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Amplification mechanism and effects of spontaneously generated coherence on the probe gain in a four-level system*

Xu Wei-Hua(徐卫华), Wu Jin-Hui(吴金辉), and Gao Jin-Yue(高锦岳)†

College of Physics, Jilin University, Changchun 130023, *China*

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We investigate the amplification mechanism in a four-level system by both the density-matrix and quantum-jump approaches. We show that the asymmetry between three-photon stimulated emission and one-photon absorption process is responsible for the inversionless amplification of the probe field. We also investigate the effects of spontaneously generated coherence (SGC) on the probe gain, and find that due to the SGC, the probe gain can be greatly enhanced and can be modulated by changing the relative phase between the applied fields.

Keywords: quantum-jump, spontaneously generated coherence
PACC: 4250, 3150

1. Introduction

Optical properties of an atomic medium may be controlled and modified by the application of a strong electromagnetic field. Phenomena associated with electromagnetically induced transparency (EIT) [1−3] and amplification without inversion (AWI) [4,5] have attracted considerable attention in recent years and offer a variety of interesting and potentially important applications. The essence of EIT is that an atomic coherence is induced in a multilevel system by a strong controlling laser field, which alters the response of the system to a probe laser field, and under the right circumstances, the absorption of a weak probe beam at the resonance frequency can be substantially reduced. The addition of incoherent pumping may lead to AWI, and it has been shown that AWI can be achieved experimentally in many different schemes.[6−9]

It is now well known that another kind of coherence, spontaneously generated coherence (SGC), can also change the steady-state response of the medium. Such coherence occurs in two cases. One occurs when a single excited state decays to a closely spaced lower doublet (the Λ-configuration),[10,11] and the other appears when a closely spaced excited doublet decays to a single ground state (the V-configuration).[12,13] It has been shown that this kind of coherence can modify significantly the absorption or spontaneous emission spectra of a near-degenerate system.[10−17] The existence of such coherence effects depends on the nonorthogonality of the two dipole matrix elements.[10,11,13]

In this paper, we investigate the gain properties and the effects of SGC on the probe gain in a four-level system proposed by Harris and Yamamoto.[18] Reference [18] described a four-level system that absorbs two photons, but does not absorb one photon. The system may function as an optical switch where a pulse of light of one frequency will cause the absorption of light at a second frequency. Yan *et al* reported an experimental observation of the nonlinear absorption based on such a system in ^{87}Rb atoms.[19] In the present paper, we show that AWI can be obtained on another transition in this system. We investigate the amplification mechanism by the quantum-jump approach.[20] We show that in this system, the asymmetry between the one-photon absorption process and the three-photon stimulated emission process

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†Correspondence should be addressed to Gao Jin-Yue. E-mail: jygao@mail.jlu.edu.cn Tel:86-431-8499069, Fax:86-431-5650478

leads to the amplification of the field considered. We also investigate the effects of SGC on the probe gain provided that the two lower levels are close-lying. We will show that, due to the existence of the SGC effects, the inversionless gain can be greatly enhanced, and the probe gain is sensitive to the relative phase between the two applied fields, and it can be remarkably modulated just by controlling the relative phase of the two fields. It is also sensitive to the angle between the dipole matrix elements of the corresponding transitions, and can be modulated by altering the angle.

This paper is organized as follows. In Sec.2, we introduce the system under study, and present the gain spectra derived from the corresponding density-matrix equations; in Sec.3, we analyse the amplification mechanism by the quantum-jump approach; in Sec.4, we investigate the effects of SGC on the inversionless gain; in Sec.5, we give a brief summary of the results.

2. The system and density-matrix equations

The closed, four-level atomic system is shown in Fig.1(a). Levels $|2\rangle$ and $|1\rangle$ are two lower levels, two fields $\boldsymbol{E}_a$ and $\boldsymbol{E}_b$ with Rabi frequencies $\Omega_a = \boldsymbol{E}_a \cdot \boldsymbol{d}_{13}/2\hbar$ and $\Omega_b = \boldsymbol{E}_b \cdot \boldsymbol{d}_{23}/2\hbar$ are used to drive transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, respectively. A third field E_d with Rabi frequency $\Omega_d = \boldsymbol{E}_d \cdot \boldsymbol{d}_{24}/2\hbar$ is applied on the optically allowed transition $|2\rangle \leftrightarrow |4\rangle$. γ_{32} and γ_{31} denote the spontaneous decay rates from levels $|3\rangle$ to levels $|2\rangle$ and $|1\rangle$, respectively, $\gamma_3 = \gamma_{32}+\gamma_{31}$; γ_{42} and γ_{41} denote the spontaneous decay rates from level $|4\rangle$ to levels $|2\rangle$ and $|1\rangle$, respectively, $\gamma_4 = \gamma_{42} + \gamma_{41}$. ω_a, ω_b and ω_d are carrier frequencies of the corresponding fields, and $\Delta_a = \omega_{31} - \omega_a$, $\Delta_b = \omega_{32} - \omega_b$ and $\Delta_d = \omega_{42} - \omega_d$ are detunings of the corresponding fields.

