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On the properties of electromagnetic waves propagating inside moving dielectric media*

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The properties of electromagnetic waves propagating inside isotropic or uniaxial dielectric media moving in an arbitrary direction are analysed. The scalar products of electromagnetic field vectors inside these moving media are investigated in the kEB system from Maxwell's equations and Lorentz-covariant constitutive relations. Several important equations are derived. They are useful in discussing problems such as the energy density and radiation pressure, which are of interest in theoretical studies and many application subjects.

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1. Introduction

The properties of electromagnetic waves propagating inside moving media are the basic theoretical problems of electromagnetic waves.[1−3] They are related to many theoretical studies and many application subjects,[3−5] such as the reflection and transmission of electromagnetic waves by moving media,[1−12] and the propagation of electromagnetic waves in the moving medium in a wave-guide[5] or in moving plasma.[4]. Due to the importance, they have drawn much attention.[4−10] Recently, we have studied problems about the reflection and transmission of the electromagnetic waves by a dielectric medium or conductor, and briefly discussed some properties of electromagnetic waves in a dielectric medium moving in an arbitrary direction.[7−10] For example, the scalar products of field vectors for the reflected wave and transmitted wave vanish under certain special conditions, but do not vanish in the general cases. In this paper, these properties are discussed in detail, and some important equations are given in the general case when the moving medium is isotropic or uniaxial.

2. Method

For the plane wave in the source-free region, Maxwell's equations become

$$\boldsymbol{k}\times\boldsymbol{E}=\omega\boldsymbol{B}, \tag{1a}$$

$$\boldsymbol{k}\cdot B=0, \tag{1b}$$

$$\boldsymbol{k}\times\boldsymbol{H}=-\omega\boldsymbol{D}, \tag{1c}$$

$$\boldsymbol{k}\cdot D=0. \tag{1d}$$

where $\boldsymbol{k}$ is the wavevector and ω is the frequency of the electromagnetic wave.

To deal with a medium in motion, two reference frames in relative motion must be considered. Assume that the medium is moving uniformly with velocity $\boldsymbol{v}$ with respect to the laboratory frame S and the frame S' is at rest with respect to the medium. In the frame S', the constitutive relations of electromagnetic waves inside the medium are as follows in the general case:

$$\boldsymbol{D}'=\bar{\varepsilon}\cdot\boldsymbol{E}', \tag{2a}$$

$$\boldsymbol{B}'=\bar{\mu}\cdot\boldsymbol{H}', \tag{2b}$$

where $\bar{\varepsilon}$ is the permittivity matrix and $\bar{\mu}$ is the permeability matrix of the medium.

Employing Lorentz transformation, the constitutive relations in the moving medium are obtained in the frame S in the $\boldsymbol{k}EB$ system,

$$\begin{bmatrix} c\boldsymbol{D} \\ \boldsymbol{H} \end{bmatrix}=\begin{bmatrix} \bar{\boldsymbol{P}} & \bar{\boldsymbol{L}} \\ \bar{\boldsymbol{M}} & \bar{\boldsymbol{Q}} \end{bmatrix}\cdot\begin{bmatrix} \boldsymbol{E} \\ c\boldsymbol{B} \end{bmatrix}, \tag{3}$$

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where $\bar{P}$, $\bar{Q}$, $\bar{L}$, $\bar{M}$ are 3×3 matrices.

3. Moving isotropic medium

In the moving isotropic medium, $\bar{\varepsilon}$ and $\bar{\mu}$ are very simple, $\bar{\varepsilon}=\varepsilon\bar{I}$, $\bar{\mu}=\mu\bar{I}$,

$$\bar{P}=\frac{\gamma^2}{c\mu}[(n^2-\beta^2)\bar{I}-(n^2-1)\boldsymbol{\beta}\boldsymbol{\beta}], \tag{4a}$$

$$\bar{Q}=\frac{\gamma^2}{c\mu}[(1-n^2\beta^2)\bar{I}+(n^2-1)\boldsymbol{\beta}\boldsymbol{\beta}], \tag{4b}$$

$$\bar{L}=\frac{\gamma^2}{\mu c}(n^2-1)\bar{\boldsymbol{\beta}}, \tag{4c}$$

$$\bar{M}=\frac{\gamma^2}{\mu c}(n^2-1)\bar{\boldsymbol{\beta}}, \tag{4d}$$

where $\boldsymbol{\beta}=\boldsymbol{v}/c$, $\gamma^2=(1-\beta^2)^{-1}$, $n=c(\mu\varepsilon)^{1/2}$. $\bar{I}$ is the unit matrix, $\overline{\boldsymbol{\beta}}$ is defined by $\bar{\boldsymbol{\beta}}\cdot\boldsymbol{A}=\boldsymbol{\beta}\times A$ on arbitrary vector $\boldsymbol{A}$, c is the velocity of light in vacuum.

