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Transparency induced by two-photon interference in a beam splitter*

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We propose a special two-photon state which is completely transparent in a 50/50 beam splitter. This effect is caused by the destructive two-photon interference and shows the signature of photon entanglement. We find that the symmetry of the two-photon spectrum plays the key role for the properties of two-photon interference.

Keywords: two-photon interference, entangled state, Bell state
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Quantum coherent trapping (QCT) is a very interesting phenomenon in quantum physics. In a quantum interaction system, when a state remains unchanged in the interaction, the effect is called quantum coherent trapping. Physically, the trapping of a quantum state occurs due to destructive quantum interference. The variations of two parts of the initial state in the interaction interfere destructively and cancel each other, so that the quantum state remains invariant. It looks like the interaction does not exist. One of the well known examples of the QCT effect is the dark state in electromagnetically induced transparency (EIT).[1] The topic has been studied intensively and has found important application in quantum information. There are not so many examples of QCT found in real quantum systems. In this work, we find QCT in two-photon interference. When two photons in a two-photon state are separately incident upon two input ports of a beamsplitter (BS), a new two-photon state is formed in the output due to the reflection and transmission. For a special kind of two-photon wavepacket incident upon a 50/50 BS, the output state is exactly the same as the input, and QCT occurs. It appears as if the BS becomes transparent for this two-photon wavepacket without any reflection. The perfect transparency for the two-photon wavepacket is induced by a destructive interference in BS, and it shows the nature of two-photon entanglement.

A single-photon wavepacket being in two orthogonal polarized modes, H and V, is written as $|\Phi_s\rangle = \sum_{\omega}[C_{\rm H}(\omega)a^{\dagger}_{\rm H}(\omega)+C_{\rm V}(\omega)a^{\dagger}_{\rm V}(\omega)]|0\rangle$. If there are two single-photon wavepackets which are un-entangled and travel in different directions designated by indices 1 and 2, the combined state is written as

$$\begin{aligned}|\Phi_{\rm two}\rangle = \sum_{\omega_1,\omega_2} &[C_{1\rm H}(\omega_1)C_{2\rm H}(\omega_2)a^{\dagger}_{1\rm H}(\omega_1)a^{\dagger}_{2\rm H}(\omega_2)\\ &+ C_{1\rm V}(\omega_1)C_{2\rm V}(\omega_2)a^{\dagger}_{1\rm V}(\omega_1)a^{\dagger}_{2\rm V}(\omega_2)\\ &+ C_{1\rm H}(\omega_1)C_{2\rm V}(\omega_2)a^{\dagger}_{1\rm H}(\omega_1)a^{\dagger}_{2\rm V}(\omega_2)\\ &+ C_{1\rm V}(\omega_1)C_{2\rm H}(\omega_2)a^{\dagger}_{1\rm V}(\omega_1)a^{\dagger}_{2\rm H}(\omega_2)]|0\rangle. \quad (1)\end{aligned}$$

A general form of a two-photon wavepacket in the two polarized modes can be described by

$$|\Phi_{\rm two}\rangle = |\Phi_{\rm HH}\rangle + |\Phi_{\rm VV}\rangle + |\Phi_{\rm HV}\rangle, \quad (2)$$

where

$$|\Phi_{pp}\rangle = \sum_{\omega_1,\omega_2} C_{pp}(\omega_1,\omega_2)a^{\dagger}_{1p}(\omega_1)a^{\dagger}_{2p}(\omega_2)|0\rangle, \quad p = {\rm H,V} \quad (3a)$$

$$\begin{aligned}|\Phi_{\rm HV}\rangle = \sum_{\omega_1,\omega_2} &[C_{\rm HV}(\omega_1,\omega_2)a^{\dagger}_{1\rm H}(\omega_1)a^{\dagger}_{2\rm V}(\omega_2)\\ &+ C_{\rm VH}(\omega_1,\omega_2)a^{\dagger}_{1\rm V}(\omega_1)a^{\dagger}_{2\rm H}(\omega_2)]|0\rangle. \quad (3b)\end{aligned}$$

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In the case of only one polarization available for a two-photon wavepacket, one has $|\Phi_{\rm two}\rangle = |\Phi_{pp}\rangle$.

