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Effect of adiabatic variation of dust charges on dust acoustic solitary waves in magnetized dusty plasmas*

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The effect of dust charging and the influence of its adiabatic variation on dust acoustic waves is investigated. By employing the reductive perturbation technique we derived a Zakharov–Kuznetsov (ZK) equation for small amplitude dust acoustic waves. We have analytically verified that there are only rarefactive solitary waves for this system. The instability region for one-dimensional solitary wave under transverse perturbations has also been obtained. The obliquely propagating solitary waves to the z-direction for the ZK equation are given in this paper as well.

Keywords: soliton, dusty plasma, Zakharov–Kuznetsov equation
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1. Introduction

Nonlinear waves, which have become ubiquitous in many branches of physics, have been studied extensively.[1−3] Since it was initiated in plasma physics, the reductive perturbation technique has been the main approach for deriving various nonlinear wave equations, such as the Korteweg de Vries (KdV) equation, Kodomtsev–Perviashvili (KP) equation, Zakharov–Kuznetsov (ZK) equation and the nonlinear Schrödinger equation (NLSE).[4] Recently, many researchers have theoretically and experimentally studied various aspects of linear and nonlinear waves in dusty plasmas. For example, the dust acoustic wave (DAW) was first predicted by Rao *et al*[5]; on the other hand, Shukla and Silin[6] showed the existence of dust-ion-acoustic waves (DIAW). Recent experiments on dusty plasmas have confirmed the existence of DAW and DIAW.[7,8] Furthermore, low-frequency electrostatic ion-acoustic and ion-cyclotron waves have also been studied in a magnetized dusty plasma.[9] A theoretical investigation has been made on the instability of propagating electrostatic solitary structure in a magnetized three-component dusty plasma, which consists of a negatively charged dust fluid, Boltzmann distributed electron and free as well as fast ion.[10,11] Recently, Nejoh[12] pointed out that the dust charge variation with some parameters would affect the characteristic collective motion of the plasma. Ma and Liu[13] and Xie *et al*[14,15] have considered this effect and showed the existence of dust acoustic solitons. Based on the same model as presented by Mamun,[10,11] we studied the low-frequency dust acoustic solitary waves in a magnetized three-component dusty plasma, but only considering the contributions of charge variations of dust grains. We find that there are only rarefactive stationary solitary waves which have been verified analytically in this paper. The solitary wave which propagates in an oblique direction is also given in this paper.

2. ZK equation and its solutions

We now consider a three-component dusty plasma consisting of massive, negatively charged dusty fluid, Boltzmann distributed electrons and ions in the presence of an external static magnetic field $\| \boldsymbol{B}_m \| \boldsymbol{k}$,

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where $\boldsymbol{k}$ is a unit vector along the z-direction. Thus, we have $n_{i0} = Z_d n_{d0} + n_{e0}$, where n_{i0}, n_{d0} and n_{e0} are the unperturbed ion, dust grains and electron number densities respectively. Z_d is the number of electrons residing on the dust grains. If we consider both the dust temperature and the charge variations of dust grains for low-frequency dust acoustic waves, the equations of motion are as follows:[14,10,16]

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u) + \frac{\partial}{\partial y}(n_d v) + \frac{\partial}{\partial z}(n_d w) = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\sigma}{n_d}\frac{\partial p_d}{\partial x} + Z_d\frac{\partial \phi}{\partial x} + \omega_{cd} Z_d v, \quad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\sigma}{n_d}\frac{\partial p_d}{\partial y} + Z_d\frac{\partial \phi}{\partial y} - \omega_{cd} Z_d u, \quad (3)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\sigma}{n_d}\frac{\partial p_d}{\partial z} + Z_d\frac{\partial \phi}{\partial z}, \quad (4)$$

$$\frac{\partial p_d}{\partial t} + u\frac{\partial p_d}{\partial x} + v\frac{\partial p_d}{\partial y} + w\frac{\partial p_d}{\partial z} + \gamma p_d\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0, \quad (5)$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = Z_d n_d + n_e - n_i, \quad (6)$$

