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Chaotic evolution of the coupled scalar fields system during inflation*

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In this paper we consider two coupled scalar fields during the inflation as a dynamical system. With the Poincaré section method, we investigate the evolution of the coupled scalar fields system. We find that the evolution of the system changes from a regular motion into a chaotic motion when the energy density and the coupling constant of the system increase.

Keywords: chaotic evolution, coupled scalar fields system
PACC: 0314, 0545,0400

1. Introduction

The evolution of the early universe is an enigma. Chaotic behaviours have been considered as an interesting phenomenon by many physicists since Lorenz [1] found the deterministic non-periodic flow in 1963. In the past decades, chaos has been one of the most important ideas to explain various nonlinear phenomena in nature. After Poincaré researched the three-body problem, many studies on chaos in celestial mechanics and astrophysics have been done.[2−7] Recently Cornish and Levin [8,9] proposed to quantify the chaos in the Mixmaster universe by calculating the dimensions of fractal basin boundaries in initial condition sets for the full dynamics. Some cosmologists are interested in the effects of anisotropy on the dynamics of the early universe; they hope to gain a better understanding of the behaviour of the model by generalizing the isotropic mini-super-space to a full super-space by adding more degrees of freedom. Bianchi cosmologies with fluid sources are such models. The matter in these models is either a perfect fluid [10] or consists of massive or massless vector fields.[11,12] Oliveira *et al*[13−16] have studied the chaotic behaviour in the Bianchi IX model. The work has been driven largely by inflation,[17,18] the most successful known mechanism for explaining the large-scale homogeneity and isotropy of the universe, and small-scale inhomogeneity and anisotropy of the universe.[19,20] According to inflationary scenarios of cosmic evolution, the universe undergoes a period of exponential expansion after it is essentially devoid of matter. At this stage the energy density is almost entirely of the large oscillating expectation value of the scalar field that drives inflation. The theory of reheating is concerned with the question how this energy is transferred from the inflaton to other fields and eventually thermalized. Early calculations assumed that the inflaton would decay to lighter particles via perturbative decay.[21,22] It was only recently realized [23] that nonperturbative mechanisms may completely dominate the reheating process. In particular, the classical phenomenon of parametric resonance may lead to explosive particle product and very rapid decay of the inflaton field.[24] This stage of the evolution, called preheating in the literature, leads to very different physics than would be obtained from slow perturbative decay.

It is interesting to investigate the dynamics of multifield models of inflation in which the inflaton is coupled to another dynamical field during inflation. These models can lead to a variety of features unavailable in the case of a single field. Such multifield scenarios include the well known hybrid inflation models.[25,26] Hybrid inflation and reheating models

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share important common threads. They both involve the coupling of two or more dynamical, interacting scalar fields. An important aspect of such systems is the possibility of mixing of the fields. We note that Minkowski spacetime is a good approximation in the latter stages of certain hybrid inflation models, and it also allows comparison with much of the original reheating literature [21−23,27−29] which often neglects the effects of expansion, allowing us to directly determine the role played by the mixing of the fields in the dynamics.

The outline of the paper is as follows. In Sec.2 we begin by considering the Lagrangian for N coupled scalar fields and set up our formalism for the quantization of the system. This is followed by an outline of the renormalization procedure. In Sec.3, we follow the Poincaré section method to investigate the evolution of the system, and we find the stochastic properties of the two coupled scalar fields during the inflation. In Sec.4 a brief conclusion is given.

2. Dynamics of the coupled fields system during inflation

We work with the following Lagrangian for real scalar fields Φ_i with $i = 1 \ldots N$:

$$L[\Phi_i] = \sum_{i=1}^{N} \frac{1}{2}\partial_\mu \Phi_i(x)\partial^\mu \Phi_i(x) - V[\Phi_i(x)], \quad (1)$$

where the potential is

$$\begin{aligned} V[\Phi_i(x)] = & \sum_{i,j,k,l=1}^{N} A_i \Phi_i + \frac{1}{2!} m_{i,j} \Phi_i \Phi_j \\ & + \frac{1}{3!} g_{ijk} \Phi_i \Phi_j \Phi_k \\ & + \frac{1}{4!} \lambda_{ijkl} \Phi_i \Phi_j \Phi_k \Phi_l. \end{aligned} \quad (2)$$