A BS performs a linear transform to two input optical beams. In the quantum regime, the bosonic commutation must be satisfied for all the field operators. So, for a lossless BS, the general transformation between the input and output field operators obeys [2]

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = S(\theta,\phi_\tau,\phi_\rho)\begin{pmatrix} a_1 \\ a_2 \end{pmatrix},$$
$$S(\theta,\phi_\tau,\phi_\rho) = \begin{pmatrix} {\rm e}^{{\rm i}\phi_\tau}\cos\theta & {\rm e}^{{\rm i}\phi_\rho}\sin\theta \\ -{\rm e}^{-{\rm i}\phi_\rho}\sin\theta & {\rm e}^{-{\rm i}\phi_\tau}\cos\theta \end{pmatrix}, \quad (4)$$

where a_i and b_i are the field annihilation operators for the input and output ports, respectively. The subscript i ($i = 1,2$) symbolizes the ports in the same propagation direction. θ characterizes both the reflection and the transmission rates: for instance, $\theta = \pi/4$ for a 50/50 BS. ϕ_τ and ϕ_ρ are two phases allowed in the unitary transformation (4). The transform of wavevectors in the S-picture corresponding to the operator transformation (4) is written as $|\Psi\rangle_{\rm out} = U|\Psi\rangle_{\rm in}$ where U has been given in Ref.[2]. In this paper, we use, alternatively, a simpler method to evaluate the output wavevector. It duplicates the method in a dynamic quantum system: when the evolution of operators is known in the H-picture, one may obtain the state evolution in the S-picture without solving the Schödinger equation.[3,4] If the initial state is known as $|\Psi\rangle_{\rm in} = f(a_1,a_1^\dagger,a_2,a_2^\dagger)|0\rangle$, the output state is obtained as

$$\begin{aligned}|\Psi\rangle_{\rm out} =& U|\Phi\rangle_{\rm in} = Uf(a_1,a_1^\dagger,a_2,a_2^\dagger)U^{-1}U|0\rangle \\ =& f(\overline{b}_1,\overline{b}_1^\dagger,\overline{b}_2,\overline{b}_2^\dagger)|0\rangle, \end{aligned} \quad (5)$$

where $\overline{b}_i \equiv Ua_iU^{-1}$ and $\overline{b}_i^\dagger \equiv Ua_i^\dagger U^{-1} (i = 1,2)$. Because $b_i = U^{-1}a_iU$ is known due to Eq.(4), one may obtain its inverse transform as

$$\begin{aligned}\begin{pmatrix} \overline{b}_1 \\ \overline{b}_2 \end{pmatrix} &= S^{-1}(\theta,\phi_\tau,\phi_\rho)\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \\ &= S(-\theta,-\phi_\tau,\phi_\rho)\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}. \end{aligned} \quad (6)$$

For a 50/50 BS, we may write

$$\begin{aligned}\overline{b}_1^\dagger(\omega) &= [{\rm e}^{{\rm i}\phi_\tau}a_1^\dagger(\omega) - {\rm e}^{-{\rm i}\phi_\rho}a_2^\dagger(\omega)]/\sqrt{2}, \\ \overline{b}_2^\dagger(\omega) &= [{\rm e}^{{\rm i}\phi_\rho}a_1^\dagger(\omega) + {\rm e}^{-{\rm i}\phi_\tau}a_2^\dagger(\omega)]/\sqrt{2}. \end{aligned} \quad (7)$$

Equation (7) is valid for a given polarization.

According to Eq.(5), one obtains the output state for the input state (2),

$$|\Psi_{\rm two}\rangle_{\rm out} = |\Psi_{\rm HH}\rangle_{\rm out} + |\Psi_{\rm VV}\rangle_{\rm out} + |\Psi_{\rm HV}\rangle_{\rm out}, \quad (8)$$

where

$$|\Psi_{pp}\rangle_{\rm out} = \sum_{\omega_1,\omega_2} C_{pp}(\omega_1,\omega_2)\overline{b}_{1p}^\dagger(\omega_1)\overline{b}_{2p}^\dagger(\omega_2)|0\rangle, \quad p = {\rm H,V} \quad (9a)$$

$$\begin{aligned}|\Psi_{\rm HV}\rangle_{\rm out} =& \sum_{\omega_1,\omega_2}[C_{\rm HV}(\omega_1,\omega_2)\overline{b}_{1\rm H}^\dagger(\omega_1)\overline{b}_{2\rm V}^\dagger(\omega_2) \\ &+ C_{\rm VH}(\omega_1,\omega_2)\overline{b}_{1\rm V}^\dagger(\omega_1)\overline{b}_{2\rm H}^\dagger(\omega_2)]|0\rangle. \end{aligned} \quad (9b)$$

Note that there is no interference between the states $|\Phi_{\rm HH}\rangle, |\Phi_{\rm VV}\rangle$ and $|\Phi_{\rm HV}\rangle$, since they are distinguishable. Using Eq.(7), one obtains

$$\begin{aligned}|\Psi_{pp}\rangle_{\rm out} =& (1/2)\sum_{\omega_1,\omega_2} C_{pp}(\omega_1,\omega_2)\{[a_{1p}^\dagger(\omega_1)a_{1p}^\dagger(\omega_2){\rm e}^{{\rm i}\phi} \\ &- a_{2p}^\dagger(\omega_1)a_{2p}^\dagger(\omega_2){\rm e}^{-{\rm i}\phi}] + [a_{1p}^\dagger(\omega_1)a_{2p}^\dagger(\omega_2) \\ &- a_{2p}^\dagger(\omega_1)a_{1p}^\dagger(\omega_2)]\}|0\rangle, \end{aligned} \quad (10)$$