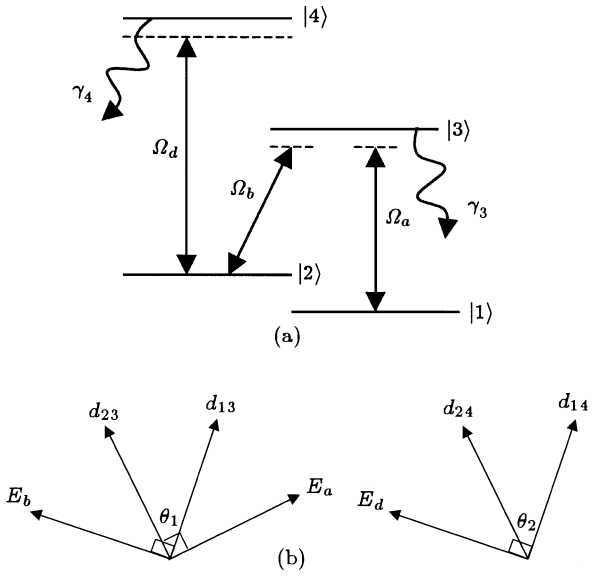


Fig.1. (a) Schematic diagram of a four-level Λ-type atomic system driven by three coherent fields. (b) The arrangement of field polarization required for a single field driving one transition if dipoles are nonorthogonal.

If the third field E_d does not exist, the two fields E_a and E_b form a standard Λ-type configuration for EIT [3] and coherent population trapping (CPT).[21,22] In such a system, the atomic medium is prepared by two coherent light fields into a coherent superposition of the lower-energy states where the population is trapped due to destructive interference between two different absorption paths, and the two fields both exhibit EIT at $\Delta_a = \Delta_b = 0$. But in the presence of the third field E_d, the properties of the medium are completely altered, for the atoms are very sensitive to the processes that alter the coherent superposition between states $|1\rangle$ and $|2\rangle$. As far as the field E_a is concerned, it will no longer exhibit EIT at $\Delta_a = \Delta_b = 0$, but instead, it will be absorbed.[18,19] Then another question arises: in the presence of the third field E_d, what happens to the field E_b? Will it also be absorbed? In the following, we will analyse the properties of the field E_b first by using the density matrix equations. Here, we refer to the fields E_b and E_a as probe field and coupling field, respectively.

In the interaction picture, the semiclassical interaction Hamiltonian of this system in a rotating-wave frame can be written as

$$\begin{aligned} H_{\mathrm{I}} =& \Delta_a |3\rangle\langle 3| + (\Delta_a - \Delta_b)|2\rangle\langle 2| \\ &+ (\Delta_a - \Delta_b + \Delta_d)|4\rangle\langle 4| \\ &- [\Omega_b |3\rangle\langle 2| + \Omega_a |3\rangle\langle 1| \\ &+ \Omega_d |4\rangle\langle 2| + \mathrm{H.c.}]. \end{aligned} \tag{1}$$

The density-matrix equations of motion in the rotating-wave approximation and the dipole approximation can be derived as follows:

$$\begin{aligned}
\dot{\rho}_{33} =& -(\gamma_{32}+\gamma_{31})\rho_{33} + \mathrm{i}\Omega_b\rho_{23} \\
& - \mathrm{i}\Omega_b^*\rho_{32} + \mathrm{i}\Omega_a\rho_{13} - \mathrm{i}\Omega_a^*\rho_{31}, \\
\dot{\rho}_{22} =& \gamma_{32}\rho_{33} + \gamma_{42}\rho_{44} - \mathrm{i}\Omega_b\rho_{23} \\
& + \mathrm{i}\Omega_b^*\rho_{32} - \mathrm{i}\Omega_d\rho_{24} + \mathrm{i}\Omega_d^*\rho_{42}, \\
\dot{\rho}_{11} =& \gamma_{31}\rho_{33} + \gamma_{41}\rho_{44} - \mathrm{i}\Omega_a\rho_{13} + \mathrm{i}\Omega_a^*\rho_{31}, \\
\dot{\rho}_{41} =& [-\Gamma_{41} - \mathrm{i}(\Delta_a - \Delta_b + \Delta_d)]\rho_{41} \\
& - \mathrm{i}\Omega_a\rho_{43} + \mathrm{i}\Omega_d\rho_{21}, \\
\dot{\rho}_{42} =& (-\Gamma_{42} - \mathrm{i}\Delta_d)\rho_{42} + \mathrm{i}\Omega_d(\rho_{22}-\rho_{44}) - \mathrm{i}\Omega_b\rho_{43}, \\
\dot{\rho}_{43} =& [-\Gamma_{43} + \mathrm{i}(\Delta_b - \Delta_d)]\rho_{43} + \mathrm{i}\Omega_d\rho_{23} \\
& - \mathrm{i}\Omega_b^*\rho_{42} - \mathrm{i}\Omega_a^*\rho_{41}, \\
\dot{\rho}_{31} =& (-\Gamma_{31} - \mathrm{i}\Delta_a)\rho_{31} + \mathrm{i}\Omega_b\rho_{21} + \mathrm{i}\Omega_a(\rho_{11}-\rho_{33}), \\
\dot{\rho}_{32} =& (-\Gamma_{32} - \mathrm{i}\Delta_b)\rho_{32} \\
& + \mathrm{i}\Omega_a\rho_{12} - \mathrm{i}\Omega_d\rho_{34} + \mathrm{i}\Omega_b(\rho_{22}-\rho_{33}) \\
\dot{\rho}_{21} =& [-\Gamma_{21} + \mathrm{i}(\Delta_b - \Delta_a)]\rho_{21} - \mathrm{i}\Omega_a\rho_{23} \\
& + \mathrm{i}\Omega_b^*\rho_{31} + \mathrm{i}\Omega_d^*\rho_{41}. \qquad (2)
\end{aligned}$$

Here Γ_{ij} are the coherence decay rates of the corresponding transition $i \leftrightarrow j$, in the radiative limit when there are no dephasing collisions; they are given by $\Gamma_{41} = \Gamma_{42} = \gamma_4/2$, $\Gamma_{31} = \Gamma_{32} = \gamma_3/2$, $\Gamma_{43} = (\gamma_3+\gamma_4)/2$, $\Gamma_{21} = 0$. The above equations are constrained by $\rho_{ij} = \rho_{ji}^*$ and $\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44} = 1$.