$$n_i = \mu \exp(-s\phi), \quad (7)$$

$$n_e = \nu \exp(\beta s \phi), \quad (8)$$

where n_d and p_d are the dimensionless number density and the pressure of the dust grains respectively; Z_d is the number charge residing on the dust grains, ϕ the electrical potential, n_i the number density of ions, n_e the number density of electrons, and u, v and w are the components of the velocity of dust grains in the x, y and z directions respectively. For simplicity, we have normalized all the quantities. The densities of ions and electrons are normalized by $Z_{d0} n_{d0}$, the dust density is normalized by n_{d0}, and Z_d by Z_{d0}. The space coordinates x, y and z are normalized by $\lambda_{Dd} = (T_{eff}/4\pi Z_{d0} n_{d0} e^2)^{1/2}$, time t is normalized by $\omega_{pd}^{-1} = (m_d/4\pi n_{d0} Z_{d0}^2 e^2)^{1/2}$, velocity by $C_d = (Z_{d0} T_{eff}/m_d)^{1/2}$, ϕ by T_{eff}/e, pressure by $n_{d0} T_d Z_{d0}$ respectively, where $\sigma = T_d/T_i$, T_d is the dust temperature. $\omega_{cd} = (Z_{d0} e B_m)/(m_d \omega_{pd})$. $\beta = T_i/T_e$. $s = 1/(\mu + \nu\beta)$. Obviously $\mu = \nu + 1$. T_{eff} is an effective temperature which is defined by $T_{eff} = T_i T_e/(\mu T_e + \nu T_i)$, where T_e and T_i are the temperatures of electrons and ions respectively. For the isothermal case $\gamma = 1$, but for the adiabatic case $\gamma \approx 3$. Moreover, we know that $Z_d = 1 + \gamma_1\phi + \gamma_2\phi^2$.[14]

We now follow the reductive perturbation technique and construct a weakly nonlinear theory for electrostatic waves with a small but finite amplitude, which leads to a scaling of the independent variables through the stretched coordinates

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \\ \tau \end{pmatrix} = \epsilon^{1/2} \begin{pmatrix} x \\ y \\ z - v_0 t \\ \epsilon t, \end{pmatrix}, \quad (9)$$

where ϵ is a small parameter measuring the weakness of the dispersion, and v_0 is the unknown wave phase velocity (to be determined later). It may be noted here that ξ, η and ζ, are all normalized to the Debye length (λ_{Dd}), τ is normalized to the dust plasma period (ω_{pd}^{-1}) and v_0 is normalized to the dust acoustic speed (C_d). We can now expand the perturbed quantities about their equilibrium values in power of ϵ as

$$\begin{pmatrix} n_d \\ u \\ v \\ w \\ \phi \\ p_d \\ Z_d \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \epsilon \begin{pmatrix} n_1 \\ \epsilon^{1/2} u_1 \\ \epsilon^{1/2} v_1 \\ w_1 \\ \phi_1 \\ p_1 \\ Z_1 \end{pmatrix} + \epsilon^2 \begin{pmatrix} n_2 \\ u_2 \\ v_2 \\ w_2 \\ \phi_2 \\ p_2 \\ Z_2 \end{pmatrix} + \cdots \quad (10)$$

Substituting Eqs.(9) and (10) into Eqs.(1)–(8) and collecting the terms in the different powers of ϵ, we have, for the lowest order,

$$\begin{pmatrix} n_1 \\ w_1 \\ p_1 \end{pmatrix} = -(1+\gamma_1) \begin{pmatrix} 1 \\ v_0 \\ \gamma \end{pmatrix} \phi_1, \quad (11)$$

$$\begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \frac{1 + \gamma\sigma(1+\gamma_1)}{\omega_{cd}} \begin{pmatrix} \dfrac{\partial \phi_1}{\partial \eta} \\ -\dfrac{\partial \phi_1}{\partial \xi} \end{pmatrix}, \quad (12)$$

where

$$v_0^2 = \gamma\sigma + \frac{1}{1+\gamma_1}. \quad (13)$$

We also find that

$$\frac{\partial u_1}{\partial \xi}+\frac{\partial v_1}{\partial \eta}=0, \tag{14}$$

$$\begin{pmatrix} u_2 \\ v_2 \end{pmatrix}=\frac{v_0}{\omega_{\rm cd}}\begin{pmatrix} \dfrac{\partial v_1}{\partial \zeta} \\ -\dfrac{\partial u_1}{\partial \zeta} \end{pmatrix}. \tag{15}$$

For the higher order of ϵ, we obtain the following equation:

$$\begin{aligned}&\frac{\partial \phi_1}{\partial \tau}+A\phi_1\frac{\partial \phi_1}{\partial \zeta}+B\frac{\partial^3\phi_1}{\partial \zeta^3}\\&+C\frac{\partial}{\partial \zeta}\left(\frac{\partial^2\phi_1}{\partial \xi^2}+\frac{\partial^2\phi_1}{\partial \eta^2}\right)=0,\end{aligned} \tag{16}$$

where

$$A=-q\left(p+\frac{\nu\beta^2-\mu}{(\mu+\nu\beta)^2}\right), \tag{17}$$

$$B=\frac{1}{2v_0(1+\gamma_1)^2}, \tag{18}$$

$$C=\frac{1}{2v_0(1+\gamma_1)^2}+\frac{v_0^3}{\omega_{\rm cd}^2}, \tag{19}$$

$$q=\frac{v_0}{2(1+3\sigma(1+\gamma_1))(1+\gamma_1)}>0, \tag{20}$$

$$p=3+3\gamma_1+2\gamma_2+12\sigma(1+\gamma_1)^3. \tag{21}$$

The two-dimensional (2D) ZK equation (16) also has a one-dimensional (1D) stationary solitary wave solution, as follows:

$$\phi_0=\phi_m{\rm sech}^2\left[\frac{\zeta-u_0\tau}{w}\right], \tag{22}$$

where $\phi_m=\dfrac{3u_0}{A}$ and $w=2\sqrt{\dfrac{B}{u_0}}$ are the amplitude and width of the soliton respectively. It is easy to find that both B and C are positive, so whether the solitary wave of Eq.(22) is compressive or rarefactive depends on whether A is positive or negative. We now try to find if A is positive or negative. Here we select $\gamma\approx 3$. It is found that

$$\frac{{\rm d}A}{{\rm d}\beta}=-\frac{2\mu\nu q(\beta+1)}{(\mu+\nu\beta)^3}.$$

Therefore at $\beta=-1$, there is a maximum value of A

$$A_{\max}=-q(p-1). \tag{23}$$

We let A_0 denote the value of A when $\beta=0$. $A_0=-q\left(p-\dfrac{1}{\mu}\right)$. A_∞ denotes the value of A when $\beta\to\infty$. $A_\infty=-q\left(p+\dfrac{1}{\nu}\right)$. Note that $A_0-A_\infty=q\left(\dfrac{1}{\mu}+\dfrac{1}{\nu}\right)>0$, and $\dfrac{{\rm d}A}{{\rm d}\beta}<0$ when $\beta>0$. So for $-\dfrac{1}{\nu}<p<\dfrac{1}{\mu}$, A can be positive, and $A=0$ at the point of $\beta_{\rm c}$

$$\beta_{\rm c}=\frac{-\mu\nu p+\sqrt{(1-p)\mu\nu}}{\nu(p\nu+1)}. \tag{24}$$

Finally we obtain the following results:

1. For $p>\dfrac{1}{\mu}$, $A_\infty<A<A_0<0$, $(\beta\geq 0)$, there are only rarefactive solitary waves.

2. For $p=\dfrac{1}{\mu}$, $A_\infty<A<0$, $(\beta\geq 0)$, there are only rarefactive solitary waves.

3. For $-\dfrac{1}{\nu}<p<\dfrac{1}{\mu}$, $0<A_\infty<A<A_0$ (when $0<\beta<\beta_{\rm c}$), there are compressive solitary waves; $A=0$ $(\beta=\beta_{\rm c})$, $A<0$ $(\beta>\beta_{\rm c})$, there are rarefactive solitary waves.

4. For $p=-\dfrac{1}{\nu}$ and $p<-\dfrac{1}{\nu}$, we find that $A>0$; for $\beta>0$, there are only compressive solitary waves. It is reported that $\gamma_1>0,\gamma_2>0$,[14,15] therefore $p>0$. We also consider the real case $\beta>0$, $\nu>0,\mu>0$. So, case 4 is not a real case. It is noted that for $p>3$ we have $\mu>1$, therefore the situation of $-\dfrac{1}{\nu}<p<\dfrac{1}{\mu}$ does not exist. It can be finally concluded that there are no compressive solitary waves. We now study the 1D solitary wave solution of Eq.(22) under transverse perturbations. It has been found that there is instability to perturbations with wavenumber k such that $0<k<k_{\rm c}$ for the 2D ZK equation, where k_c is a certain critical wavenumber.[17,18] Here we study a similar problem for a 3D ZK equation to find the instability region and the critical wavenumbers $k_{y'c}$ and $k_{z'c}$. We use the following transformations:

$$x'=\sqrt{u_0}(\zeta-u_0\tau), \tag{25}$$

$$y'=\sqrt{u_0}\xi, \tag{26}$$

$$z'=\sqrt{u_0}\eta, \tag{27}$$

$$t'=u_0\sqrt{u_0}\tau, \tag{28}$$

and then the ZK equation (16) becomes

$$\begin{aligned}&\frac{\partial \phi_1}{\partial t'}-\frac{\partial \phi_1}{\partial x'}+\frac{A}{u_0}\phi_1\frac{\partial \phi_1}{\partial x'}+B\frac{\partial^3\phi_1}{\partial x'^3}\\&+C\left(\frac{\partial^3\phi_1}{\partial x'\partial y'^2}+\frac{\partial^3\phi_1}{\partial x'\partial z'^2}\right)=0.\end{aligned} \tag{29}$$