where $\phi = \phi_\tau + \phi_\rho$. In the summation, when the frequency variables, ω_1 and ω_2, are taken in the whole frequency space, (ω_1,ω_2) and (ω_2,ω_1) contribute to two indistinguishable states which should be added together. For doing this, we may take $\sum_{\omega_1,\omega_2} = \sum_{\omega_1<\omega_2} + \sum_{\omega_1=\omega_2} + \sum_{\omega_1>\omega_2}$, and then exchange the variables ω_1 and ω_2 in the last summation. As a result, Eq.(10) is written as

$$\begin{aligned}|\Psi_{pp}\rangle_{\rm out} =& (1/2)\Bigg(\sum_{\omega_1<\omega_2}\{[C_{pp}(\omega_1,\omega_2) + C_{pp}(\omega_2,\omega_1)] \\ &\times [a_{1p}^\dagger(\omega_1)a_{1p}^\dagger(\omega_2){\rm e}^{{\rm i}\phi} - a_{2p}^\dagger(\omega_1)a_{2p}^\dagger(\omega_2){\rm e}^{-{\rm i}\phi}] \\ &+ [C_{pp}(\omega_1,\omega_2) - C_{pp}(\omega_2,\omega_1)] \\ &\times [a_{1p}^\dagger(\omega_1)a_{2p}^\dagger(\omega_2) - a_{1p}^\dagger(\omega_2)a_{2p}^\dagger(\omega_1)]\} \\ &+ \sum_\omega C_{pp}(\omega,\omega)[(a_{1p}^\dagger(\omega))^2{\rm e}^{{\rm i}\phi} \\ &- (a_{2p}^\dagger(\omega))^2{\rm e}^{-{\rm i}\phi}]\Bigg)|0\rangle. \end{aligned} \quad (11)$$

In the above equation, the first and last terms describe the states in which two photons exit from the same port. The second term describes the state in which two photons exit from different ports, causing a 'click-click' in the coincidence measurement. Simi-

larly, Eq.(9b) is obtained as

$$\begin{aligned}|\Psi_{\rm HV}\rangle_{\rm out} =&(1/2)\sum_{\omega_1,\omega_2}\{[C_{\rm HV}(\omega_1,\omega_2)+C_{\rm VH}(\omega_2,\omega_1)]\\ &\times[a^\dagger_{1\rm H}(\omega_1)a^\dagger_{1\rm V}(\omega_2)e^{i\phi}\\ &-a^\dagger_{2\rm H}(\omega_1)a^\dagger_{2\rm V}(\omega_2)e^{-i\phi}]\\ &+[C_{\rm HV}(\omega_1,\omega_2)-C_{\rm VH}(\omega_2,\omega_1)]\\ &\times[a^\dagger_{1\rm H}(\omega_1)a^\dagger_{2\rm V}(\omega_2)-a^\dagger_{2\rm H}(\omega_1)a^\dagger_{1\rm V}(\omega_2)]\}|0\rangle.\end{aligned} \tag{12}$$

Again, the first term and the second term describe the states in which two photons exit from the same port or different ports, respectively.

Equations (11) and (12) imply that the symmetry of the spectra may result in constructive or destructive interference. The symmetric spectra read as

$$C_{pp}(\omega_1,\omega_2)\equiv C_{pp}(\omega_2,\omega_1),\qquad p=\rm H,V, \tag{13a}$$

$$C_{\rm HV}(\omega_1,\omega_2)\equiv C_{\rm VH}(\omega_2,\omega_1). \tag{13b}$$

In this case, the second terms in Eqs.(11) and (12) vanish. It causes the so-called perfect 'coalescence interference' (CI). That is, two photons coalesce together, thus the coincidence probability is null. This effect has attracted attention in the literature.[5,6] In contrast, the anti-symmetric spectra satisfy the condition

$$C_{pp}(\omega_1,\omega_2)\equiv -C_{pp}(\omega_2,\omega_1),\qquad p=\rm H,V, \tag{14a}$$

$$C_{\rm HV}(\omega_1,\omega_2)\equiv -C_{\rm VH}(\omega_2,\omega_1). \tag{14b}$$

