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Newtonian forms for the motion of particles and photons in dilaton spacetime*

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Using Newtonian forms for equations of motion, we consider the motion of particles and photons in the dilaton spacetime. Some classical gravitational effects, such as the bending of light rays, the perihelion advance of a planet, the delay of radar echo, and the gravitational redshifts, have been investigated. The results show that the gravitational effects arising from the dilaton can be observed provided that the dilaton is large enough.

Keywords: Newtonian forms, effective index of gravitation, dilaton spacetime
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1. Introduction

The gravitational field was first regarded by Eddington as an optical medium,[1] and this idea has been developed in more detail by others, such as Plebański [2] and de Felice.[3] James Evans [4,5] used the variational principle to derive an equation, with the form $F = ma$, governing the shape of a light ray in a medium of varying refractive index, and solved some problems in gradient-index optics. Evans *et al*[6] and Nandi and Islam [7] demonstrated that the Newtonian forms, $F = ma$, can also be used to describe the motion of particles and photons in the Schwarzschild spacetime, and applied these equations to some problems of physical interest. The approach can also be used to discuss the general relativistic effects on the quantum interference of de Broglie waves associated with thermal neutrons.[8−14]

We all know that the four-dimensional dilaton charged spacetimes obtained in the low-energy effective field theory describing strings have qualitative different properties [15,16] from those that appear in the ordinary Einstein gravitational theory. Therefore, it is worth investigating whether or not the Newtonian forms can be extended to dilaton charged spacetimes, and whether or not the dilaton presents observable gravitational effects. The purpose of this paper is to investigate the motion of particles and photons in dilaton spacetime by applying the Newtonian forms, and then present some interesting effects resulting from dilaton charge.

This paper is organized as follows. In Sec.2, the important formula of Nandi's work is listed for convenience. In Sec.3, we derive equations of motion for both particles and photons in dilaton spacetime. In Sec.4, we use the new equations to show some concrete application, involving the tests of some classic effects in dilaton spacetime. Some concluding remarks are given in the last section.

2. Relativistic particle mechanics in Newtonian form

In many metrics of physical interest, the space part of the line element may be written in isotropic form through transformation of coordinates. If, in addition, the metric has no explicit time dependence, the line element may have the form

$$\begin{aligned} \mathrm{d}s^2 =& \Omega^2(\boldsymbol{r})c_0^2\mathrm{d}t^2 - \Phi^{-2}(\boldsymbol{r}) \\ & \times [\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2] \\ =& \Omega^2(\boldsymbol{r})c_0^2\mathrm{d}t^2 - \Phi^{-2}(\boldsymbol{r})|\mathrm{d}\boldsymbol{r}|^2, \end{aligned} \tag{1}$$

where $\boldsymbol{r} = (r, \theta, \phi)$, Ω and Φ are functions of the isotropic coordinates r, θ, ϕ, and c_0 is the vacuum

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speed of light. The coordinate speed of light, $c(\boldsymbol{r})$, is determined by putting $\mathrm{d}s^2 = 0$:

$$c(\boldsymbol{r}) = \left|\frac{\mathrm{d}(\boldsymbol{r})}{\mathrm{d}t}\right| = c_0\,\Phi(\boldsymbol{r})\Omega(\boldsymbol{r}). \tag{2}$$

Thus the effective index of refraction for light in the gravitational field is

$$n(\boldsymbol{r}) = \Phi^{-1}(\boldsymbol{r})\Omega^{-1}(\boldsymbol{r}). \tag{3}$$

This concept of optical mechanical analogy can be used to recast the geodesic equation for both massive particles and photons into a single exact Newton form of equation of motion, which is given by [6]

$$\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}A^2} = \nabla\left(\frac{1}{2}n^4v^2\right), \qquad \text{(for massive particles)}, \tag{4}$$

$$\frac{d^2\boldsymbol{r}}{dA^2} = \nabla\left(\frac{1}{2}n^2c_0^2\right), \qquad \text{(for light)}, \tag{5}$$

where for particles

$$v = c_0 n^{-1}[1 - c_0^4\Omega^4/H^2]^{1/2}, \tag{6}$$

and for light ($H \longrightarrow \infty$)

$$v = c_0 n^{-1}, \tag{7}$$

and A is a stepping parameter of the path, which is defined by

$$\left|\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}A}\right| = n^2 v, \qquad \text{(for massive particles)}. \tag{8}$$