From Eqs.(3) and (4), $c\boldsymbol{D}$ and $\boldsymbol{H}$ can be expressed by $\boldsymbol{E}$ and $c\boldsymbol{B}$,

$$\begin{aligned}c\boldsymbol{D}=&\frac{\gamma^2}{\mu c}(n^2-\beta^2)\boldsymbol{E}-\frac{\gamma^2}{\mu c}(n^2-1)\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E})\\&+\frac{\gamma^2}{\mu c}(n^2-1)(\boldsymbol{\beta}\times c\boldsymbol{B}),\end{aligned} \tag{5a}$$

$$\begin{aligned}\boldsymbol{H}=&\frac{\gamma^2}{\mu c}(n^2-1)(\boldsymbol{\beta}\times\boldsymbol{E})+\frac{\gamma^2}{\mu c}(1-n^2\beta^2)c\boldsymbol{B}\\&+\frac{\gamma^2}{\mu c}(n^2-1)\boldsymbol{\beta}(\boldsymbol{\beta}\cdot c\boldsymbol{B}).\end{aligned} \tag{5b}$$

When β=0, (5a) and (5b) become $\boldsymbol{D}=\varepsilon\boldsymbol{E}$ and $\boldsymbol{B}=\mu\boldsymbol{H}$. When n=1, they become $\boldsymbol{D}=\varepsilon_0\boldsymbol{E}$ and $\boldsymbol{B}=\mu_0\boldsymbol{H}$. Where ε_0 is the permittivity and μ_0 is the permeability of vacuum.

Dot-multiplying (5) by the wavevector $\boldsymbol{k}$, then we obtain

$$\boldsymbol{k}\cdot\boldsymbol{E}=\frac{n^2-1}{n^2-\beta^2}[(\boldsymbol{\beta}\cdot\boldsymbol{k})(\boldsymbol{\beta}\cdot\boldsymbol{E})+\boldsymbol{\beta}\cdot(\boldsymbol{k}\times c\boldsymbol{B})], \tag{6a}$$

$$\boldsymbol{k}\cdot\boldsymbol{H}=\frac{\gamma^2}{\mu c}(n^2-1)[(\boldsymbol{\beta}\cdot\boldsymbol{k})(\boldsymbol{\beta}\cdot c\boldsymbol{B})-\boldsymbol{\beta}\cdot(\boldsymbol{k}\times\boldsymbol{E})]. \tag{6b}$$

Combining (5a) and (5b) with Maxwell's equation (1), the following equations are obtained:

$$\boldsymbol{E}\cdot\boldsymbol{B}=\boldsymbol{D}\cdot\boldsymbol{H}=0, \tag{7a}$$

$$\boldsymbol{E}\cdot\boldsymbol{D}=\boldsymbol{B}\cdot\boldsymbol{H}=\frac{1}{\omega}\boldsymbol{k}\cdot\boldsymbol{S}, \tag{7b}$$

$$\boldsymbol{E}\cdot\boldsymbol{H}=-c\boldsymbol{D}\cdot c\boldsymbol{B}=\frac{\gamma^2}{\mu c}(n^2-1)(\boldsymbol{\beta}\cdot\boldsymbol{E})(\boldsymbol{\beta}\cdot c\boldsymbol{B}). \tag{7c}$$

Where $\boldsymbol{S}$ is the Poynting's vector, $\boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H}$.

4. Moving uniaxial medium

In the moving uniaxial medium, $\bar{\varepsilon}$ and $\bar{\mu}$ are rather complex. If the axis is along the z-direction of the coordinate system,

$$\bar{\varepsilon}=\begin{bmatrix}\varepsilon&0&0\\0&\varepsilon&0\\0&0&\varepsilon_3\end{bmatrix}, \tag{8a}$$

$$\bar{\mu}=\begin{bmatrix}\mu&0&0\\0&\mu&0\\0&0&\mu_3\end{bmatrix}. \tag{8b}$$

where the subscript '3' denotes the component along the z-direction, etc.