Equation (14a) implies $C_{pp}(\omega,\omega)=0$. It has been shown in Eqs.(11) and (12) that under condition (14) the two output photons never go together. Precisely, Eqs.(11) and (12) are respectively reduced to

$$\begin{aligned}|\Psi_{pp}\rangle_{\rm out} =&\sum_{\omega_1<\omega_2}C_{pp}(\omega_1,\omega_2)[a^\dagger_{1p}(\omega_1)a^\dagger_{2p}(\omega_2)\\ &-a^\dagger_{1p}(\omega_2)a^\dagger_{2p}(\omega_1)]|0\rangle\\ =&\sum_{\omega_1,\omega_2}C_{pp}(\omega_1,\omega_2)a^\dagger_{1p}(\omega_1)a^\dagger_{2p}(\omega_2)|0\rangle\\ =&|\Phi_{\rm pp}\rangle\end{aligned} \tag{15}$$

and

$$\begin{aligned}|\Psi_{\rm HV}\rangle_{\rm out} =&\sum_{\omega_1,\omega_2}C_{\rm HV}(\omega_1,\omega_2)[a^\dagger_{1\rm H}(\omega_1)a^\dagger_{2\rm V}(\omega_2)\\ &-a^\dagger_{2\rm H}(\omega_1)a^\dagger_{1\rm V}(\omega_2)]|0\rangle\\ =&\sum_{\omega_1,\omega_2}[C_{\rm HV}(\omega_1,\omega_2)a^\dagger_{1\rm H}(\omega_1)a^\dagger_{2\rm V}(\omega_2)\\ &+C_{\rm VH}(\omega_2,\omega_1)a^\dagger_{2\rm H}(\omega_1)a^\dagger_{1\rm V}(\omega_2)]|0\rangle\\ =&|\Phi_{\rm HV}\rangle.\end{aligned} \tag{16}$$

That is, the output state is identical to the input state. Therefore, the quantum state of the two-photon wavepacket satisfying condition (14) is the eigenstate with the unit eigenvalue for the interaction of a 50/50 BS. Note that the eigenstate can be any two-photon wavepacket satisfying condition (14). Physically, this two-photon wavepacket is perfectly transparent in passing through the BS. However, conditions (14a) and (14b) are sufficient and necessary for Eqs.(15) and (16), respectively.

We first consider the case of two un-entangled single-photon wavepackets. If the two independent wavepackets are identical, $C_{1p}(\omega)\equiv C_{2p}(\omega)$ (p=H,V), the symmetric condition (13) is satisfied and the perfect CI effect occurs. As a matter of fact, the experiments have shown that the CI may occur for both entangled and un-entangled two-photon wavepackets,[5−7] so that it is not the evidence of entanglement. In contrast, we can prove that the transparency effect cannot be realized for two independent single-photon wavepackets. Assuming the spectrum of un-entangled wavepacket (1) satisfying condition (14a), one has

$$\begin{aligned}\frac{C_{1p}(\omega_1)}{C_{2p}(\omega_1)}\equiv&-\frac{C_{1p}(\omega_2)}{C_{2p}(\omega_2)}\\ =&\text{a constant independent of }\omega_i.\end{aligned} \tag{7}$$

It is immediate to presume a null spectrum. Therefore, the transparent two-photon wavepacket must be in entanglement.

Then, we inspect the Bell states in the monochromatic beams. If two photons with frequencies Ω_1 and Ω_2 are in the same polarization, the Bell states can be written as

$$\begin{aligned}|\Phi^\pm\rangle =&(1/\sqrt{2})\sum_{\omega_1,\omega_2}[\delta(\omega_1-\Omega_1)\delta(\omega_2-\Omega_1)\\ &\pm\delta(\omega_1-\Omega_2)\delta(\omega_2-\Omega_2)]a^\dagger_1(\omega_1)a^\dagger_2(\omega_2)|0\rangle,\end{aligned} \tag{18a}$$

$$\begin{aligned}|\Psi^\pm\rangle =&(1/\sqrt{2})\sum_{\omega_1,\omega_2}[\delta(\omega_1-\Omega_1)\delta(\omega_2-\Omega_2)\\ &\pm\delta(\omega_1-\Omega_2)\delta(\omega_2-\Omega_1)]a^\dagger_1(\omega_1)a^\dagger_2(\omega_2)|0\rangle.\end{aligned} \tag{18b}$$

It can be readily checked that $|\Phi^\pm\rangle$ and $|\Psi^+\rangle$ have the symmetric spectra, and $|\Psi^-\rangle$ the anti-symmetric spectrum. This is also true for the Bell states with

the polarization entanglement,

$$
\begin{aligned}
|\Phi^{\pm}\rangle =&(1/\sqrt{2}) \\
&\times \sum_{\omega_1,\omega_2} [\delta(\omega_1-\Omega_{\rm H})\delta(\omega_2-\Omega_{\rm H}) \\
&\times a^{\dagger}_{1{\rm H}}(\omega_1)a^{\dagger}_{2{\rm H}}(\omega_2) \\
&\pm \delta(\omega_1-\Omega_{\rm V})\delta(\omega_2-\Omega_{\rm V}) \\
&\times a^{\dagger}_{1{\rm V}}(\omega_1)a^{\dagger}_{2{\rm V}}(\omega_2)]|0\rangle, \qquad (19a)\\
|\Psi^{\pm}\rangle =&(1/\sqrt{2}) \\
&\times \sum_{\omega_1,\omega_2} [\delta(\omega_1-\Omega_{\rm H})\delta(\omega_2-\Omega_{\rm V}) \\
&\times a^{\dagger}_{1{\rm H}}(\omega_1)a^{\dagger}_{2{\rm V}}(\omega_2) \\
&\pm \delta(\omega_1-\Omega_{\rm V})\delta(\omega_2-\Omega_{\rm H}) \\
&\times a^{\dagger}_{1{\rm V}}(\omega_1)a^{\dagger}_{2{\rm H}}(\omega_2)]|0\rangle. \qquad (19b)
\end{aligned}
$$