We assume that there are higher-order transverse perturbations to the 1D stationary solitary wave

$$\begin{aligned}&\phi_1(x',y',z',t')\\&=\phi_0(x')+[\delta\phi(x'){\rm e}^{{\rm i}(k'_y y'+k'_z z')+\omega t'}],\end{aligned} \tag{30}$$

where $\phi_0(x') = \phi_m \mathrm{sech}^2 \dfrac{x'}{2\sqrt{B}}$, $\phi_m = \dfrac{3u_0}{A}$, and ω is the growth rate. If $\omega > 0$, there is instability for the solitary wave, otherwise the solitary wave is stable. Substituting Eq.(30) into Eq.(29), we obtain

$$B\frac{\partial^3 \delta\phi}{\partial x'^3} + \left[\frac{A}{u_0}\phi_0 - 1 - (k_{y'}^2 + k_{z'}^2)C\right]\frac{\partial \delta\phi}{\partial x'} + \left(\omega + \frac{A}{u_0}\frac{\partial \phi}{\partial x'}\right)\delta\phi = 0. \quad (31)$$

After one integration of Eq.(31) at $\omega = 0$, and requiring that $\delta\phi$ and its derivatives tend to zero as $x' \to 0$, we obtain the following associated Legendre equation:

$$\frac{\mathrm{d}}{\mathrm{d}T}\left[(1-T^2)\frac{\mathrm{d}\delta\phi}{\mathrm{d}T}\right] + \left[N(N+1) - \frac{\lambda}{1-T^2}\right]\delta\phi = 0, \quad (32)$$

where $T = \tanh \dfrac{x'}{2\sqrt{B}}$, $N = 3$, $\lambda = 4(1 + Ck_{y'}^2 + Ck_{z'}^2)$. In order to obtain a real solution of $\delta\phi$, λ should satisfy $\lambda = 2^2, 3^2$. For $\lambda = 4$, $k_{y'\mathrm{c}} = k_{z'\mathrm{c}} = 0$. For $\lambda = 9$,

$$k_{y'\mathrm{c}}^2 + k_{z'\mathrm{c}}^2 = \frac{5}{\dfrac{2}{v_0(1+\gamma_1)^2} + \dfrac{4v_0^3}{\omega_{\mathrm{cd}}^2}}. \quad (33)$$

This indicates that there are real solutions of $k_{y'\mathrm{c}}$ and $k_{z'\mathrm{c}}$. When $k_{y'}^2 + k_{z'}^2 < k_{y'\mathrm{c}}^2 + k_{z'\mathrm{c}}^2$, the 1D solitary waves are unstable under transverse perturbations.

In order to find a solitary wave solution propagating obliquely to the z direction for the ZK equation (16), we let

$$\phi_1 = \phi_1(\chi) = \phi_1(\zeta + k_\xi \xi + k_\eta \eta - M\tau). \quad (34)$$

Substituting Eq.(34) into Eq.(16), we obtain

$$-M\frac{\partial \phi_1}{\partial \chi} + A\frac{\partial}{\partial \chi}\left(\frac{\phi_1^2}{2}\right) + (B + Ck_\xi^2 + Ck_\eta^2)\frac{\partial^3 \phi_1}{\partial \chi^3} = 0. \quad (35)$$

It is found that there is a soliton solution for Eq.(35),

$$\phi_1 = \phi_m \mathrm{sech}^2\left[\frac{\chi}{w}\right], \quad (36)$$

where $\phi_m = \dfrac{3u_0}{A}$, $w = 2\sqrt{\dfrac{B + Ck_\xi^2 + Ck_\eta^2}{u_0}}$ are the amplitude and width of the soliton respectively. This indicates that the amplitude of the oblique solitary waves is the same as that for 1D solitary wave, but the width is different. The width of the solitary wave for the oblique one will increase as the coefficients of both B and C increase. It also increases as $k_{y'}$ and $k_{z'}$ increase. So the width of a solitary wave propagating obliquely to the z direction is larger than that of the 1D one propagating in the z direction.

3. Conclusions

We have considered the effect of dust charging and the influence of its adiabatic variation on dust acoustic waves, and found that a ZK equation is the governing equation for small amplitude dust acoustic waves. After investigating this ZK equation, it seems that there are only rarefactive solitary waves for this system. The instability region for a 1D solitary wave under transverse perturbations has been obtained. The results are: when $k_{y'}^2 + k_{z'}^2 < \dfrac{5}{\dfrac{2}{v_0(1+\gamma_1)^2} + \dfrac{4v_0^3}{\omega_{\mathrm{cd}}^2}}$, the 1D solitary waves are unstable under transverse perturbations. The obliquely propagating solitary wave to the z direction for the ZK equation is given in this paper. The amplitude and the width are $\phi_m = 3u_0/A$, $w = 2\sqrt{\dfrac{B + Ck_\xi^2 + Ck_\eta^2}{u_0}}$ respectively, which are different from those of waves propagating in the z direction.

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