In this case, the formulae are much more complex, so the derivation process is omitted and the results are given as follows directly:

$$\begin{aligned}&(n^2-\beta^2)\boldsymbol{k}\cdot\boldsymbol{E}\\=&(n^2-1)[(\boldsymbol{\beta}\cdot\boldsymbol{k})(\boldsymbol{\beta}\cdot\boldsymbol{E})+\boldsymbol{\beta}\cdot(\boldsymbol{k}\times c\boldsymbol{B})]\\&-\mu c^2(\varepsilon_3-\varepsilon)\left[\left(\frac{1}{\gamma}-1\right)^2\frac{\beta_3^2}{\beta^4}(\boldsymbol{\beta}\cdot\boldsymbol{E})(\boldsymbol{\beta}\cdot\boldsymbol{E})\right.\\&\left.+\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}(E_3\boldsymbol{\beta}\cdot\boldsymbol{k}+k_3\boldsymbol{\beta}\cdot\boldsymbol{E})+k_3E_3\right]\\&-\mu c^2(\varepsilon_3-\varepsilon)(\beta_1cB_2-\beta_2cB_1)\\&\times\left[\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}(\boldsymbol{\beta}\cdot\boldsymbol{k})+k_3\right]\\&-\left(\frac{\mu}{\mu_3}-1\right)(\beta_1k_2-\beta_2k_1)\\&\times\left[\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}\boldsymbol{\beta}\cdot c\boldsymbol{B}+cB_3\right]\\&+(\frac{\mu}{\mu_3}-1)(\beta_1k_2-\beta_2k_1)(\beta_1E_2-\beta_2E_1),\end{aligned} \tag{9a}$$

$$\begin{aligned}&\frac{\mu c}{\gamma^2}\boldsymbol{k}\cdot\boldsymbol{H}\\=&(n^2-1)[(\boldsymbol{\beta}\cdot\boldsymbol{k})(\boldsymbol{\beta}\cdot c\boldsymbol{B})-\boldsymbol{\beta}\cdot(\boldsymbol{k}\times\boldsymbol{E})]\\&+\left(\frac{\mu}{\mu_3}-1\right)\left[\left(\frac{1}{\gamma}-1\right)^2\frac{\beta_3^2}{\beta^4}(\boldsymbol{\beta}\cdot\boldsymbol{k})(\boldsymbol{\beta}\cdot c\boldsymbol{B})\right.\\&\left.+\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}(cB_3\boldsymbol{\beta}\cdot\boldsymbol{k}+k_3\boldsymbol{\beta}\cdot c\boldsymbol{B})+k_3cB_3\right]\\&-\left(\frac{\mu}{\mu_3}-1\right)(\beta_1E_2-\beta_2E_1)\\&\times\left[\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}\boldsymbol{\beta}\cdot\boldsymbol{k}+k_3\right]\\&-\mu c^2(\varepsilon_3-\varepsilon)(\beta_1k_2-\beta_2k_1)\\&\times\left[\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}\boldsymbol{\beta}\cdot\boldsymbol{E}+E_3\right]\\&-\mu c^2(\varepsilon_3-\varepsilon)(\beta_1k_2-\beta_2k_1)(\beta_1cB_2-\beta_2cB_1).\end{aligned} \tag{9b}$$

When μ_3=μ and ε_3=ε, Eq.(9) returns to Eq.(6) immediately.

The scalar products of electromagnetic field vectors can be obtained in the same way as follows:

$$\boldsymbol{E}\cdot\boldsymbol{B}=\boldsymbol{D}\cdot\boldsymbol{H}=0, \tag{10a}$$

$$\boldsymbol{E}\cdot\boldsymbol{D}=\boldsymbol{B}\cdot\boldsymbol{H}=\frac{1}{\omega}\boldsymbol{k}\cdot\boldsymbol{S}, \tag{10b}$$

$$\begin{aligned}\frac{\mu c}{\gamma^2}\boldsymbol{E}\cdot\boldsymbol{H}=&(n^2-1)(\boldsymbol{\beta}\cdot\boldsymbol{E})(\boldsymbol{\beta}\cdot c\boldsymbol{B})+\left(\frac{\mu}{\mu_3}-1\right)\left[\left(\frac{1}{\gamma}-1\right)^2\frac{\beta_3^2}{\beta^4}(\boldsymbol{\beta}\cdot\boldsymbol{E})(\boldsymbol{\beta}\cdot c\boldsymbol{B})\right.\\&\left.+\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}(cB_3\boldsymbol{\beta}\cdot\boldsymbol{E}+E_3\boldsymbol{\beta}\cdot c\boldsymbol{B})+E_3cB_3\right]-\left(\frac{\mu}{\mu_3}-1\right)(\beta_1E_2-\beta_2E_1)\\&\times\left[\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}\boldsymbol{\beta}\cdot\boldsymbol{E}+E_3\right]-\mu c^2(\varepsilon_3-\varepsilon)(\beta_1E_2-\beta_2E_1)\left[\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}\boldsymbol{\beta}\cdot\boldsymbol{E}+E_3\right]\\&-\mu c^2(\varepsilon_3-\varepsilon)(\beta_1E_2-\beta_2E_1)(\beta_1cB_2-\beta_2cB_1),\end{aligned} \tag{10c}$$