For two fields, the orthogonal rotation matrix can be written in terms of a single mixing angle θ. The matrix has the form

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \quad (10)$$

The motion equations read as

$$\ddot{\varphi} + m_\varphi^2 \varphi + \frac{\lambda}{6}\psi^3 + \frac{g^2}{2}\varphi\psi^2 = 0, \quad (11)$$

Note that $m_{i,j}$, g_{ijk}, λ_{ijkl} are symmetric in each index, but are generally non-diagonal, resulting in the mixing of the different fields. In what follows, subscripts and superscripts $i \ldots n$ run over the values $1 \ldots N$ and we use a convention in which summation is assumed over repeated subscript indices, but not superscript indices.

Here we consider a two-field case. It is worthwhile to look at a few details of such systems. We begin with a system of two real scalar fields φ and ψ,

$$L = \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{1}{2}(\partial_\mu \psi)^2 - V(\varphi, \psi) \quad (3)$$

with the potential

$$\begin{aligned} V(\varphi, \psi) = & \frac{1}{2} m_\varphi^2 \varphi^2 + \frac{1}{2} m_\psi^2 \psi^2 \\ & + \frac{\lambda}{4!}\varphi^4 + \frac{\kappa}{4!}\psi^4 + \frac{g^2}{4}\varphi^2\psi^2. \end{aligned} \quad (4)$$

This Lagrangian has the form of Eq.(1) with

$$\Phi_1 = \varphi, \Phi_2 = \psi, \quad (5)$$

$$m_{11} = m_\varphi^2, m_{22} = m_\psi^2, m_{12} = m_{21} = 0, \quad (6)$$

$$\lambda_{1111} = \lambda, \lambda_{2222} = \kappa, \lambda_{1122} = g^2, \lambda_{1112} = \lambda_{1222} = 0, \quad (7)$$

$$A_i = 0, g_{ijk} = 0. \quad (8)$$

The remaining components of λ_{ijkl} are determined by the fact that it is symmetric in each of its indices.

The mass matrix M is

$$M = \begin{pmatrix} m_\varphi^2 + \lambda\varphi^2/2 + g^2\psi^2/2 & g^2\varphi\psi \\ g^2\varphi\psi & m_\psi^2 + \kappa\psi^2/2 + g^2\varphi^2/2 \end{pmatrix}. \quad (9)$$

$$\ddot{\psi} + m_\psi^2 \psi + \frac{\kappa}{6}\varphi^3 + \frac{g^2}{2}\psi\varphi^2 = 0. \quad (12)$$

The total energy density of the system can be expressed as

$$\begin{aligned} \varepsilon = & \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\dot{\psi}^2 + \frac{1}{2} m_\varphi^2 \varphi^2 + \frac{1}{2} m_\psi^2 \psi^2 \\ & + \frac{\lambda}{4!}\varphi^4 + \frac{\kappa}{4!}\psi^4 + \frac{g^2}{4}\varphi^2\psi^2. \end{aligned} \quad (13)$$

3. Simulation experiments

We will numerically analyse the behaviour of the mechanical system with the potential $V(\varphi, \psi)$ given in Eq.(4) which leads to the dynamical equations (11) and (12). At first let us investigate the evolution of the dynamical system for various choices of initial conditions. The values of the parameters are $m_\varphi^2 = m_\psi^2 = 2, \lambda = \kappa = 12, g^2 = \varepsilon = 10$ for Fig.1. From this figure we can see the stochastic evolution of fields in the phase space $(\dot{\varphi}, \varphi)$ and (φ, ψ), and from the last curve in Fig.1 where we plot the evolution of the field φ for two different initial conditions, we can find that the evolution of the field sensitively depends on the initial conditions.