Here we have assumed that the two photons of different polarizations may have different frequencies. Therefore, for both cases, the three 'symmetric' Bell states, $|\Phi^{\pm}\rangle$ and $|\Psi^{+}\rangle$, carry out the perfect CI, and the anti-symmetric Bell state, $|\Psi^{-}\rangle$, is perfectly transparent in a 50/50 BS.

In the spontaneous parametric down-conversion (SPDC), a pair of converted photons can be entangled, and this is a good source to observe the transparency effect. For SPDC of type I, one may have a symmetric spectrum

$$Q(\omega_1,\omega_2) = g(\omega_1+\omega_2-2\Omega)f(\omega_1-\Omega)f(\omega_2-\Omega), \quad (20)$$

where Ω is the central frequency for both beams, and f describes the spectral profile for each single photon.[8] This symmetric two-photon wavepacket can perform a perfect CI as was observed in experiments.[5,6] When $g(x) = 1$, Eq.(20) describes two independent single-photon wavepackets, i.e. an un-entangled two-photon wavepacket, which can show perfect CI also.[7] Now, we want to show that an anti-symmetric spectrum can be produced by introducing suitable phase shifts to the beams. Figure 1 is the schematic diagram of the two-photon interference caused by introducing different paths for the two beams. We assume a two-photon wavepacket described by Eq. (20). Let beam 1 to be passing a 50/50 BS1, then the two-photon state after BS1 is written as

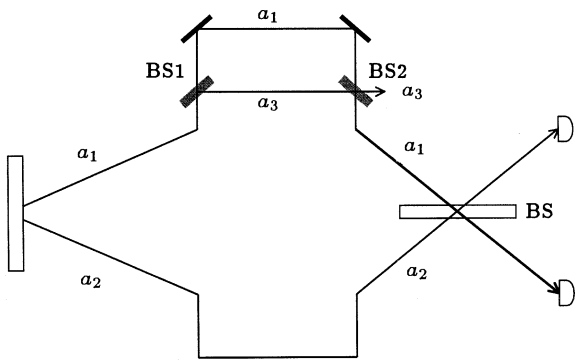


Fig.1. Schematic diagram of two-photon interference by introducing different paths to the two beams.

$$
\begin{aligned}
&(1/\sqrt{2})\sum_{\omega_1,\omega_2} Q(\omega_1,\omega_2)\bar{b}^{\dagger}_1(\omega_1)a^{\dagger}_2(\omega_2)|0\rangle \\
=&(1/\sqrt{2})\sum_{\omega_1,\omega_2} Q(\omega_1,\omega_2)[a^{\dagger}_1(\omega_1){\rm e}^{{\rm i}\phi_{1\tau}} \\
&- a^{\dagger}_3(\omega_1){\rm e}^{-{\rm i}\phi_{1\rho}}]a^{\dagger}_2(\omega_2)|0\rangle, \qquad (21)
\end{aligned}
$$

where $\phi_{1\tau}$ and $\phi_{1\rho}$ are the two phases introduced by BS1. By taking into account the three beams, a_1, a_3 and a_2, travelling the different paths, L_l, L_s and z_2, respectively, the phases to be introduced in Eq.(21) are

$$
\begin{aligned}
&(1/\sqrt{2})\sum_{\omega_1,\omega_2} Q(\omega_1,\omega_2)[a^{\dagger}_1(\omega_1){\rm e}^{{\rm i}(\omega_1 L_l/c+\phi_{1\tau})} \\
&- a^{\dagger}_3(\omega_1){\rm e}^{{\rm i}(\omega_1 L_s/c-\phi_{1\rho})}]a^{\dagger}_2(\omega_2){\rm e}^{{\rm i}\omega_2 z_2/c}|0\rangle. \qquad (22)
\end{aligned}
$$

