perturbation for U affects not only the evolution of U, but also the evolution of V via the coupling effect between the two beams. However, although both solitons cannot remain invariant with propagation distance, each of them tends to evolve into a solitary beam after a short distance, which means this pair is stable against the 50% relative amplitude perturbation in one solitary component. If we take a larger amplitude perturbation, such as $\delta r = 500\%$, both solitons from this pair cannot propagate stably, as shown in Fig.4. The evolutions of the peak intensity and intensity FWHM of U and V for different values of δr are shown in Fig.5, from which we can see that this holographic soliton pair is stable against small amplitude perturbations in one solitary component.

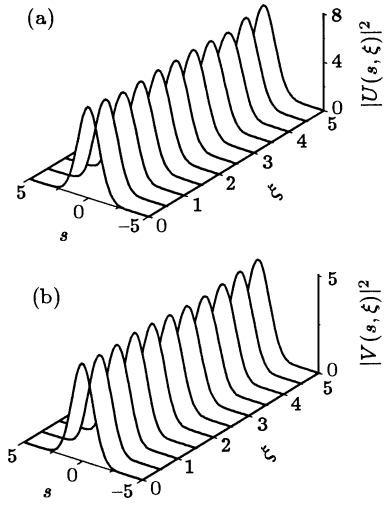


Fig.3. Effects of a small amplitude perturbation in one solitary component on the evolutions of both solitary beams from a bright holographic soliton pair with $r = 10$ and $\sigma = 4$. Input conditions: (a) $U = [r(1+\delta r)/2]^{1/2}\exp\{-s^2/[\omega_0(1+\delta s)]^2\}$ with $r = 10$, $\omega_0 = 1.49$, $\delta r = 50\%$ and $\delta s = 0$. (b) V is an exact bright holographic soliton solution with $r = 10$ and $\sigma = 4$.

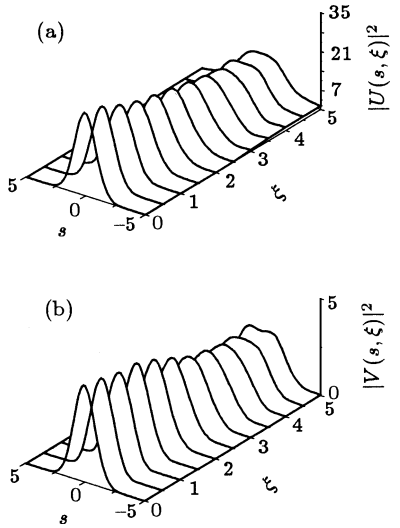


Fig.4. Effects of a large amplitude perturbation in one solitary component on the evolutions of both solitary beams from a bright holographic soliton pair with $r = 10$ and $\sigma = 4$. Input conditions: (a) $U = [r(1+\delta r)/2]^{1/2}\exp\{-s^2/[\omega_0(1+\delta s)]^2\}$ with $r = 10$, $\omega_0 = 1.49$, $\delta r = 500\%$ and $\delta s = 0$. (b) V is an exact bright holographic soliton solution with $r = 10$ and $\sigma = 4$.