From Eq.(8), it is easy to find that the relation between A and t is given by

$$\mathrm{d}A = \frac{\mathrm{d}t}{n^2}. \tag{9}$$

Equation (8) can also be treated as a conservation condition of energy

$$\frac{1}{2}\left|\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}A}\right|^2 - \frac{1}{2}n^4v^2 = 0. \tag{10}$$

The left-hand side of Eq.(4), $\mathrm{d}^2\boldsymbol{r}/\mathrm{d}A^2$, takes the form of an acceleration, and the right-hand side of the equation can be considered as a force. The function $-\frac{1}{2}n^4v^2$ plays the role of a potential energy function. The corresponding velocity is $\mathrm{d}\boldsymbol{r}/\mathrm{d}A$. Thus, the analogy of the kinetic energy is $|\mathrm{d}\boldsymbol{r}/\mathrm{d}A|^2$, and the total energy is the sum of the potential and kinetic energy given by Eq.(10).

3. Equations of motion for particles and photons in dilaton spacetime

The dilaton in heterotic string theory has a linear coupling to F^2, so every solution with nonzero $F_{\mu\nu}$ must have a nonconstant dilaton. Thus the Reissner–Nordström solution, which describes charged black holes in general relativity, is not even an approximate solution of string theory. Starting from the four-dimensional low-energy Lagrangian obtained from string theory, $S = \int \mathrm{d}^4x\sqrt{-g}[-R + 2(\nabla\phi)^2 + \mathrm{e}^{-2\phi}F^2]$, where $F_{\mu\nu}$ is the Maxwell field associated with a $U(1)$ subgroup of $E_8 \times E_8$ or $\mathrm{Spin}(32)/Z_2$. Garfinkle *et al*[17] found a solution for a statically charged black hole in the string theory

$$\begin{aligned}\mathrm{d}s^2 =& c_0^2\left(1 - \frac{2m}{r'}\right)\mathrm{d}t^2 - \left(1 - \frac{2m}{r'}\right)^{-1}\mathrm{d}r'^2 \\ & - r'(r' - a)(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2) \\ =& c_0^2\Omega^2(\boldsymbol{r})\mathrm{d}t^2 - \Phi^{-2}(\boldsymbol{r}) \\ & \times (\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)),\end{aligned} \tag{11}$$

where

$$m = MG/C_0^2, \tag{12}$$

$$a = Q^2\mathrm{e}^{2\phi_0}/m, \tag{13}$$

and M is the mass of the central gravitational body, G is the gravitational constant, Q is the magnetic charge, and ϕ_0 is the asymptotic constant value of the dilaton. The presence of the dilaton was found to have important consequence.[15–17]

Here, and hereafter, $(t, r',\ \theta,\ \phi)$ are the standard coordinates, and $(t, r, \theta,\ \phi)$ are the isotropic coordinates. Equating the angular and the radial parts of two line elements, we have

$$r'(r' - a) = \Phi^{-2}r^2, \tag{14}$$

$$\left(1 - \frac{2m}{r'}\right)^{-1}\mathrm{d}r'^2 = \Phi^{-2}\mathrm{d}r^2. \tag{15}$$

Dividing Eq.(15) by Eq.(14) to eliminate Φ, then integrating and considering that at large radial distances r and r' must be asymptotically equal, we obtain

$$\begin{aligned}2r =& r' - \frac{1}{2}(2m + a) \\ & + [r'^2 - (2m + a)r' + 2ma]^{1/2}.\end{aligned} \tag{16}$$

The inverse transformation is

$$r' = r + \frac{1}{2}(2m + a) + (2m - a)^2/16r. \tag{17}$$

Using these transformations, we have

$$\begin{aligned}\Omega^2(r) =&[1-(2m-a)^2/16r^2]^2\\ &\times[1+(2m+a)/2r\\ &+(2m-a)^2/16r^2]^{-2},\end{aligned}\tag{18}$$

$$\Phi^{-2}(r) =[1+(2m+a)/2r+(2m-a)^2/16r^2]^2.\tag{19}$$

The horizons of the spacetime occur at $r_s=(2m-a)/4$, and the effective index of refraction is described by

$$\begin{aligned}n(r) =&[1+(2m+a)/2r+(2m-a)^2/16r^2]^2\\ &\times[1-(2m-a)^2/16r^2]^{-1}.\end{aligned}\tag{20}$$