As is known, the absorption (or gain) coefficient on transition $|3\rangle \leftrightarrow |2\rangle$ is proportional to the imaginary part of ρ_{23}, which can be obtained numerically from Eqs.(2). We plot the gain (absorption) coefficient $\mathrm{Im}(\rho_{23})$ as a function of the detuning Δ_b in Fig.2(a). In our notation, $\mathrm{Im}(\rho_{23}) > 0$ means that the probe field is amplified. It can be found that, in the presence of the field E_d, amplification occurs at the wings of the resonance region under the selected parameters. In order to discern if the amplification belongs to inversionless gain, we derive the population distribution in the dressed state in the following.

Under the action of the coherent field Ω_a, dressed sublevels $|a+\rangle$ and $|a-\rangle$ can be expressed as

$$\begin{aligned}
|a+\rangle &= \sin\alpha\,|3\rangle + \cos\alpha\,|1\rangle\,, \\
|a-\rangle &= \cos\alpha\,|3\rangle - \sin\alpha\,|1\rangle\,. \qquad (3a)
\end{aligned}$$

Under the action of the coherent field Ω_d, dressed sublevels $|d+\rangle$ and $|d-\rangle$ can be expressed as

$$\begin{aligned}
|d+\rangle &= \sin\beta\,|4\rangle + \cos\beta\,|2\rangle\,, \\
|d-\rangle &= \cos\beta\,|4\rangle - \sin\beta\,|2\rangle\,, \qquad (3b)
\end{aligned}$$

where $\tan 2\alpha = -\Omega_a/\Delta_a$ and $\tan 2\beta = -\Omega_d/\Delta_d$. The eigenenergies of $E_{a\pm}$ and $E_{d\pm}$ are $[-\Delta_a \pm \sqrt{\Delta_a^2+4\Omega_a^2}]/2$ and $[-\Delta_d \pm \sqrt{\Delta_d^2+4\Omega_d^2}]/2$, respectively.

The population distributions in the dressed sublevels are

$$\begin{aligned}
\rho_{a+a+} =& \rho_{33}\sin^2\alpha + \rho_{11}\cos^2\alpha \\
& + (\rho_{13}+\rho_{31})\sin\alpha\cos\alpha, \\
\rho_{a-a-} =& \rho_{33}\cos^2\alpha + \rho_{11}\sin^2\alpha \\
& - (\rho_{13}+\rho_{31})\sin\alpha\cos\alpha, \\
\rho_{d+d+} =& \rho_{44}\sin^2\beta + \rho_{22}\cos^2\beta \\
& + (\rho_{24}+\rho_{42})\sin\beta\cos\beta, \\
\rho_{d-d-} =& \rho_{44}\cos^2\beta + \rho_{22}\sin^2\beta \\
& - (\rho_{24}+\rho_{42})\sin\beta\cos\beta.
\end{aligned}$$

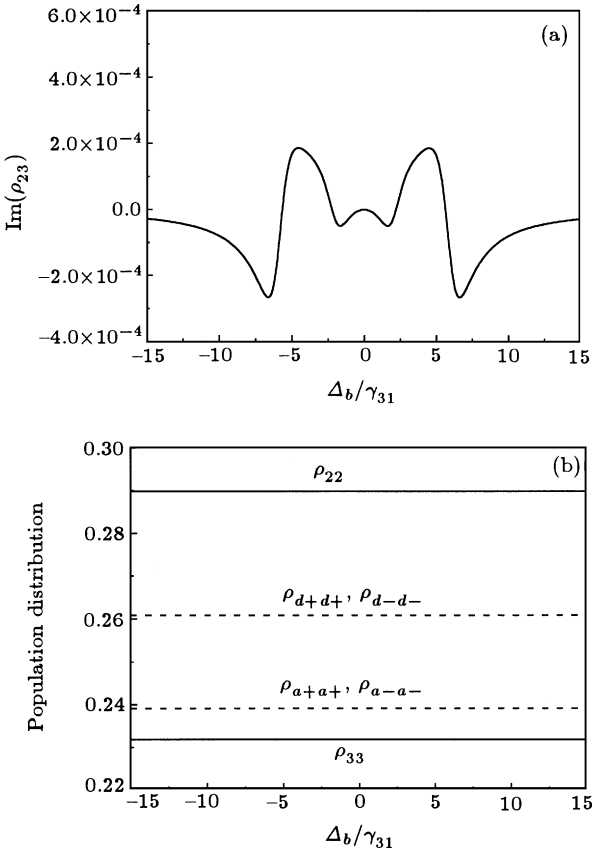


Fig.2. (a) Gain (absorption) coefficient $\mathrm{Im}(\rho_{23})$of the probe field E_b as a function of the probe detuning Δ_b. (b) Population distribution in bare and dressed states. The parameters are $\gamma_{32} = \gamma_{41} = \gamma_{42} = \gamma_{31}$, $\Delta_c = \Delta_d = 0$, $\Omega_a = 4\gamma_{31}$, $\Omega_d = 2\gamma_{31}$, $\Omega_b = 0.01\gamma_{31}$ (In this paper, parameters γ_{32}, γ_{41}, γ_{42}, Δ_b, Ω_a, Ω_d, Ω_b are all in units of γ_{31}).

We plot the population distribution of both the bare and dressed states in Fig.2(b). This shows that

with the selected parameters, there is no population inversion in both the dressed and bare states, i.e. the amplification occurs when $\rho_{33} < \rho_{22}$ and $\rho_{a+a+}, \rho_{a-a-} < \rho_{d+d+}, \rho_{d-d-}$ are fulfilled simultaneously.