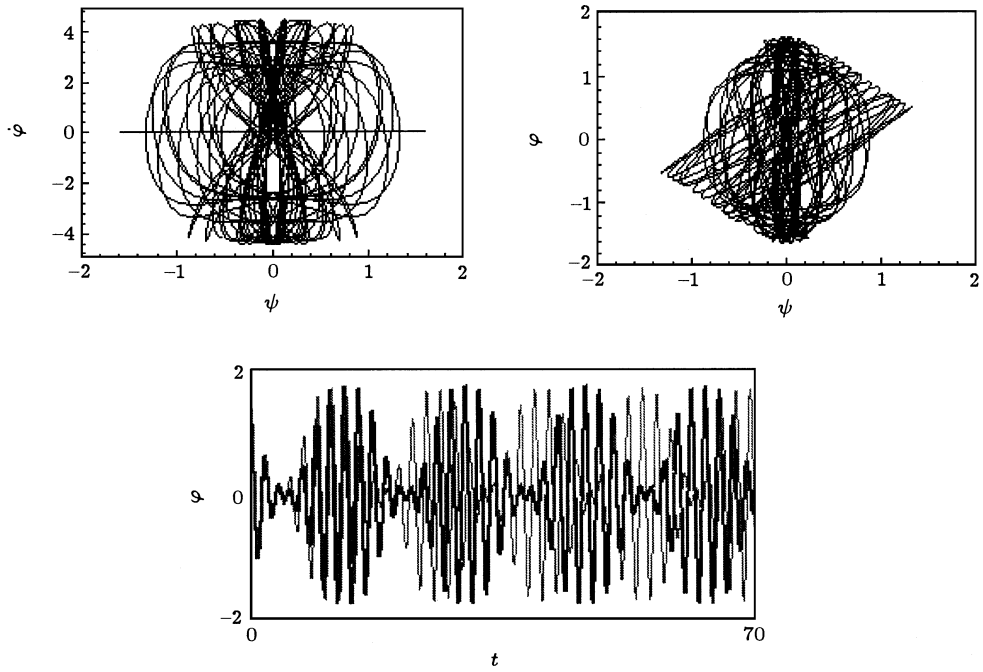


Fig.1. The evolution of the fields in the phase space and the evolution of the fields for $m_\varphi^2 = m_\psi^2 = 2$, $\lambda = \kappa = 12$, $g^2 = \varepsilon = 10$.

Based on the nonlinear dynamical theory, if the motion is not chaotic, the plotted points form a closed curve in the Poincaré section of the 2D $(\dot{\varphi}, \varphi)$ plane, because a regular orbit will move on a torus in the phase space and the curve is a cross section of the torus. If the orbit is chaotic, some of those tori will be broken and the Poincaré section does not consist of a set of closed curves, but the points will be distributed randomly in the allowed region to form a chaotic sea. From the distribution of the points in the Poincaré section, we can judge whether the motion is chaotic or not. We consider the interacting fields as an effective mechanical system with coordinates φ, ψ and the term $\frac{g^2}{4}\varphi^2\psi^2$ as a perturbation of the unperturbed system described by the Lagrangian

$$\varepsilon_0 = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\dot{\psi}^2 + \frac{1}{2}m_\varphi^2\varphi^2 + \frac{1}{2}m_\psi^2\psi^2 + \frac{\lambda}{4!}\varphi^4 + \frac{\kappa}{4!}\psi^4. \qquad (14)$$

In order to investigate the evolution of the unperturbed system described by Eq.(14), we plot the Poincaré sections in Fig.2 for the values of the parameters $m_\varphi^2 = m_\psi^2 = 2, \lambda = \kappa = 12$, $\varepsilon_0 = 0.1$ (a) and $\varepsilon_0 = 10$(b). From the Poincaré sections in Fig.2, we find that all trajectories of the unperturbed system described by the Lagrangian ε_0 are either periodical or quasi-periodical. In order to describe the role of the perturbation, the Kolmogerov–Arnold–Moser (KAM) theorem [30] can be applied which states that, if the perturbation is analytical and small enough, the part of the phase space spanned by our system can be divided into two regions of nonvanishing volumes, one of them being small compared to the other and tending to zero in the limit of a vanishing perturbation. The larger region consists of invariant embedded tori, densely covered with trajectories. In other words, for the majority of possible initial conditions, trajectories are of the Lissajous type, similar to the oscillator case. However, there exists a comparatively small region, corresponding to a small set of initial conditions,

where trajectories are completely chaotic and may go astray from the neighbouring 'restricted' trajectories. This region has a nonzero volume and it may be called the 'instability region'.