Let $u\equiv\dfrac{1}{r}$ and $u'\equiv\dfrac{1}{r'}$, then it is easy to show that

$$\mathrm{d}u = n\mathrm{d}u' \quad \text{or} \quad \mathrm{d}r=\Phi\Omega^{-1}\mathrm{d}r',\tag{21}$$

$$u=\Phi^{-1}u' \quad \text{or} \quad r=\Phi r'.\tag{22}$$

Then we have

$$\begin{aligned}\Phi^2(u') =&\frac{1}{4}\{1-\frac{1}{2}(2m+a)u'\\ &+[1-(2m+a)u'+2mau'^2]^{1/2}\}^2,\end{aligned}\tag{23}$$

$$\Omega^2(u') =1-(2m+a)u'+2mau'^2,\tag{24}$$

$$n(u') =\Phi^{-1}(u')\Omega^{-1}(u').\tag{25}$$

The singularities of $n(r)$, or equivalently, the horizons of the spacetime, occur at $r_s=\dfrac{1}{4}(2m-a)$, provided that $2m>a$. The expression for $n(r)$ is valid in the region $r>r_s$. If $2m=a$, $r_s=0$, the event horizon shrinks to zero size. In this case, $n(r)=\left(1+\dfrac{2m}{r}\right)^2$, which is regular everywhere for $r>0$. If $2m<a$, the function $n(r)$ is not singular anywhere since r_s becomes imaginary.

In spherical symmetric spacetimes, n, v, Ω and Φ are functions of the radical coordinate only. The orbit of light or a massive particle lies in a plane containing the force centre and there is a constant of the motion analogous to the angular momentum. Let ϕ be measured in the plane motion, then from Eq.(10) we know

$$h\equiv r^2\frac{\mathrm{d}\phi}{\mathrm{d}A}=\text{const.}\tag{26}$$

It is interesting to note that h is related to the classical mechanical angular momentum per unit mass $h_0\left(\equiv r^2\dfrac{\mathrm{d}\phi}{\mathrm{d}t}\right)$ by

$$h=n^2h_0.\tag{27}$$

From the conservation condition of energy (10), we may write out $\left|\dfrac{\mathrm{d}r}{\mathrm{d}A}\right|^2$ in plane–polar coordinates. Then A can be eliminated by means of Eq.(26). The orbit shape $\phi(r)$ is thereby reduced to an integral expression

$$\phi=h\int^r r^{-2}[n^4v^2-h^2/r^2]^{-1/2}\mathrm{d}r.\tag{28}$$

We can also obtain another useful form of the orbit equation:

$$h^2[(\mathrm{d}u/\mathrm{d}\phi)^2+u^2]-n^4v^2=0.\tag{29}$$

Let $u=1/r$, then differentiating with respect to u, we obtain

$$\frac{\mathrm{d}^2u}{\mathrm{d}\phi^2}+u=h^{-2}\frac{\mathrm{d}}{\mathrm{d}u}(n^4v^2/2).\tag{30}$$

Inserting Eq.(6) for $v(r)$ in the right-hand side of Eq.(29), we find

$$\left(\frac{\mathrm{d}u}{\mathrm{d}\phi}\right)^2+u^2-(c_0/h)^2n^2[1-c_0^4H^{-2}\Omega^2]=0.\tag{31}$$

Using Eqs.(21) and (22), Eq.(31) can be rewritten as

$$\left(\frac{\mathrm{d}u'}{\mathrm{d}\phi}\right)^2+u'^2\Omega^2-(c_0/h)^2[1-c_0^4H^{-2}\Omega^2]=0.\tag{32}$$

Substituting Eq.(24) for $\Omega^2(u')$, then differentiating with respect to ϕ, we obtain

$$\begin{aligned}&\frac{\mathrm{d}^2u'}{\mathrm{d}\phi^2}+u'-\frac{(2m+a)c_0^6}{2h^2H^2}\\ =&-\frac{2mac_0^6}{h^2H^2}u'+\frac{3}{2}(2m+a)u'^2-4mau'^3.\end{aligned}\tag{33}$$

The differential equation (33) describes the motion of massive particles and photons in dilaton spacetime.

