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Acceleration of a test particle in the gravitational field of central mass with a large number of magnetic monopoles*

Wang Yong-Jiu(王永久)† and Cao Deng(曹 登)

Institute of Physics, Hunan Normal University, Changsha 410081, *China*

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In a Kerr–Newman–Kasuya field we have obtained expressions of the acceleration effect and also studied the special case ($v^i = 0$). We find that in this field, the acceleration of a falling particle has both radial and transverse components. The acceleration effects of electric charge q and magnetic charge ϕ are also discussed, separately.

Keywords: Kerr–Newman–Kasuya field, magnetic monopoles, acceleration
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1. Introduction

Rubakov [1] and Callan separately presented the process of the decay of nucleon, $\rho M \to \pi^0 e^+ M$, whose reaction cross section reaches the same order of magnitude as the usual strong interaction, $\sigma \sim 10^{-26}\mathrm{cm}^2$. But Wilczek [2] forwarded a different view in an estimation that the cross section of this kind of catalyzed decay should be lower by 10 orders. Dimopolos *et al*[3] and Kolb *et al*[4] separately studied a problem that a neutron star [5] captures magnetic monopoles so that the decay of nucleons is impelled. They stated that once magnetic monopoles in the interstellar space are captured by a neutron star, they will remain in it for a long time to catalyze the decay of neutrons, whose products will transform into x-rays or ultraviolet photons to be emitted by several means (such as the annihilation of the e^+ and e^- within the Fermi surface of a neutron star into photon and $\pi^0 \to \gamma + \gamma$). They assumed that the background x-rays and ultraviolet photons are all produced in this way from neutron stars in the universe. From observations they estimated the upper limit of luminosity of the x-rays and ultraviolet photons as below.

Dimopolos:

$$\frac{<\sigma\beta>}{10^{-27}\mathrm{cm}^2}\,\Phi_{\mathrm{M}} \leq 10^{-25}\mathrm{cm}^{-2}\cdot\mathrm{s}^{-1}\cdot\mathrm{sr}^{-1}, \qquad (1)$$

Kolb:

$$\sigma\,\Phi_{\mathrm{M}} \leq 5\times 10^{-49}\cdot\mathrm{s}^{-1}\cdot\mathrm{sr}^{-1}. \qquad (2)$$

In Eq.(1) $\beta c - 0.3c$ is the Fermi velocity of a neutron in a neutron star. If the analysis by Rubakov and Callan is correct, the upper limit of the flow of magnetic monopoles in interstellar space is

$$(\Phi_{\mathrm{M}})_{\mathrm{up}} \sim 10^{-25} - 10^{-22}\mathrm{cm}^{-2}\cdot\mathrm{s}^{-1}\cdot\mathrm{sr}^{-1}. \qquad (3)$$

This upper limit is 6–9 orders lower than the Parker [6] limit, $\Phi_0 \leq 10^{-16}\mathrm{cm}^{-2}\cdot\mathrm{s}^{-1}\cdot\mathrm{sr}^{-1}$, and 12–15 orders lower than the flow estimated from the events of magnetic monopoles as $6\times 10^{-10}\mathrm{cm}^{-2}\cdot\mathrm{s}^{-1}\cdot\mathrm{sr}^{-1}$ by Cabrera.[7]

According to the estimation by Dimopolos *et al* in which they assumed that in the initial period of the formation of a neutron star, the number of the magnetic monopoles within it could be neglected, the number of the magnetic monopoles captured by a neutron star at the age of τ years is

$$\begin{aligned} N &\cong 4\pi^2 R^2 \frac{V_{\mathrm{es}}^2}{V_0^2}\,\Phi_{\mathrm{M}}\tau \\ &\cong 3\times 10^{20}\left(-\frac{10^{-3}c}{V_0}\right)^2\left(\frac{\Phi_{\mathrm{M}}}{\Phi_0}\right)\left(\frac{\tau}{10^{10}\mathrm{years}}\right), \end{aligned} \qquad (4)$$

where Φ_0 is the Parker limit of the flow of magnetic monopoles; R is the radius of a neutron ($\sim 10^6$cm); $V_{\mathrm{es}}^2 = 2GM/R$ is the escaping velocity at the surface of

*Project supported by the National Basic Research Program of China (No 2003CB716300).
†E-mail: wyj@hunnu.edu.cn

a neutron star; $V_0 \sim 10^{-3}c$ is the typical velocity of magnetic monopoles in interstellar space, and $\Phi_{\rm M}$ is the flow of magnetic monopoles. Here we assume that the positive magnetic monopoles are dominant in the Milky Way system. (Because in the very initial period of the universe the magnetic field may have been very strong, the separation of the positive and negative magnetic monopoles was possible.) Because the magnetic charge of a magnetic monopole is q=137e/2, the neutron star at the age of τ years has the magnetic charge

$$\begin{aligned} q_m =& Nq \sim 10^{13}\left(\frac{10^{-3}c}{V_0}\right)^2\left(\frac{\Phi_{\rm M}}{\Phi_0}\right)\left(\frac{\tau}{10^{10}\text{years}}\right) \\ =& A\left(\frac{\tau}{10^{14}\text{years}}\right) \quad \text{(Tesla unit)}, \end{aligned} \tag{5}$$

where A is related to the flow of magnetic monopoles $\Phi_{\rm M}/\Phi_0$.