Then, beams a_1 and a_3 are mixed in the BS2. After BS2, the two-photon state is obtained as

$$
\begin{aligned}
&(1/\sqrt{2})\sum_{\omega_1,\omega_2} Q(\omega_1,\omega_2)[\bar{b}^{\dagger}_1(\omega_1){\rm e}^{{\rm i}(\omega_1 L_l/c+\phi_{1\tau})} \\
&- \bar{b}^{\dagger}_3(\omega_1){\rm e}^{{\rm i}(\omega_1 L_s/c-\phi_{1\rho})}]a^{\dagger}_2(\omega_2){\rm e}^{{\rm i}\omega_2 z_2/c}]|0\rangle \\
=&(1/2)\sum_{\omega_1,\omega_2} Q(\omega_1,\omega_2)\{[a^{\dagger}_1(\omega_1){\rm e}^{{\rm i}\phi_{2\tau}} \\
&- a^{\dagger}_3(\omega_1){\rm e}^{-{\rm i}\phi_{2\rho}}]{\rm e}^{{\rm i}(\omega_1 L_l/c+\phi_{1\tau})} \\
&- [a^{\dagger}_1(\omega_1){\rm e}^{{\rm i}\phi_{2\rho}} + a^{\dagger}_3(\omega_1){\rm e}^{-{\rm i}\phi_{2\tau}}]{\rm e}^{{\rm i}(\omega_1 L_s/c-\phi_{1\rho})}\} \\
&\times a^{\dagger}_2(\omega_2){\rm e}^{{\rm i}\omega_2 z_2/c}|0\rangle, \qquad (23)
\end{aligned}
$$

where $\phi_{2\tau}$ and $\phi_{2\rho}$ are the two phases introduced by BS2. Because beam a_3 has no concern with the two-photon interference in the final BS, we may omit it in Eq.(23), so that the two-photon state ready for interference in the final BS is written as

$$
\begin{aligned}
&(1/2)\sum_{\omega_1,\omega_2} Q(\omega_1,\omega_2)[{\rm e}^{{\rm i}(\omega_1 L_l/c+\phi_{1\tau}+\phi_{2\tau})} \\
&- {\rm e}^{{\rm i}(\omega_1 L_s/c-\phi_{1\rho}+\phi_{2\rho})}]{\rm e}^{{\rm i}\omega_2 z_2/c}a^{\dagger}_1(\omega_1)a^{\dagger}_2(\omega_2)]|0\rangle \\
=&{\rm i}{\rm e}^{{\rm i}\phi_-}\sum_{\omega_1,\omega_2} Q(\omega_1,\omega_2){\rm e}^{{\rm i}(\omega_1 z_1+\omega_2 z_2)/c} \\
&\times \sin(\omega_1\Delta L/c+\phi_+)a^{\dagger}_1(\omega_1)a^{\dagger}_2(\omega_2)|0\rangle, \qquad (24)
\end{aligned}
$$

where $z_1 = (L_l + L_s)/2$, $\Delta L = (L_l - L_s)/2$, $\phi_+ = (\phi_{1\tau} + \phi_{2\tau} + \phi_{1\rho} - \phi_{2\rho})/2$ and $\phi_- = (\phi_{1\tau} + \phi_{2\tau} - \phi_{1\rho} + \phi_{2\rho})/2$. Substituting the symmetric spectrum (20) into Eq.(24), one obtains

$$\begin{aligned} &\mathrm{i}\mathrm{e}^{\mathrm{i}\phi_-} \sum_{\nu_1,\nu_2} g(\nu_1+\nu_2) f(\nu_1) f(\nu_2) \mathrm{e}^{\mathrm{i}(\nu_1 z_1 + \nu_2 z_2)/c} \\ &\times \mathrm{e}^{\mathrm{i}\Omega(z_1+z_2)/c} \sin(\nu_1 \Delta L/c + \alpha) a_1^\dagger(\nu_1) a_2^\dagger(\nu_2)|0\rangle, \quad (25) \end{aligned}$$

where $\nu_i = \omega_i - \Omega$, $(i = 1, 2)$ and $\alpha \equiv \Omega \Delta L/c + \phi_+$. If one sets $z_1 = z_2 \equiv z$, and $g(\nu_1 + \nu_2) \to \delta(\nu_1 + \nu_2)$, Eq.(25) becomes

$$\begin{aligned} &\mathrm{i}\mathrm{e}^{\mathrm{i}\phi_-} \mathrm{e}^{\mathrm{i}2\Omega z/c} \sum_{\nu_1,\nu_2} \delta(\nu_1+\nu_2) f(\nu_1) f(\nu_2) \\ &\times \mathrm{e}^{\mathrm{i}(\nu_1+\nu_2) z/c} \sin(\nu_1 \Delta L/c + \alpha) a_1^\dagger(\nu_1) a_2^\dagger(\nu_2)|0\rangle. \quad (26) \end{aligned}$$

The δ-function, which denotes a perfect phase matching in SPDC, assures the anti-symmetry and symmetry of the above spectrum as long as α is taken as $n\pi$ and $(n + 1/2)\pi$, respectively. This method was reported in Refs.[9] and [10], in which the authors wanted to show that two-photon interference occurs even if the two photons 'do not meet'. The experiment [10] shows both the dip and peak in the interference patterns by changing the path difference ΔL. The peak reflects the full coincidence probability. In the present theory, this can be understood as the transparency of the two-photon state.