4. Some applications

In this section, we utilize Eqs.(26) and (33) to study some interesting gravitational effects in the dilaton spacetime, such as the bending of light rays, the perihelion advance of a planet, the radar echo delay, and the gravitational redshifts.

4.1. Bending of light rays

The equation of motion for a light ray can be derived from the limitation of $H\longrightarrow\infty$ in Eq.(33), which can be explicitly expressed as

$$\frac{\mathrm{d}^2u'}{\mathrm{d}\phi^2}+u'=\frac{3}{2}(2m+a)u'^2-4mau'^3.\tag{34}$$

If the right-hand side of Eq.(34) is temporarily put equal to zero, we obtain the straight-line solution

$$u' = \frac{\sin\phi}{R},$$

where R is the distance of closest approach to the origin. Substituting the zeroth-order solution $\sin\phi/R$ for u' in the right-hand side of Eq.(34), and solving the resulting differential equation for $u'(\phi)$, we obtain the solution of first order:

$$u' = \frac{\sin\phi}{R} + \frac{3(2m+a)}{4R^2}\left[1+\frac{1}{3}\cos 2\phi\right] + \frac{3ma}{2R^3}\phi\cos\phi - \frac{ma}{8R^3}\sin 3\phi. \quad (35)$$

As $r' \longrightarrow \infty$, $u' \longrightarrow 0$, and $\phi \longrightarrow \phi_\infty$. The angle ϕ_∞ may be small practically. Thus Eq.(35) reduces to

$$\frac{\phi_\infty}{R} + \frac{2m+a}{R^2} + \frac{9ma\phi_\infty}{8R^3} = 0. \quad (36)$$

The total deflection is $\triangle\phi_\infty = 2|\phi_\infty|$, that is

$$\triangle\phi_\infty \simeq \frac{2(2m+a)}{R}\left(1-\frac{9ma}{8R^2}\right). \quad (37)$$

Effects $\frac{2a}{R} - \frac{9ma(2m+a)}{4R^3}$ result from dilaton a. It is obvious that the effect will be observed provided that the parameter a is large enough. If the celestial body is an extreme black hole, i.e. $2m = a$, the bending of the light ray is almost twice as much as that for the general black hole. For the case of $a = 0$, the result (37) reduces to the bending effect of light rays in the Schwarzschild spacetime, $\triangle\phi_\infty = 4m/R$.[18]

4.2. Perihelion advance of a planet

If we treat some of the terms on the right side of Eq.(33) as perturbations, no precision will be lost by replacing the constants of the motion h and H by their classical limits: $H^2 \simeq C_0^4$, $h \simeq h_0$ (at sufficiently large r). Thus we have

$$\frac{\mathrm{d}^2u'}{\mathrm{d}\phi^2} + u' - \frac{(2m+a)c_0^2}{2h_0^2} = -\frac{2mac_0^2}{h_0^2}u' + \frac{3}{2}(2m+a)u'^2 - 4mau'^3. \quad (38)$$

If we temporarily put the terms in u'^2 and u'^3 equal to zero, we obtain a differential equation that may be solved exactly:

$$\frac{\mathrm{d}^2u'}{\mathrm{d}\phi^2} + k^2u' = \frac{1}{\alpha_0}, \quad (39)$$

where

$$k^2 = 1 + 2mac_0^2/h_0^2, \quad (40)$$

$$\alpha_0 = 2h_0^2/(2m+a)c_0^2. \quad (41)$$

The solution is a precise ellipse

$$u' = \alpha^{-1}(1 + e\cos(k\phi)), \quad (42)$$

where e is the eccentricity of the ellipse and α is the semi-latus rectum

$$\alpha = \alpha_0\left(1 + \frac{2mac_0^2}{h_0^2}\right). \quad (43)$$

The advance of the perihelion per revolution of the planet on its orbit is

$$\delta_1 = -\frac{2\pi mac_0^2}{h_0^2} = -\frac{\pi(2m+a)}{\alpha_0}\frac{4ma}{(2m+a)^2}. \quad (44)$$