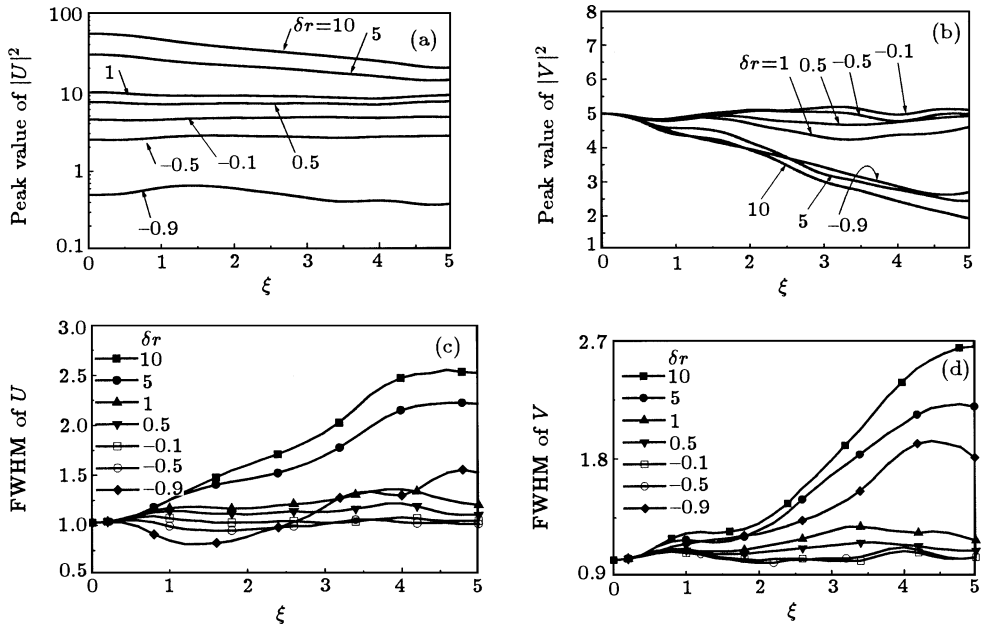


Fig.5. Evolutions of the peak intensity and intensity FWHM of both solitons from a bright soliton pair with $r = 10$ and $\sigma = 4$ for different values of relative amplitude perturbation in U. Input conditions: $U = [r(1+\delta r)/2]^{1/2}\exp\{-s^2/[\omega_0(1+\delta s)]^2\}$ with $r = 10$, $\omega_0 = 1.49$, and $\delta s = 0$ and V is an exact bright holographic soliton solution with $r = 10$ and $\sigma = 4$. (a) Peak intensity of U. (b) Peak intensity of V. (c) FWHM of U. (d) FWHM of V.

We then consider the effect of the width perturbation in one solitary component on the evolutions of both solitons from a bright holographic soliton pair with $r = 10$ and $\sigma = 4$. Figure 6 gives an example with a width perturbation δs =10%, from which we can see that although the width perturbation makes both beams propagate in the medium without a constant shape, each of the two beams tends to evolve into a solitary beam over a short distance. That is, this pair is stable against a 10% width perturbation. The evolutions of the peak intensity and intensity FWHM of both solitons from this pair for different values of δs are shown in Fig.7, from which we can see that this holographic soliton pair is stable against small width perturbations in one solitary component.

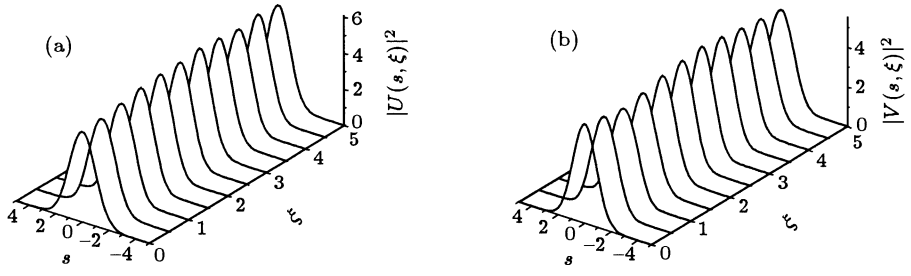


Fig.6. Effects of a width perturbation in one solitary component on the evolutions of both solitons from a bright holographic soliton pair with $r = 10$ and $\sigma = 4$. Input conditions: (a) $U = [r(1+\delta r)/2]^{1/2}\exp\{-s^2/[\omega_0(1+\delta s)]^2\}$ with $r = 10$, $\omega_0 = 1.49$, $\delta r = 0\%$ and $\delta s = 10\%$. (b) V is an exact bright holographic soliton solution with $r = 10$ and $\sigma = 4$.