3. Gain mechanism: quantum-jump analysis

In what follows, we use the quantum-jump approach to analyse the gain mechanism. This approach is a theoretical tool for analysing the time evolution of an atomic system driven by coherent laser fields and subjected to dissipative processes.[23–26] This allows one to calculate the respective contributions of the various physical processes responsible for the absorption or amplification of the probe field and to get new insight into the relevant physical mechanisms.

In the standard rotating wave approximation, the states of the total system of atom plus laser photons can be grouped into different manifolds ξ (N_a, N_d, N_b) of four states:

$$\begin{aligned}&\xi(N_a, N_d, N_b)\\ =&\{|4, N_a, N_d, N_b\rangle, |1, N_a+1, N_d+1, N_b-1\rangle,\\ &|2, N_a, N_d+1, N_b\rangle, |3, N_a, N_d+1, N_b-1\rangle\}. \quad (5)\end{aligned}$$

In the above equation, N_i denotes the photon number of the field E_i. In this approach, the time evolution of an atom is pictured as consisting of a series of coherent evolution periods separated by quantum-jumps occurring at random time. The quantum jumps are governed by dissipative processes, such as the spontaneous-emission process or the incoherent stimulated emission process. In the absence of these dissipative processes the system would remain forever in the same manifold, in which an atom can be transferred from one of the four states to another, with the corresponding coupling being characterized by the Rabi frequencies of the three fields. The system can leave a manifold ξ only by a quantum jump that brings it into a neighbouring manifold $\xi^{|}$. Thus, if the system begins its coherent evolution in the state $|i\rangle$ of the manifold ξ it remains in this manifold until a quantum jump takes it from state $|j\rangle$ of ξ into a new initial state $|i^{|}\rangle$ of a different manifold $\xi^{|}$. This coherent evolution from state $|i\rangle$ to state $|j\rangle$ in a given manifold is called the coherent period (i, j). Each coherent evolution period (i, j) in a given manifold is characterized by the state i of ξ in which the system enters ξ, after the quantum jump of entry; and by the state j of ξ from which the system leaves ξ, through the quantum jump of exit. The time-averaged evolution from $|i\rangle$ to $|j\rangle$ is described by the Schrödinger equation of the atom plus laser photons system with an effective non-Hermitian Hamiltonian.

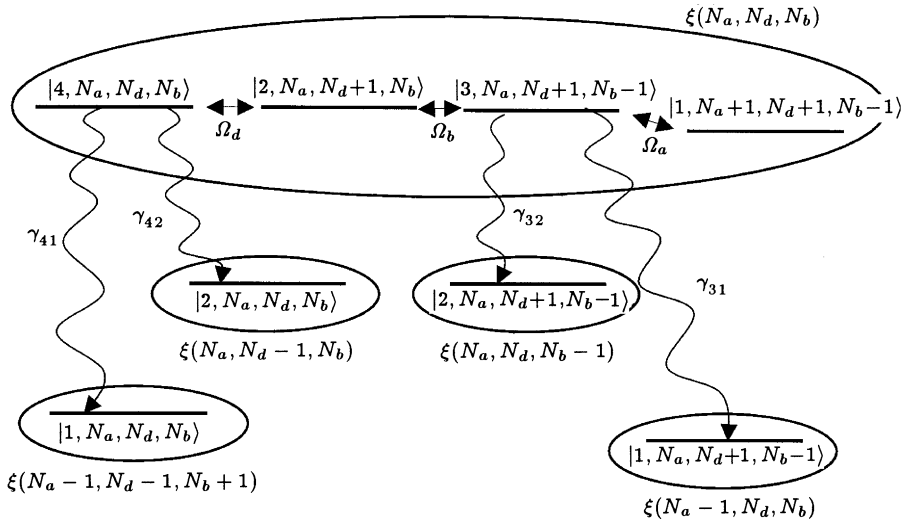


Fig.3. Different manifolds $\xi(N_a, N_d, N_b)$ of the atom plus laser photon system and the allowed quantum-jumps. The coherent evolution in a given manifold is characterized by the Rabi frequencies Ω_a, Ω_d, Ω_b; the dissipative processes are governed by the quantum jumps.

Figure 3 depicts the different manifolds of the atom plus laser photon system and the allowed quantum-jumps, from which we can see that the quantum jumps can take place only when the atom is in states $|3\rangle$ or $|4\rangle$, so the final state of a coherent period must be either $|3\rangle$ or $|4\rangle$. From Fig.3 we also can find

that the initial state of a coherent period must be $|1\rangle$ or $|2\rangle$. So, there are only four different periods in each manifold, namely

Period $(1,4)$ with $\Delta N_a = -1, \Delta N_d = -1, \Delta N_b = 1$,

Period $(2,4)$ with $\Delta N_a = 0, \Delta N_d = -1, \Delta N_b = 0$,

Period $(1,3)$ with $\Delta N_a = -1, \Delta N_d = 0, \Delta N_b = 0$,

Period $(2,3)$ with $\Delta N_a = 0, \Delta N_d = 0, \Delta N_b = -1$.

Of them only two periods (1,4) and (2,3) contribute to changing the probe field photon number. The period (2,3) refers to the one-photon absorption process, and the period (1,4) refers to the three-photon stimulated emission process.