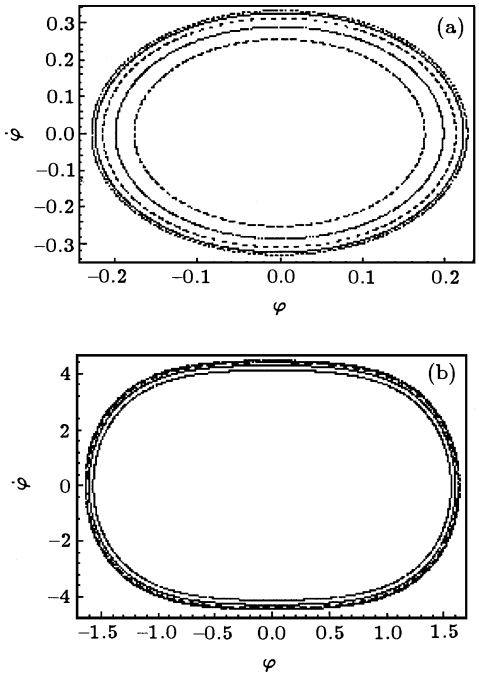


Fig.2. Pioncaré section $(\dot{\varphi}, \varphi)$ across the plane $\psi = 0, \dot{\psi} > 0$ for $m_\varphi^2 = m_\psi^2 = 2$, $\lambda = \kappa = 12$, $\varepsilon_0 = 0.1$ (a) and $\varepsilon_0 = 10$(b).

In order to further investigate the evolution of two coupled scalar fields during inflation in detail, let us follow Ref.[31] and seek numerical solutions with Lagrangian ε and plot the Poincaré sections for $\psi = 0$ under the condition $\dot{\psi} > 0$. In Fig.3 we present the Poincaré sections for different values of Lagrangian with the same coupling constant. Each picture contains a set of trajectories with equal ε. If the set of intersection points for the given trajectory is restricted to a point, the motion is periodical; if they lie along the same curve, the motion is quasi-periodical. For the stochastic motion, the intersection points cover densely a finite area. Figure 3(a) corresponds to comparatively low energy density; it is easily seen that all the trajectories form closed curves. According to the KAM theorem, this means that practically the entire phase space consists of tori, which corresponds to a quasi-periodical motion. For the higher energy densities such as in Figs.3(b)–(d), the situation is completely different, and there arise regions of ergodic behaviour, though there remain some small regions of toroidal structure. It should be clearly understood that the dispersed points in Fig.3(c) and (d) form a chaotic sea which means that the trajectories completely change into chaotic evolution.