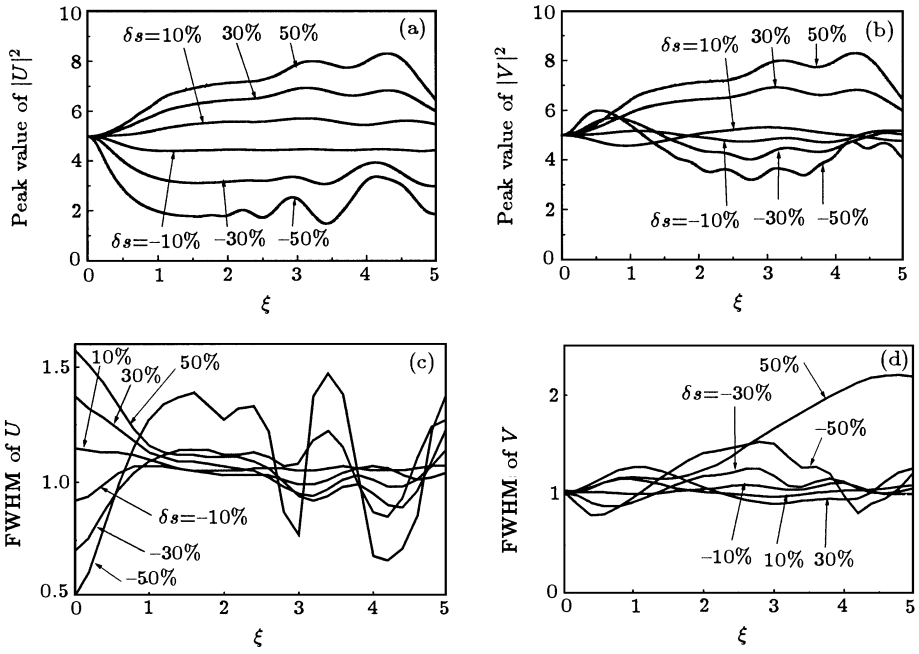


Fig.7. Evolutions of the peak intensity and intensity FWHM of both solitons from a soliton pair with $r = 10$ and $\sigma = 4$ for different values of relative width perturbation for U. Input conditions: $U = [r(1+\delta r)/2]^{1/2}\exp\{-s^2/[\omega_0(1+\delta s)]^2\}$ with $r = 10$, $\omega_0 = 1.49$, and $\delta r = 0$ and V is an exact bright holographic soliton solution with $r = 10$ and $\sigma = 4$. (a) Peak intensity of U. (b) Peak intensity of V. (c) FWHM of U. (d) FWHM of V.

4. Soliton stability for two perturbed beams

In this section, we investigate the dynamical evolution of bright holographic soliton pairs in biased PR crystals when both solitons from a pair are perturbed in amplitude or width. We pay attention on the sign of perturbation in both beams and we mainly consider three cases: i.e. both beams have a positive perturbation, both beams have a negative perturbation and one beam has a positive perturbation while the other beam has a negative one. At first, we determine a bright soliton pair with two solitary components U and V from Eqs.(3) and (5) with $\sigma = 4$ and $r = 10$, which have been shown in Fig.2. The intensity FWHM of the two solitons is W_0 =1.75. We then perturb both beams, U and V, by approximately matching them into Gaussian beams given by

$$U = [r(1+\delta r_1)/2]^{1/2}\exp\{-s^2/[\omega_0(1+\delta s_1)]^2\}, \quad (8a)$$

$$V = [r(1+\delta r_2)/2]^{1/2}\exp\{-s^2/[\omega_0(1+\delta s_2)]^2\}, \quad (8b)$$

where $\delta r_j = \Delta r_j/r$ and $\delta s_j = \Delta s_j/\omega_0$ (j=1,2) denote the relative amplitude and width perturbations and Δr_j and Δs_j (j=1,2) denote the absolute ones, respectively. When $r_0 = 10$, ω_0=0.85 and $W_0 = 1.49$, we have $\Delta r_j = 0$ and $\Delta s_j = 0$ (j=1,2), and Eqs.(8) are slightly different from those exact bright holographic-soliton solutions with $\sigma = 4$ and $r = 10$ in their maximum amplitudes, widths and functional forms. Taking U and V as the input beams, the evolutions of the two input beams in the medium can be analysed by numerically solving Eqs.(2) with $\sigma = \sigma_0$ and a given set of δr_j and δs_j (j=1,2), from which we can obtain the information about the effects of the amplitude and width perturbations on the stability of the two solitary beams.