For SPDC of type II, the two-photon state with polarization entanglement can be generated in the overlap between the o-ray cone and the e-ray cone.[11] We denote by $f_{\mathrm{H}}(\omega - \Omega_{\mathrm{H}})$ and $f_{\mathrm{V}}(\omega - \Omega_{\mathrm{V}})$ the spectral profiles for the o-ray and e-ray, with the central frequencies Ω_{H} and Ω_{V}, respectively. The two-photon wavepacket with polarization entanglement is described by

$$\begin{aligned} &\sum_{\omega_1,\omega_2} g(\omega_1 + \omega_2 - \Omega_p) \\ &\times [f_{\mathrm{H}}(\omega_1 - \Omega_{\mathrm{H}}) f_{\mathrm{V}}(\omega_2 - \Omega_{\mathrm{V}}) a_{1\mathrm{H}}^\dagger(\omega_1) a_{2\mathrm{V}}^\dagger(\omega_2) \\ &+ \mathrm{e}^{\mathrm{i}\alpha} f_{\mathrm{V}}(\omega_1 - \Omega_{\mathrm{V}}) f_{\mathrm{H}}(\omega_2 - \Omega_{\mathrm{H}}) a_{1\mathrm{V}}^\dagger(\omega_1) a_{2\mathrm{H}}^\dagger(\omega_2)]|0\rangle, \\ & \quad (27) \end{aligned}$$

where $g(\omega_1 + \omega_2 - \Omega_p)$ is the phase matching function. The phase α depends on the crystal birefringence, but it can be set as desired by inserting a wave plate in one path of the beam.[11] For $\alpha = 0$ and $\alpha = \pi$, Eq.(27) satisfies conditions (13b) and (14b), respectively. Unlike the frequency entanglement shown above, the symmetry of spectra is not relevant to the particular form of function g. In order to simulate experimental observation, we assume that beam 1(2) travels a path $z_1(z_2)$. The spectra of the input two-photon state are then written as

$$\begin{aligned} C_{\mathrm{HV}}(\omega_1, \omega_2) =& g(\omega_1 + \omega_2 - \Omega_p) f_{\mathrm{H}}(\omega_1 - \Omega_{\mathrm{H}}) \\ &\times f_{\mathrm{V}}(\omega_2 - \Omega_{\mathrm{V}}) \mathrm{e}^{\mathrm{i}(\omega_1 z_1 + \omega_2 z_2)/c}, \\ C_{\mathrm{VH}}(\omega_1, \omega_2) =& \mathrm{e}^{\mathrm{i}\alpha} g(\omega_1 + \omega_2 - \Omega_p) f_{\mathrm{V}}(\omega_1 - \Omega_{\mathrm{V}}) \\ &\times f_{\mathrm{H}}(\omega_2 - \Omega_{\mathrm{H}}) \mathrm{e}^{\mathrm{i}(\omega_1 z_1 + \omega_2 z_2)/c}. \quad (28) \end{aligned}$$

According to Eq.(12), the coincidence probability of 'click-click' detection is given by

$$\begin{aligned} P_{\mathrm{c}} =& (1/2) \int\int |C_{\mathrm{HV}}(\omega_1, \omega_2) - C_{\mathrm{VH}}(\omega_2, \omega_1)|^2 \mathrm{d}\omega_1 \mathrm{d}\omega_2 \\ =& \int\int |g(\nu_1 + \nu_2) f_{\mathrm{H}}(\nu_1) f_{\mathrm{V}}(\nu_2)|^2 \\ &\times \{1 - \cos[(\nu_2 - \nu_1)\Delta z/c \\ &+ (\Omega_{\mathrm{V}} - \Omega_{\mathrm{H}})\Delta z/c - \alpha]\} \mathrm{d}\nu_1 \mathrm{d}\nu_2, \quad (29) \end{aligned}$$

where $\nu_1 = \omega_1 - \Omega_{\mathrm{H}}, \nu_2 = \omega_2 - \Omega_{\mathrm{V}}$, and $\Delta z = z_2 - z_1$ is the path difference. To simplify the calculation, we assume $g(x)$ to be an even function, $g(x) = g(-x)$, and $f_{\mathrm{H}}(x) = f_{\mathrm{V}}(x) = \exp[-x^2/(2\sigma^2)]$ where σ is the bandwidth for both o- and e-beams. The exact expression for the coincidence probability is obtained as

$$\begin{aligned} P_{\mathrm{c}} =& (1/2)\{1 - \cos[(\Omega_{\mathrm{V}} - \Omega_{\mathrm{H}})\Delta z/c - \alpha] \\ &\times \mathrm{e}^{-\frac{1}{2}\sigma^2 \Delta z^2/c^2}\}. \quad (30) \end{aligned}$$