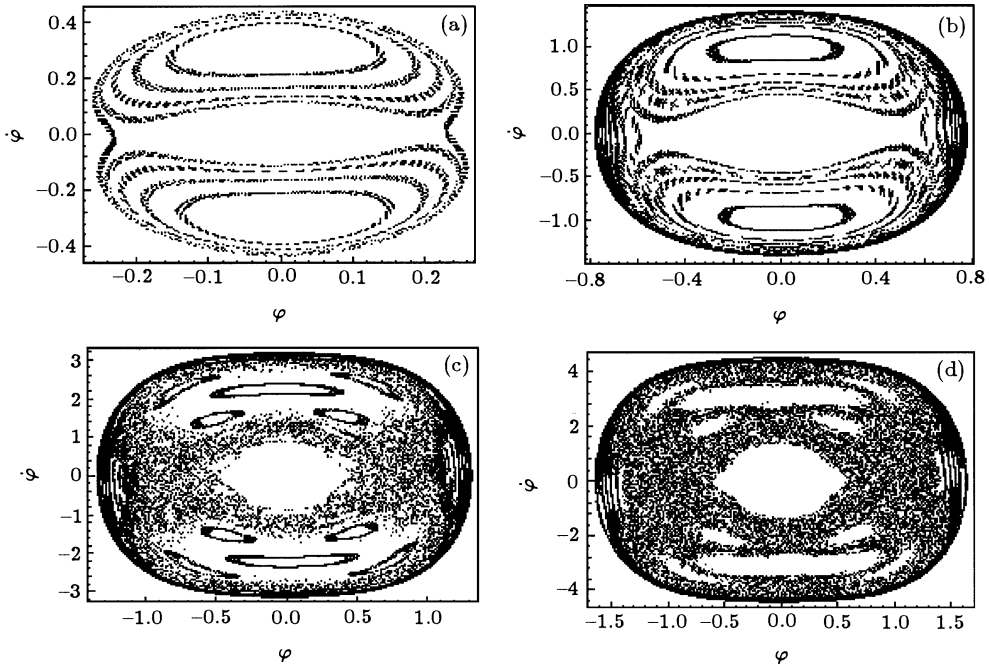


Fig.3. Pioncaré section $(\dot{\varphi}, \varphi)$ across the plane $\psi = 0, \dot{\psi} > 0$ for $m_\varphi^2 = m_\psi^2 = 2$, $\lambda = \kappa = 12$, $g = 10$ and $\varepsilon = 0.1$ (a), $\varepsilon = 1$(b), $\varepsilon = 5$(c) and $\varepsilon = 10$(d).

In Fig.4 we present the Poincaré sections for different values of the coupling constant with the same energy density. Figure 4(a) corresponds to a comparatively small coupling constant; it is easily seen that all the trajectories form closed curves. According to the KAM theorem, this means that practically the entire phase space consists of tori, which corresponds to a quasi-periodical motion. For greater coupling constants such as in Figs.4(b)–(d), the situation is completely different and there arise regions of ergodic behaviour. It should be clearly understood that the dispersed points in Figs.4(c) and (d) form a chaotic sea which means that the trajectories completely change into chaotic evolution.

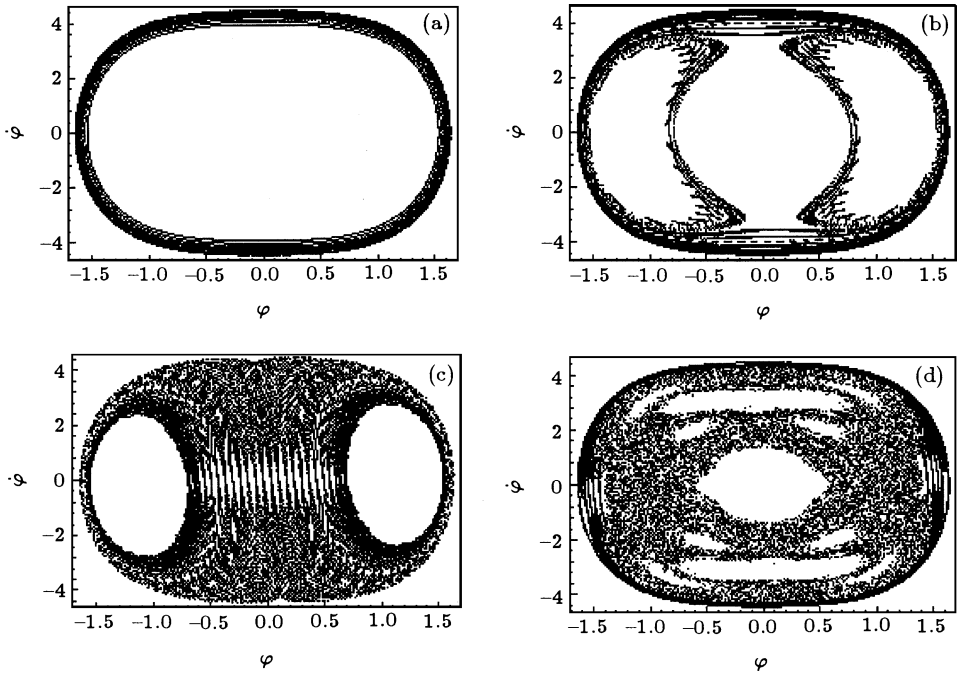


Fig.4. Pioncaré section $(\dot{\varphi}, \varphi)$ across the plane $\psi = 0, \dot{\psi} > 0$ for $m_\varphi^2 = m_\psi^2 = 2$, $\lambda = \kappa = 12$, $\varepsilon = 10$ and $g = 0.1$(a), $g = 1$(b), $g = 2$(c) and $g = 10$(d).