At first, we consider the case of amplitude perturbations. Putting amplitude perturbations into U and V simultaneously, the evolution of the two beams in the medium are shown in Fig.8 for $\delta r_1 = \delta r_2 = 5$,$\delta r_1 = \delta r_2 = -50\%$ as well as $\delta r_1 = 5$ and $\delta r_2 = -50\%$, from which we can see that when the perturbations for both solitary beams have the same sign, the two beams are stable against the perturbations, whereas when they have different signs, the two beams are unstable.

We then consider the case of width perturbations. Putting width perturbations into U and V simultaneously, the evolution of the two beams in the medium is shown in Fig.9 for $\delta s_1 = \delta s_2 = 10\%$, $\delta s_1 = \delta s_2 = -10\%$ as well as $\delta s_1 = -10\%$ and $\delta s_2 = 10\%$, from which we can see that both beams exhibit a stability property similar to that when only one beam is perturbed, no matter whether both perturbations have the same or different signs.

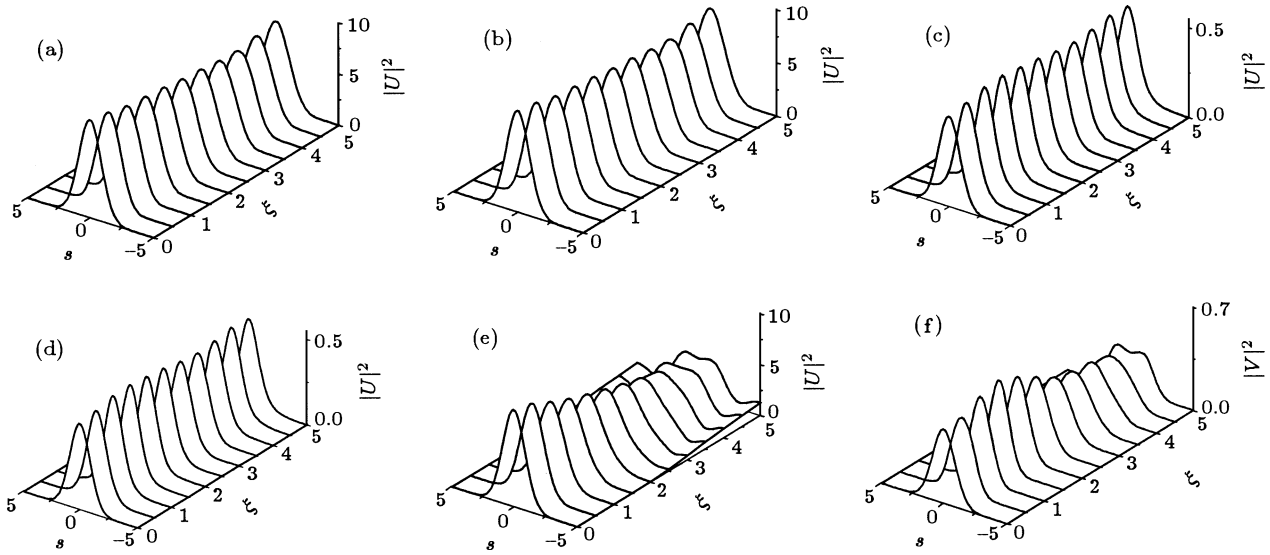


Fig.8. Effects of an amplitude perturbation in both solitary components on the evolutions of both solitons from a bright holographic soliton pair with $r = 10$ and $\sigma = 4$. Input conditions: $U = [r(1 + \delta