$$\begin{aligned}-\frac{\mu c}{\gamma^2}c\boldsymbol{D}\cdot c\boldsymbol{B}=&(n^2-1)(\boldsymbol{\beta}\cdot\boldsymbol{E})(\boldsymbol{\beta}\cdot c\boldsymbol{B})-\mu c^2(\varepsilon_3-\varepsilon)\left[\left(\frac{1}{\gamma}-1\right)^2\frac{\beta_3^2}{\beta^4}(\boldsymbol{\beta}\cdot c\boldsymbol{B})(\boldsymbol{\beta}\cdot\boldsymbol{E})\right.\\&\left.+\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}(cB_3\boldsymbol{\beta}\cdot\boldsymbol{E}+E_3\boldsymbol{\beta}\cdot c\boldsymbol{B})+E_3cB_3\right]-\mu c^2(\varepsilon_3-\varepsilon)(\beta_1cB_2-\beta_2cB_1)\\&\times\left[\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}\boldsymbol{\beta}\cdot c\boldsymbol{B}+cB_3\right]-\left(\frac{\mu}{\mu_3}-1\right)(\beta_1cB_2-\beta_2cB_1)\left[\left(\frac{1}{\gamma}-1\right)\frac{\beta_3}{\beta^2}\boldsymbol{\beta}\cdot c\boldsymbol{B}+cB_3\right]\\&+\left(\frac{\mu}{\mu_3}-1\right)(\beta_1E_2-\beta_2E_1)(\beta_1cB_2-\beta_2cB_1).\end{aligned} \tag{10d}$$

where $\boldsymbol{S}$ is the Poynting vector, $\boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H}$. When $\mu_3=\mu$ and $\varepsilon_3=\varepsilon$, Eqs.(10c) and (10d) are equivalent.

5. Discussion

From Eqs.(6)–(10), the properties of electromagnetic waves in moving media can be discussed in detail as follows.

(1) From (6) and (10), it can be obtained that $\boldsymbol{k}\cdot\boldsymbol{E}=\boldsymbol{k}\cdot\boldsymbol{H}=0$ when β=0 or n=1. This means that the propagating direction of the electromagnetic wave is perpendicular to the direction of $\boldsymbol{E}$ and $\boldsymbol{H}$ when electromagnetic waves propagate in the rest media or in vacuum.[7,8]

(2) From (7a) and (10a), it can be obtained that $\boldsymbol{E}\cdot\boldsymbol{B}=\boldsymbol{D}\cdot\boldsymbol{H}=0$. This means that $\boldsymbol{E}$ is perpendicular to $\boldsymbol{B}$ and $\boldsymbol{D}$ is perpendicular to $\boldsymbol{H}$ in the general case.[7,8]

(3) From (7b) and (10b), it can be obtained that $\boldsymbol{E}\cdot\boldsymbol{D}$ and $\boldsymbol{B}\cdot\boldsymbol{H}$ do not vanish in the general case. The results are helpful to calculate the energy density, momentum density and Maxwell stress tensor of the electromagnetic waves in moving media, and the radiation pressure exerted on moving media by the electromagnetic waves.[2,9,10,11]

(4) From (7c), (10c) and (10d), it can be obtained that $\boldsymbol{E}\cdot\boldsymbol{H}$ and $\boldsymbol{D}\cdot\boldsymbol{B}$ do not vanish in the general case. They are equivalent in a moving isotropic medium but not equivalent in a moving uniaxial medium in the general case. When β=0 or n=1, $\boldsymbol{E}\cdot\boldsymbol{H}=\boldsymbol{D}\cdot\boldsymbol{B}=0$. This means that $\boldsymbol{E}$ is perpendicular to $\boldsymbol{H}$ and $\boldsymbol{D}$ is perpendicular to $\boldsymbol{B}$ when electromagnetic waves propagate in the rest media or in vacuum.

In the studies of electromagnetic waves, the electric field vectors and magnetic field vectors always appear symmetrically in many formulae. The formulae given in this paper are highly symmetrical. The formulae not only make it simple to derive and calculate some quantities about electromagnetic waves in moving media and in many application subjects, but also reveal the nature of electromagnetic waves.

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