4. Conclusions

In this paper we have investigated the evolution of the two coupled scalar fields system during inflation. By numerical analysis we found that the evolution of coupled scalar fields system may change from a regular motion into a chaotic motion when the energy density of the system and the coupling constant increase. This means that the evolution of field system becomes more chaotic in the earlier universe.

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References

[1] Lorenz E N 1963 *J. Atoms. Sci.* **20** 215
[2] Moser T 1978 *Math. Intelligen.* **1** 65
[3] Chen J H Wang Y J 2003 *Chin. Phys. Lett.* **20** 972
[4] Chen J H Wang Y J 2001 *Acta Phys. Sin.* **50** 1833
[5] Chen J H Wang Y J 2003 *Chin. Phys.* **12** 836
[6] Chen J H Wang Y J 2003 *Commun. Theor. Phys.* **40** 301
[7] Wisdom J 1987 *Proc. R. Soc. Lond.* A **413** 109
[8] Cornish J N and Levin J J 1997 *Phys. Rev. Lett.* **78** 998
[9] Cornish J N and Levin J J 1997 *Phys. Rev.* D **55** 7489
[10] Stewart J M and Ellis G F R 1968 *J. Math. Phys.* **9** 1072
[11] Ford L H 1989 *Phys. Rev.* D **40** 967
[12] Thorne K S 1967 *Astrophys. J.* **148** 151
[13] de Oliveira H P, Damiao Soares I and Stuchi T J 1997 *Phys. Rev.* D **56** 730
[14] Monerat G A, de Oliveira H P and Damiao Soares I 1998 *Phys. Rev.* D **58** 063504
[15] Tonini E V 2001 *Phys. Rev.* D **63** 063502

[16] de Oliveira H P, Ozorio de Almeida A M, Damiao Soares I and Tonini E V gr-qc/0202047
[17] Guth A 1981 *Phys. Rev.* D **23** 347
[18] Kolb E W and Turner M S 1990 *The Early Universe* (Redwood City, CA: Addison-Wesley)
[19] Kamionkowski M and Kosowsky A 1999 *Annu. Rev. Nucl. Part. Sci.* **49** 77
[20] Chen J H Wang Y J *Commun. Theor. Phys.* to be published
[21] Dolgov A D and Linde A D 1982 *Phys. Lett.* B **116** 329
[22] Albbott L F, Farhi E and Wise M 1982 *Phys. Lett.* B **117** 29
[23] Traschen J and Brandenberger R H 1990 *Phys. Rev.* D **42** 2491
[24] Kofman L Linde A D and Starobinskii A A 1994 *Phys. Rev. Lett.* **73** 3195
[25] Linde A D 1990 *Phys. Lett.* B **129** 177
[26] Copeland E J, Liddle A R, Lyth D H, Stewart E D and Wands D 1994 *Phys. Rev.* D **49** 6410
[27] Albrecht A, Steinhardt P J, Turner MS and Wilczek F 1982 *Phys. Rev. Lett.* **48** 1437
[28] Boyanovsky D, De Vega H J, Holman R, Lee D S and Singh A 1995 *Phys. Rev.* D **51** 4419
[29] Sthanov Y ,Traschen J and Brandenberger R 1995 *Phys. Rev.* D **51** 5438
[30] Kolmogorov A N 1954 *Dokl. Akad. Nauk. SSSR.* **98** 27
Arnold V I 1963 *Usp. Mat. -Nauk.* **18** 13; Moser J, Nachr Akad Wiss Goett II 1962 *Math. -Phys. Kl.* **11a** 1
see also Arnold V I 1974 *Mathematical Methods of Classical Mechanics* (Moscow: Nauka) (in Russian)
[31] Henon M and Heiles C 1964 *Astron. J.* **69** 73