r_1)/2]^{1/2}\exp\{-s^2/[\omega_0(1 + \delta s_1)]^2\}$ with $\delta s_1 = 0$ and $V = [r(1 + \delta r_2)/2]^{1/2}\exp\{-s^2/[\omega_0(1 + \delta s_2)]^2\}$ with $\delta s_2 = 0$ as well as $r = 10$ and $\omega_0 = 1.49$. (a) U for $\delta r_1 = \delta r_2 = 5$; (b) V for $\delta r_1 = \delta r_2 = 5$; (c) U for $\delta r_1 = \delta r_2 = -50\%$; (d) V for $\delta r_1 = \delta r_2 = -50\%$; (e) U for $\delta r_1 = 5$ and $\delta r_2 = -50\%$; and (f) V for $\delta r_1 = 5$ and $\delta r_2 = -50\%$.

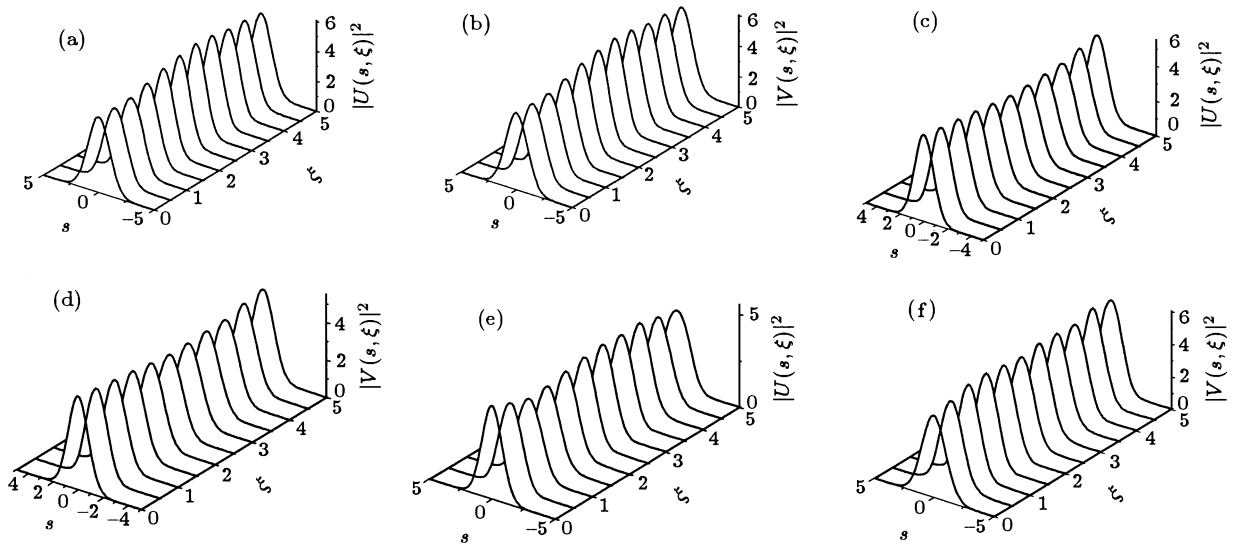


Fig.9. Effects of a width perturbation in both solitary components on the evolutions of both solitons from a bright holographic soliton pair with $r = 10$ and $\sigma = 4$. Input conditions: $U = [r(1 + \delta r_1)/2]^{1/2}\exp\{-s^2/[\omega_0(1 + \delta s_1)]^2\}$ with $\delta r_1 = 0$ and $V = [r(1 + \delta r_2)/2]^{1/2}\exp\{-s^2/[\omega_0(1 + \delta s_2)]^2\}$with $\delta r_2 = 0$ as well as $r = 10$ and $\omega_0 = 1.49$. (a) U for $\delta s_1 = \delta s_2 = 10\%$; (b) V for $\delta s_1 = \delta s_2 = 10\%$; (c) U for $\delta s_1 = \delta s_2 = -10\%$; (d) V for $\delta s_1 = \delta s_2 = -10\%$; (e) U for $\delta s_1 = -10\%$ and $\delta s_2 = 10\%$; and (f) V for $\delta s_1 = -10\%$ and $\delta s_2 = 10\%$.