Figure 2 shows the coincidence probability as a function of the normalized path difference for different phases α, in which Fig.2(a) is for the equal central frequency $\Omega_{\mathrm{H}} = \Omega_{\mathrm{V}}$, and Figs.2(b) and (c) are for the frequency differences $\Omega_{\mathrm{V}} - \Omega_{\mathrm{H}} = \sigma$ and 10σ, respectively. In two-photon interference with polarization entanglement, the forms of interference patterns depend on the phase α. Though two photons entering two input ports of BS are in the different polarization modes, the dip of the interference pattern indicating CI still appears. However, the peak displays the perfect transparency. It is also interesting that, for $\alpha = \pi/2$, there is no interference ($P_{\mathrm{c}} = 1/2$), at least at the balanced position as shown in Fig.2(b). This is due to the probability amplitudes involved in the interference being out of phase, though the input two-photon state is in entanglement. Figure 2(c) shows an interference fringe when a large frequency difference, $\Omega_{\mathrm{V}} - \Omega_{\mathrm{H}} = 10\sigma$, is taken, and the transparency effect occurs at the balanced position because of $\alpha = \pi$. The fringe will shift when changing the phase α.

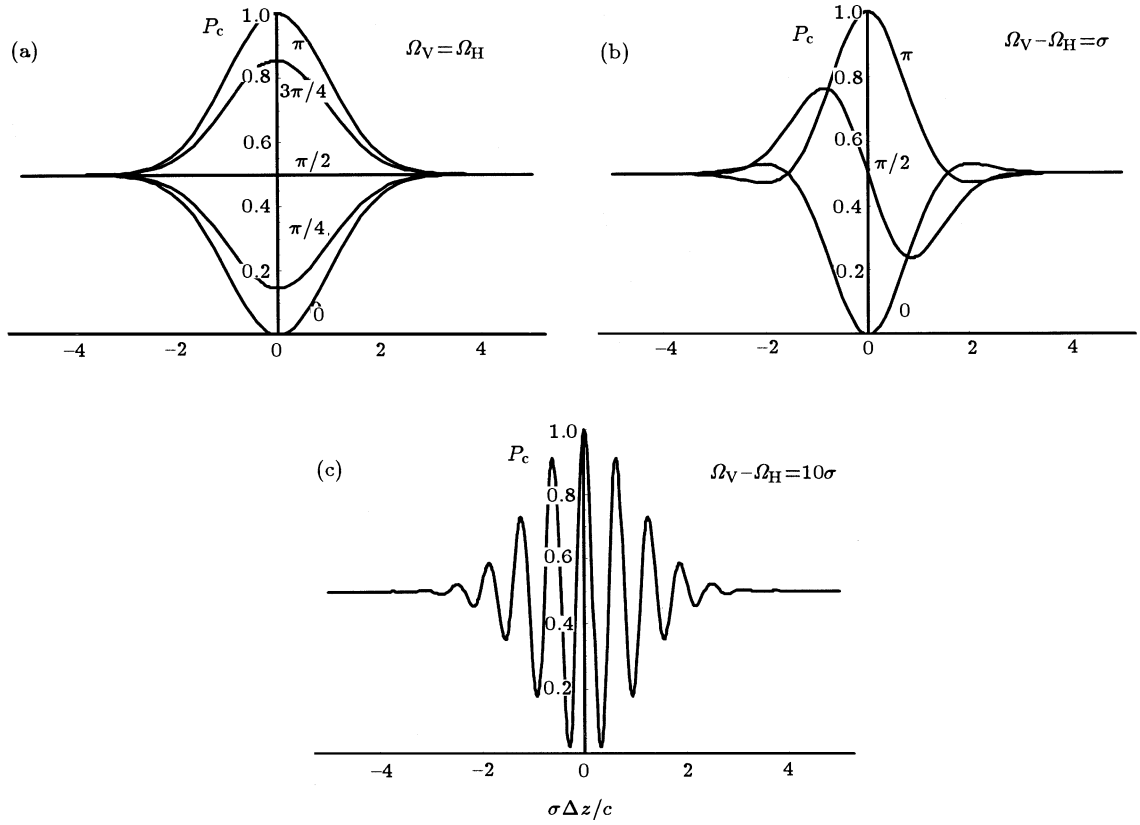


Fig.2. Coincidence probability as a function of normalized path difference for (a) $\Omega_H = \Omega_V$, and $\alpha = 0, \pi/4, \pi/2, 3\pi/4$ and π; (b) $\Omega_V - \Omega_H = \sigma$, and $\alpha = 0, \pi/2$, and π; (c) $\Omega_V - \Omega_H = 10\sigma$, and $\alpha = \pi$.

In conclusion, we find that the symmetry of two-photon spectrum dominates the behaviour of two-photon interference in a BS. When the spectrum is symmetric, which can be acquired for both the entangled and un-entangled two-photon wavepackets, the perfect CI effect occurs. However, when the spectrum is anti-symmetric, it shows the QCT effect in the two-photon interference. The two-photon wavepacket is perfectly transparent in passing the BS. This two-photon transparent state must be in entanglement. We show that the two-photon entangled state with a symmetric spectrum may convert to the two-photon transparent state which has an anti-symmetric spectrum, and it therefore reveals the nature of two-photon entanglement.

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