A fundamental quantity here is the probability of a period (i,j), i.e. the probability that a random choice among all periods of the stochastic sequence gives as a result the period (i,j), we denote this as $P(i,j)$. Since each period corresponds to a well-defined change in the photon numbers, the probabilities $P(i,j)$ are directly connected with the absorption or the amplification processes. The probability $P(i,j)$ is related to two factors:

$$P(i,j) = P(i)P(j|i), \tag{6}$$

where $P(i)$ refers to the probability of a randomly chosen period starting from the state $|i\rangle$, $P(j|i)$ refers to the conditional probability that, given that the period has started from $|i\rangle$, it ends in the state $|j\rangle$.

$$H_{\text{eff}} = \begin{bmatrix} 0 & 0 & \Omega_a & 0 \\ 0 & \Delta_a - \Delta_b & \Omega_b & \Omega_d \\ \Omega_a & \Omega_b & -\frac{\gamma_3}{2}\mathrm{i} + \Delta_a & 0 \\ 0 & \Omega_d & 0 & -\frac{\gamma_4}{2}\mathrm{i} + (\Delta_a - \Delta_b + \Delta_d) \end{bmatrix}. \tag{10}$$

The Schrödinger equation associated with the above effective Hamiltonian leads to the following differential equations for the probability amplitude $c_{ij}(\tau) = \langle j| \exp(-\mathrm{i}H_{\text{eff}}\tau/\hbar) |i\rangle$, which describes the coherent evolution within the manifold from the state $|i\rangle$ at time $\tau = 0$:

$$\begin{aligned}
\dot{c}_{i4}(\tau) &= -\left(\frac{\gamma_4}{2} + \mathrm{i}(\Delta_a - \Delta_b + \Delta_d)\right) c_{i4}(\tau) + \mathrm{i}\Omega_d c_{i2}(\tau), \\
\dot{c}_{i3}(\tau) &= -\left(\frac{\gamma_3}{2} + \mathrm{i}\Delta_a\right) c_{i3}(\tau) + \mathrm{i}\Omega_b c_{i2}(\tau) + \mathrm{i}\Omega_a c_{i1}(\tau), \\
\dot{c}_{i2}(\tau) &= \mathrm{i}\Omega_d c_{i4}(\tau) + \mathrm{i}\Omega_b c_{i3}(\tau) - \mathrm{i}(\Delta_a - \Delta_b) c_{i2}(\tau), \\
\dot{c}_{i1}(\tau) &= \mathrm{i}\Omega_a c_{i3}(\tau), \\
c_{ij}(\tau = 0) &= \delta_{ij}.
\end{aligned} \tag{11}$$

We first calculate the probability $P(i)$, which satisfies two conditions:

$$\sum_i P(i) = 1, \quad P(i) = \sum_j P(j) Q(\text{in} : i|\text{in} : j), \tag{7}$$

where $Q(\text{in} : i|\text{in} : j)$ denotes the conditional probability to start a period from the state $|i\rangle$, given that the previous period has started from the state $|j\rangle$. Taking $\Omega_b \ll \gamma_{41}, \gamma_{42}, \gamma_{31}, \gamma_{32}, \Omega_a, \Omega_d$, we have

$$\begin{aligned}
Q(\text{in} : i|\text{in} : 4) &= \frac{\gamma_{41}}{\gamma_4}\delta_{i1} + \frac{\gamma_{42}}{\gamma_4}\delta_{i2}, \\
Q(\text{in} : i|\text{in} : 3) &= \frac{\gamma_{31}}{\gamma_3}\delta_{i1} + \frac{\gamma_{32}}{\gamma_3}\delta_{i2}, \\
Q(\text{in} : i|\text{in} : 2) &= \frac{\gamma_{41}}{\gamma_4}\delta_{i1} + \frac{\gamma_{42}}{\gamma_4}\delta_{i2}, \\
Q(\text{in} : i|\text{in} : 1) &= \frac{\gamma_{31}}{\gamma_3}\delta_{i1} + \frac{\gamma_{32}}{\gamma_3}\delta_{i2}.
\end{aligned} \tag{8}$$

From Eqs.(7) and (8) we derive

$$P(1) = \frac{\gamma_{41}\gamma_3}{\gamma_{32}\gamma_4 + \gamma_{41}\gamma_3}, \tag{9a}$$

$$P(2) = \frac{\gamma_{32}\gamma_4}{\gamma_{32}\gamma_4 + \gamma_{41}\gamma_3}, \tag{9b}$$

$$P(3) = P(4) = 0. \tag{9c}$$

In order to calculate the conditional probability $P(j|i)$, we first write the effective non-Hermitian Hamiltonian which governs the time-averaged evolution of the atom from $|i\rangle$ to $|j\rangle$:

The conditional probabilities satisfy

$$P(j|i) = \gamma_j \int_0^\infty |c_{ij}|^2 \mathrm{d}\tau, \tag{12}$$

where γ_j are the rates of the dissipative processes taking population out of states $|j\rangle$, and $\gamma_3 = \gamma_{31} + \gamma_{32}$, $\gamma_4 = \gamma_{41} + \gamma_{42}$.

Using Eqs.(6), (9) and (12), we finally obtain

$$P(1,4) = \frac{\gamma_{41}\gamma_3}{\gamma_{32}\gamma_4 + \gamma_{41}\gamma_3}\gamma_4 \int_0^\infty |c_{14}|^2 \mathrm{d}\tau, \tag{13a}$$

$$P(2,3) = \frac{\gamma_{32}\gamma_4}{\gamma_{32}\gamma_4 + \gamma_{41}\gamma_3}\gamma_3 \int_0^\infty |c_{23}|^2 \mathrm{d}\tau. \tag{13b}$$

The mean change of the number of probe photons per period is

$$\langle \Delta N_b \rangle = P(1,4) - P(2,3). \tag{14}$$

Using numerical calculation, we present in Fig.4 $\langle \Delta N_b \rangle$ and $P(i,j)$ as functions of Δ_b with the same atomic parameters as those used in Fig.2. We can see that $\langle \Delta N_b \rangle$ present a total agreement between the quantum-jump approach and the density-matrix equations. This shows clearly that the asymmetry between the three-photon stimulated emission process and the one-photon absorption process leads to the gain at the wings of the resonant region.