5. Conclusion

In this paper, we have numerically investigated the coupling effects between two solitons from a holographic soliton pair when one or both initial input solitary beams are perturbed and the stability properties of the two solitons against these perturbations are discussed. Our results show that when any one of the two bright solitary beams from a bright soliton pair is perturbed in amplitude or width, both beams can be affected by the perturbation via the coupling effects between the beams, thus resulting in both beams propagating in the medium without a constant shape; however, both beams are still stable against small perturbations. When the two bright solitary beams from a bright soliton pair are simultaneously perturbed in

amplitude, for some given absolute values of the perturbations, the two beams are stable against these perturbations if both solitary beams are perturbed with the same sign, whereas they are unstable when perturbed with different signs. When the two bright solitary beams from a bright soliton pair are simultaneously perturbed in width, both beams have a stability property similar to that when only one beam is perturbed no matter whether both perturbations have the same or different signs.

References

[1] Trillo S and Torruellas W 2001 *Spatial Solitons* (Berlin: Springer)
[2] Segev M, Valley G C, Crosignani B, Porto P D *et al* 1994 *Phys. Rev. Lett.* **73** 3211
[3] Christodoulides D N and Carvalho M I 1995 *J. Opt. Soc. Am.* B **12** 1628
[4] Valley G C, Segev M, Crosignani B *et al* 1994 *Phys. Rev.* A **50** R4457
[5] Segev M, Valley G C, Bashaw M C *et al* 1997 *J. Opt. Soc. Am.* B **14** 1772
[6] Andrade-Lucio J A, Iturbe-Castillo M D and Marquez-Aguilar P A 1998 *Opt. Quantum Electron.* **30** 829
[7] Liu J S and Lu K Q 1998 *Acta Phys. Sin.* **47** 1509 (in Chinese)
[8] Liu J S and Lu K Q 1999 *J. Opt. Soc. Am.* B **16** 550
[9] Liu J S 2001 *Chin. Phys.* **10** 1037
[10] Liu J S and Zhang D Y 2000 *Chin. Phys.* **9** 667
[11] Liu J S 2001 *Acta Phys. Sin.* **50** 880 (in Chinese)
[12] Liu J S and Hao Z H 2002 *Chin. Phys.* **11** 254
[13] Hou C F, Yuan B H, Sun X D *et al* 2000 *Acta Phys. Sin.* **49** 1969 (in Chinese)
[14] Hao Z H and Liu J S 2002 *Acta Phys. Sin.* **51** 818 (in Chinese)
[15] Cohen O, Carmon T, Segev M *et al* 2002 *Opt. Lett.* **22** 2031
[16] Barthelemy A, Maneuf S and Froehly C 1985 *Opt. Commun.* **55** 201
[17] Vaupel M, Seror C and Dykstra R 1997 *Opt. Lett.* **22** 1470
[18] Liu J S 2003 *Phys. Rev.* E **68** 026607
[19] Liu J S 2003 *J. Opt. Soc. Am.* B **20** 1732
[20] Chen Z G, Segev M, Coskun T H *et al* 1997 *J. Opt. Soc. Am.* B **14** 3066
[21] Chen Z, Segev M, Coskun T H *et al* 1996 *Opt. Lett.* **21** 1436
[22] Christodoulides D N, Singh R S, Carvalho M I *et al* 1996 *Appl. Phys. Lett.* **68** 1763
[23] Liu J S 2002 *Phys. Lett.* A **300** 213
[24] Liu J S and Hao Z H 2002 *Phys. Rev.* E **65** 066601
[25] Liu J S and Hao Z H 2003 *Phys. Lett.* A **309** 44
[26] Hao Z H and Liu J S 2002 *Acta Phys. Sin.* **51** 2772 (in Chinese)
[27] Hao Z H and Liu J S *SPIE Proc.* 2002 **4918** 326
[28] Hao Z H and Liu J S 2003 *J. Mod. Opt.* **50** 1967
[29] Liu J S and Hao Z H 2003 *Chin. Phys.* **12** 1124