The terms in u'^2 and u'^3 may be treated as perturbations. One inserts Eq.(42) in these two terms on the right-hand side of Eq.(38) and solves the resulting equation, the term in u'^2, acting alone, produces

$$\delta_2 = \frac{6\pi(2m+a)^2c_0^2}{4h_0^2} = \frac{3\pi(2m+a)}{\alpha_0}. \quad (45)$$

Combining the results (44) and (45), we find the total perihelion advance δ of the planet per revolution:

$$\delta = \delta_1 + \delta_2 = \frac{3\pi(2m+a)}{\alpha_0}\left[1 - \frac{4ma}{3(2m+a)^2}\right]. \quad (46)$$

The advance of perihelion per revolution resulting from dilaton a is $\frac{3\pi a}{\alpha_0} - \frac{4\pi ma}{\alpha_0(2m+a)}$. If a is large enough, the effect can be observed. For the case $a = 0$, Eq.(46) gives the advance of perihelion per revolution in Schwarzschild spacetime.[18]

4.3. Radar echo delay

We now consider the propagation in time of radar pulses in dilaton spacetime. Without loss of general properties, we can assume that the motion takes place in the $\theta = \pi/2$ plane. Writing the conservation equation of energy (10) in plane–polar coordinates and making use of Eqs.(7) and (26), we obtain

$$\left(\frac{\mathrm{d}r}{\mathrm{d}A}\right)^2 + \frac{h^2}{r^2} - n^2c_0^2 = 0. \quad (47)$$

In order to evaluate the constant of the motion, h, we let r_0 denote the distance of approach of the ray to the centre of the gravitational body. When $r = r_0$, we have $\mathrm{d}r/\mathrm{d}A = 0$. Thus Eq.(47) gives

$$h = r_0n(r_0)c_0, \quad (48)$$

which is analogous to the classical-mechanical expression $r_0 v(r_0)$.

By using Eqs.(21), (22), and (9), we have the equation of motion for the radar pulse

$$\left(\frac{\mathrm{d}r'}{\mathrm{d}t}\right)^2 = \Omega^4(r')c_0^2\left[1-\frac{r_0'^2}{r'^2}\frac{\Omega^2(r')}{\Omega^2(r_0')}\right]. \tag{49}$$

The time of travel from r_0 to r' is then

$$\Delta t = c_0^{-1}\int_{r_0'}^{r'}\Omega^{-2}(r')\left[1-\frac{r_0'^2}{r'^2}\frac{\Omega^2(r')}{\Omega^2(r_0'^2)}\right]^{-1/2}\mathrm{d}r'. \tag{50}$$

Using Eq.(24) to write out $\Omega(r')$ and $\Omega(r_0')$, then expending to first order, we obtain

$$\begin{aligned}\Delta t =& c_0^{-1}\int_{r_0'}^{r'}(1-\frac{r_0'^2}{r'^2})^{-1/2}\\ &\times\left[1+\frac{2m+a}{r'}+\frac{(2m+a)r_0'}{2r'(r'+r_0')}-\frac{3ma}{r'^2}\right]\mathrm{d}r'\\ \simeq& c_0^{-1}(r'^2-r_0'^2)^{1/2}+\frac{2m+a}{c_0}\\ &\times\ln\left[\frac{r'}{r_0'}+\frac{(r'^2-r_0'^2)^{1/2}}{r_0'}\right]\\ &+\frac{2m+a}{2c_0}\left[\frac{r'-r_0'}{r'+r_0'}\right]^{1/2}\\ &+\frac{3ma}{r_0'c_0}\sin^{-1}\left(\frac{r_0'}{r'}\right)-\frac{3\pi ma}{2r_0'c_0}.\end{aligned} \tag{51}$$

The first term in the right-hand side of Eq.(51) is the transit time of light in Euclidean space. The $\Delta t(r_0, r')$ due to general-relativistic effects is the sum of the remaining terms, and the terms containing a resulting from dilaton a. When $a = 0$, we gain the delay of radar echo in Schwarzschild spacetime.[18]