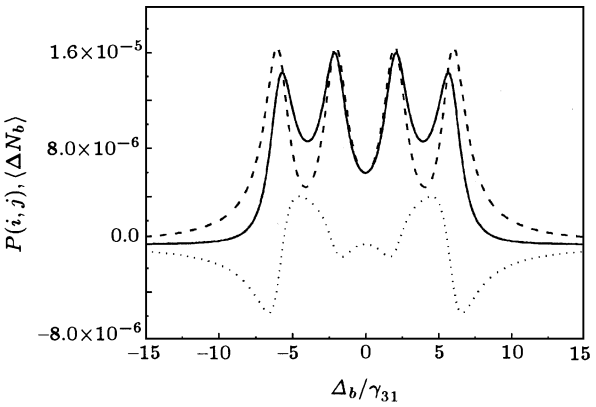


Fig.4. The probability of the coherent evolution period (i,j) between all possible periods, $P(i,j)$, and the mean change of the probe field photon number per period, $\langle \Delta N_b \rangle$, as a function of the probe detuning Δ_b. Solid curve represents three-photon stimulated emission process $P(1,4)$, dashed curve represents one-photon absorption process $P(2,3)$, dotted curve represents $\langle \Delta N_b \rangle$. The parameters are the same as those used in Fig.2. Mean change of the probe field photon number per period $\langle \Delta N_b \rangle$ shows full agreement with the result of the density-matrix method in Fig.2.

4. Effects of SGC on the probe gain

In this system, if the two lower levels are close-lying, the spontaneous emission from excited state $|3\rangle$ ($|4\rangle$) to the lower state $|1\rangle$ can strongly affect the neighbouring transition $|3\rangle \to |2\rangle$ ($|4\rangle \to |2\rangle$),and then interference due to spontaneous emission can arise. The existence of this SGC depends on the nonorthogonality of the two dipole matrix elements, which can be obtained from the mixing of the levels arising from internal fields [27] or external fields.[28−30] It should be noted that the near-degenerated system considered here is different from the ordinary degenerate system. In the ordinary degenerate system, there exists no interference effect, for the ordinary degenerate sublevels are absolutely unrelated and it is impossible for the same vacuum modes to couple them to another level.

The basic equations describing spontaneous emission from excited states to two close-lying lower states are given by (cf. Fig.1 with $\boldsymbol{E}_a = \boldsymbol{E}_b = \boldsymbol{E}_d = 0$)

$$
\begin{aligned}
\dot{\rho}_{33} &= -(\gamma_{32}+\gamma_{31})\rho_{33},\\
\dot{\rho}_{22} &= -\gamma_{32}\rho_{33}+\gamma_{42}\rho_{44},\\
\dot{\rho}_{11} &= \gamma_{31}\rho_{33}+\gamma_{41}\rho_{44},\\
\dot{\rho}_{41} &= -\Gamma_{41}\rho_{41},\\
\dot{\rho}_{43} &= -\Gamma_{43}\rho_{43},\\
\dot{\rho}_{42} &= -\Gamma_{42}\rho_{42},\\
\dot{\rho}_{31} &= -\Gamma_{31}\rho_{31},\\
\dot{\rho}_{32} &= -\Gamma_{32}\rho_{32},\\
\dot{\rho}_{21} &= (\sqrt{\gamma_{31}\gamma_{32}}\rho_{33}\cos\theta_1+\sqrt{\gamma_{41}\gamma_{42}}\rho_{44}\cos\theta_2)\mathrm{e}^{\mathrm{i}\Omega t}. \qquad (15)
\end{aligned}
$$

Here the term $\sqrt{\gamma_{31}\gamma_{32}}\rho_{33}\cos\theta_1$ represents the quantum interference effect between the spontaneous emission pathways from $|3\rangle$ to $|2\rangle$ and $|1\rangle$, and the term $\sqrt{\gamma_{41}\gamma_{42}}\rho_{44}\cos\theta_2$ represents the quantum interference effect between the spontaneous emission pathways from $|4\rangle$ to $|2\rangle$ and $|1\rangle$. $\theta_1(\theta_2)$ is the angle between the two induced dipole moments $\boldsymbol{d}_{13}$ and $\boldsymbol{d}_{23}$ ($\boldsymbol{d}_{14}$ and $\boldsymbol{d}_{24}$). $\hbar\Omega$ is the energy spacing between the two ground levels. It should be noted that only for small Ω are the effects of generated coherence between $|2\rangle$ and $|1\rangle$ important, for large Ω the rapid oscillation in ρ_{21} will average out any such effects.[11−13]

Using the Hamiltonian (1) we obtain the complete set of equations of motion:

$$
\begin{aligned}
\dot{\rho}_{33} &= -(\gamma_{32}+\gamma_{31})\rho_{33}+\mathrm{i}\Omega_b\rho_{23}\\
&\quad -\mathrm{i}\Omega_b^*\rho_{32}+\mathrm{i}\Omega_a\rho_{13}-\mathrm{i}\Omega_a^*\rho_{31},\\
\dot{\rho}_{22} &= \gamma_{32}\rho_{33}+\gamma_{42}\rho_{44}-\mathrm{i}\Omega_b\rho_{23}\\
&\quad +\mathrm{i}\Omega_b^*\rho_{32}-\mathrm{i}\Omega_d\rho_{24}+\mathrm{i}\Omega_d^*\rho_{42},\\
\dot{\rho}_{11} &= \gamma_{31}\rho_{33}+\gamma_{41}\rho_{44}-\mathrm{i}\Omega_a\rho_{13}+\mathrm{i}\Omega_a^*\rho_{31},\\
\dot{\rho}_{41} &= [-\Gamma_{41}-\mathrm{i}(\Delta_a-\Delta_b+\Delta_d)]\rho_{41}\\
&\quad -\mathrm{i}\Omega_a\rho_{43}+\mathrm{i}\Omega_d\rho_{21},\\
\dot{\rho}_{42} &= (-\Gamma_{42}-\mathrm{i}\Delta_d)\rho_{42}+\mathrm{i}\Omega_d(\rho_{22}-\rho_{44})-\mathrm{i}\Omega_b\rho_{43},\\
\dot{\rho}_{43} &= [-\Gamma_{43}+\mathrm{i}(\Delta_b-\Delta_d)]\rho_{43}+\mathrm{i}\Omega_d\rho_{23}\\
&\quad -\mathrm{i}\Omega_b^*\rho_{42}-\mathrm{i}\Omega_a^*\rho_{41},\\
\dot{\rho}_{31} &= (-\Gamma_{31}-\mathrm{i}\Delta_a)\rho_{31}+\mathrm{i}\Omega_b\rho_{21}+\mathrm{i}\Omega_a(\rho_{11}-\rho_{33}),\\
\dot{\rho}_{32} &= (-\Gamma_{32}-\mathrm{i}\Delta_b)\rho_{32}+i\Omega_a\rho_{12}-\mathrm{i}\Omega_d\rho_{34}\\
&\quad +\mathrm{i}\Omega_b(\rho_{22}-\rho_{33}),\\
\dot{\rho}_{21} &= [-\Gamma_{21}+\mathrm{i}(\Delta_b-\Delta_a)]\rho_{21}-\mathrm{i}\Omega_a\rho_{23}+\mathrm{i}\Omega_b^*\rho_{31}\\
&\quad +\mathrm{i}\Omega_d^*\rho_{41}+\eta(\sqrt{\gamma_{31}\gamma_{32}}\rho_{33}\cos\theta_1\\
&\quad +\sqrt{\gamma_{41}\gamma_{42}}\rho_{44}\cos\theta_2). \qquad (16)
\end{aligned}
$$

The above equations include the SGC effect characterized by the term $\eta(\sqrt{\gamma_{31}\gamma_{32}}\rho_{33}\cos\theta_1+\sqrt{\gamma_{41}\gamma_{42}}\rho_{44}\cos\theta_2)$; if the two lower levels lie so closely

that the SGC effect has to be taken into account, then $\eta = 1$, otherwise $\eta = 0$. Note that the field polarizations are so arranged that one field drives only one transition, as shown in Fig.1(b). θ_1, θ_2 are always nonzero, though they could be small.

The existence of the SGC effects makes the system quite sensitive to the phases of the applied fields, so the Rabi frequencies have to be treated as complex parameters. We use ϕ_a, ϕ_b and ϕ_d to denote phases of the corresponding fields, and rewrite $\Omega_a = G_a e^{-i\phi_a}$, $\Omega_b = G_b e^{-i\phi_b}$, $\Omega_d = G_d e^{-i\phi_d}$, $\sigma_{31} = \rho_{31} e^{i\phi_a}$, $\sigma_{32} = \rho_{32} e^{i\phi_b}$, $\sigma_{42} = \rho_{42} e^{i\phi_d}$, $\sigma_{41} = \rho_{41} e^{i(\phi_d + \phi_a - \phi_b)}$, $\sigma_{43} = \rho_{43} e^{i(\phi_d - \phi_b)}$, $\sigma_{21} = \rho_{21} e^{i(\phi_a - \phi_b)}$, $\sigma_{ii} = \rho_{ii}$. Then we get equations about σ_{ij} which are found to be identical with Eq.(16) except that η is replaced by $\eta e^{i\Phi}$ (where $\Phi = \phi_a - \phi_b$, which is the relative phase between the coupling field E_a and the probe field E_b), Ω_a, Ω_b and Ω_d are replaced by G_a, G_b and G_d, respectively. G_a, G_b and G_d are treated as real parameters. For simplicity, we assume $\theta_1 = \theta_2 = \theta$ in the following calculations.

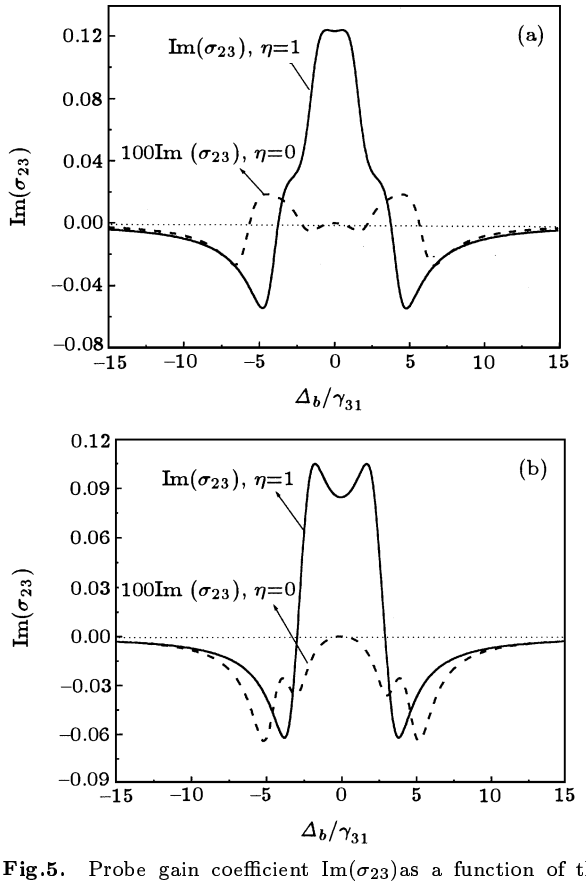


Fig.5. Probe gain coefficient Im(σ_{23})as a function of the probe detuning Δ_b for $\eta = 0$ (solid curve) and $\eta = 1$ (dashed curve). The parameters are $G_b = 0.01\gamma_{31}\sin\theta$, $G_a = 4\gamma_{31}\sin\theta$, $\theta = \pi/4$, $\Phi = \pi$, $G_d = 2\gamma_{31}\sin\theta$ in (a) and $G_d = \gamma_{31}\sin\theta$ in (b). Other parameters are the same as in Fig.2.

In Fig.5(a), with $\theta = \pi/4$, $\Phi = \pi$, $G_b = 0.01\gamma_{31}\sin\theta$, $G_a = 4\gamma_{31}\sin\theta$, $G_d = 2\gamma_{31}\sin\theta$, and other parameters the same as in Fig.2, we plot the probe gain (or absorption) coefficient Im(σ_{23}) as a function of the probe detuning Δ_b for cases not considering the SGC effect ($\eta = 0$) or considering the SGC effect ($\eta = 1$). This shows that, when the SGC effect is included, the probe gain is dramatically enhanced, and the gain peak appears at the resonance region where there was no gain previously. In Fig.5(b), with the smaller value of G_d, there is no gain at all in the whole region when $\eta = 0$, but when the SGC is included, the gain dramatically appears. So the gain (absorption) properties can be greatly affected by the spontaneously generated coherence in this system, SGC plays an important role in achieving the large probe gain.

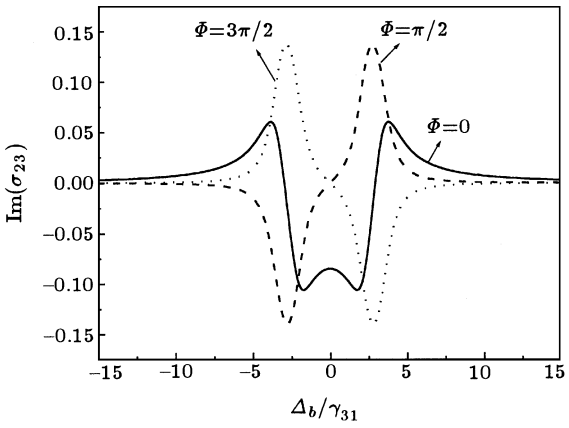


Fig.6. Probe gain coefficient Im(σ_{23}) as a function of Δ_b for three different values of Φ. Other parameters are the same as those in Fig.5(b).

According to Eqs.(16), the gain (absorption) properties of this system depend on the relative phase Φ between the coupling field E_a and the probe field E_b. (Please note that the gain-absorption properties are irrelevant to the phase of the coherent field E_d, which can be found from Eqs.(16)). In Fig.6, we plot probe gain versus probe detuning Δ_b for three different values of Φ, other parameters being the same as in Fig.5(b). It is found that the probe gain is quite sensitive to the relative phase Φ. With different Φ, we get different gain profiles. The dashed curve corresponding to $\Phi = \pi/2$ can be obtained from the dotted curve corresponding to $\Phi = 3\pi/2$ just by replacing Im(σ_{23}) by $-$Im(σ_{23}), and the solid curve corresponding to $\Phi = 0$ in Fig.6 can be obtained from the solid curve corresponding to $\Phi = \pi$ in Fig.5 (b) just by replacing Im(σ_{23}) by $-$Im(σ_{23}).

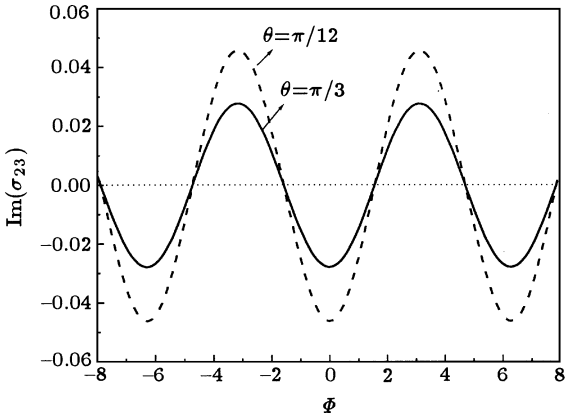


Fig.7. Probe gain coefficient $\mathrm{Im}(\sigma_{23})$ as a function of Φ with $\eta = 1$ at $\Delta_b = 0$ for $\theta = \pi/3$ (solid curve) and $\theta = \pi/12$ (dashed curve). Other parameters are the same as those in Fig.5(b).

In order to get a deeper insight into the dependence of the probe gain on the relative phase Φ, in Fig.7 we plot the probe gain coefficient $\mathrm{Im}(\sigma_{23})$ vs the relative phase Φ for $\theta = \pi/3$ and $\theta = \pi/12$ at $\Delta_b = 0$. This shows that the probe gain is a periodic function of the relative phase Φ with the period of 2π. With a fixed detuning Δ_b, we can dramatically modulate the amplitude of the probe gain just by changing the relative phase between the probe and coupling fields, which can completely change the absorption (gain) properties of the system: e.g., we can make the probe field from being intensely absorbed to being greatly amplified at $\Delta_b = 0$ just by changing the relative phase Φ from 0 to π. From Fig.7, it also can be found that the amplitude of the probe gain is also affected by the parameter θ.

5. Conclusions

In summary, we have investigated the gain properties in a four-level system by both density-matrix and quantum-jump approaches. The quantum-jump approach gives the same results as the density-matrix approach and provides us with the underlying physical mechanisms of the gain process in this system. We show that the gain is originated from the asymmetry between the three-photon stimulated emission and the one-photon absorption process. We also investigated the effects of SGC on the probe gain, and found that the gain can be greatly enhanced by the effect of SGC and can be modulated by the relative phase Φ between the two fields and the angle θ between the two induced dipole moments. The configuration discussed in this paper exists in many real atomic systems, such as Rb, Na etc., and the above results can be experimentally observed.

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