4.4. Gravitational redshifts

The gravitational redshifts are not dynamical effects, so we do not need to solve an equation of motion in order to work them out. However, it is interesting to study the gravitational redshifts by using the effective index of refraction. Analogous to the ordinary optics: $\lambda(\boldsymbol{r_1})n(\boldsymbol{r_1}) = \lambda(\boldsymbol{r_2})n(\boldsymbol{r_2})$, we have a connection between $|\Delta\boldsymbol{r}|$ and n at different points of spacetime in general relativity.[6]

$$|\Delta\boldsymbol{r_1}|n(\boldsymbol{r_1}) = |\Delta\boldsymbol{r_2}|n(\boldsymbol{r_2}), \tag{52}$$

where $|\Delta\boldsymbol{r}|$ is the distance between successive crests of a light pulse located at $\boldsymbol{r}$. The metric length λ of the pulse at point $\boldsymbol{r}$ is obtained by applying metric (1) to the coordinate length $|\Delta\boldsymbol{r}|$ of the pulse, which is

$$\lambda(\boldsymbol{r}) = \Phi^{-1}(\boldsymbol{r})|\Delta\boldsymbol{r}|. \tag{53}$$

Thus we have

$$\Phi(\boldsymbol{r_1})\lambda(\boldsymbol{r_1})n(\boldsymbol{r_1}) = \Phi(\boldsymbol{r_2})\lambda(\boldsymbol{r_2})n(\boldsymbol{r_2}), \tag{54}$$

or by using Eq.(3), we have the usual gravitational redshift relation

$$\lambda(\boldsymbol{r_1})\Omega^{-1}(\boldsymbol{r_1}) = \lambda(\boldsymbol{r_2})\Omega^{-1}(\boldsymbol{r_2}). \tag{55}$$

Now let us apply it to the case of the dilaton spacetime. Given the source of light located at $\boldsymbol{r_1}$ and an observer at $\boldsymbol{r_2}$, sufficiently far from the central gravitational body so that we may put $\Omega(\boldsymbol{r_2}) \simeq 1$, then, with the use of Eq.(24), Eq.(55) gives

$$\begin{aligned}z \equiv&(\lambda(\boldsymbol{r_2})-\lambda(\boldsymbol{r_1}))/\lambda(\boldsymbol{r_1})\\ =&[1-(2m+a)/r'+2ma/r'^2]^{-1/2}-1\\ \simeq&\frac{m}{r'}+\frac{a}{2r'}.\end{aligned} \tag{56}$$

Compared with the Schwarzschild metric, dilaton a results in a redshift $\frac{a}{2r'}$ in the dilaton spacetime.

5. Conclusions

The Newtonian forms and the conservation condition of energy for the geodesic equations offer some practical advantages for the calculation of gravitational effects. By using this new method the equations of motion for both the massless particles and the massive particles in the dilaton spacetime have been established. As examples of the applications of the equation, we use it to discuss the motion of massive particles and photons in the dilaton spacetime. Some interesting gravitational effects resulting from dilaton a are found: (1) the bending of light rays is given by $\Delta\phi_a = \frac{2a}{R} - \frac{9ma(2m+a)}{4R^3}$; (2) the perihelion advance of the planet per revolution is $\Delta\delta_a = \frac{3\pi a}{\alpha_0} - \frac{4\pi ma}{\alpha_0(2m+a)}$; (3) the delay of radar echo is $\Delta t_a = \frac{a}{c_0}\ln\left[\frac{r'}{r_0'}+\frac{(r'^2-r_0'^2)^{1/2}}{r_0'}\right] + \frac{a}{2c_0}\left(\frac{r'-r_0'}{r'+r_0'}\right)^{1/2} + \frac{3ma}{r_0'c_0}\sin^{-1}\left(\frac{r_0'}{r'}\right) - \frac{3\pi ma}{2r_0'c_0}$ and (4) the gravitational redshift is $z_a = \frac{a}{2r'}$. All the results show that the gravitational effects arising from the dilaton can be observed provided that the dilaton is large enough. By observing these effects, it is possible to test the string theory.

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