Dirac [8] pointed out that quantum mechanics does not preclude the existence of magnetic monopoles and Schwinger [9] proposed the dyon, i.e. a pole possessing both electric and magnetic charges.[10] In the paper of Kasuya,[11] an exact solution of a rotating dyon black hole in the Tomimatsu–Sato–Yamazaki spacetime was presented. Now, by using the Lorentz coordinates and comoving coordinates, we study the acceleration effect [12] in a Kerr–Newman–Kasuya field.

2. Gravitational field of central mass with a large number of magnetic monopoles

We take $G = c = 1$. The stationary axisymmetric dyon solution is a straightforward extension of the Tomimatsu–Sato–Yamazaki solutions in Boyer–Lindquist coordinates,

$$\begin{aligned} {\rm d}s^2 =& -\frac{\Sigma}{\Delta}{\rm d}r^2 - \Sigma {\rm d}\theta^2 - \frac{B_\delta \Delta}{A_\delta}\sin^2\theta {\rm d}\varphi^2 \\ &+ \frac{A_\delta}{B_\delta}\left[{\rm d}t - \frac{2aC_\delta}{\sigma A_\delta}\sin^2\theta {\rm d}\varphi\right]^2, \end{aligned} \tag{6}$$

where

$$\begin{aligned} \Sigma =& \frac{(m\sigma)^{2\delta}B_\delta}{\delta^2(m^2-a^2-q^2-\phi^2)^{\delta-1}} \\ &\times\left(\frac{\delta^2\Delta}{m^2-a^2-q^2-\phi^2}+\sin^2\theta\right)^{1-\delta^2}, \end{aligned} \tag{7}$$

$$\Delta = (r-m)^2 - \frac{1}{\delta^2}(m^2-a^2-q^2-\phi^2), \tag{8}$$

and $a = J/m$.[13]

With an arbitrary integer distortion parameter $\delta = 1$, the stationary axisymmetric dyon expression in the Kerr–Newman–Kasuya metric is given by

$$A_1 = \frac{1}{(m\delta)^2}[\Sigma - (2mr - q^2 - \phi^2)], \tag{9}$$

$$B_1 = \frac{\Sigma}{(m\sigma)^2}, \tag{10}$$

$$C_1 = \frac{\sigma}{2(m\sigma)^2}(2mr - q^2 - \phi^2), \tag{11}$$

where $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = r^2 + a^2 + q^2 + \phi^2 - 2mr$.

Thus we have

$$\begin{aligned} {\rm d}s^2 =& \left(1 - \frac{2mr - q^2 - \phi^2}{\Sigma}\right){\rm d}t^2 - \frac{\Sigma}{\Delta}{\rm d}r^2 - \Sigma{\rm d}\theta^2 \\ &- \left[\frac{(2mr - q^2 - \phi^2)a^2\sin^2\theta}{\Sigma} + r^2 + a^2\right]\sin^2\theta{\rm d}\varphi^2 \\ &+ \frac{2a\sin^2\theta}{\Sigma}(2mr - q^2 - \phi^2){\rm d}\varphi{\rm d}t, \end{aligned} \tag{12}$$

and

$$g^{00} = \frac{a^4 + 2r^4 - a^2(q^2 + \phi^2 - 2mr - 3r^2) + a^2\Delta\cos 2\theta}{\Delta(a^2 + 2r^2 + a^2\cos 2\theta)}, \tag{13}$$

$$g^{03} = g^{30} = -\frac{2a(q^2 + \phi^2 - 2mr)}{\Delta(a^2 + 2r^2 + a^2\cos 2\theta)}, \tag{14}$$

$$g^{11} = -\frac{\Delta}{\Sigma}, \tag{15}$$

$$g^{22} = -\frac{1}{\Sigma}, \tag{16}$$

$$g^{33} = -\frac{2(q^2 + \phi^2 - 2mr + \Sigma)\csc^2\theta}{\Delta(a^2 + 2r^2 + a^2\cos 2\theta)}, \tag{17}$$

otherwise $g^{\mu\nu} = g_{\mu\nu} = 0$. So we have

$$\Gamma^i_{00} = \frac{1}{2}g^{i\lambda}(2g_{\lambda 0,0} - g_{00,\lambda}), \tag{18}$$

$$\Gamma^i_{01} = \frac{1}{2}g^{i\lambda}(g_{\lambda 0,1} + g_{\lambda 1,0}), \tag{19}$$

$$\Gamma^i_{11} = \frac{1}{2}g^{i\lambda}(2g_{\lambda 1,1} - g_{11,\lambda}), \tag{20}$$

where i=1,2,3. Substituting Eqs.(13)–(17) into Eqs.(18)–(20), we have

$$\begin{aligned}\Gamma^1_{00} =& -\frac{1}{2}g^{11}g_{00,1}\\ =& -\frac{\Delta[r(q^2+\phi^2-mr)+a^2m\cos^2\theta]}{\Sigma^3}, \qquad (21)\\ \Gamma^2_{00} =& -\frac{1}{2}g^{22}g_{00,2}\\ =&\frac{1}{2}\frac{a^2(q^2+\phi^2-2mr)\sin 2\theta}{\Sigma^3}, \qquad (22)\\ \Gamma^3_{01} =&\frac{1}{2}g^{30}g_{00,1}+\frac{1}{2}g^{33}g_{30,1}\\ =& -\frac{4a[r(q^2+\phi^2-mr)+a^2m\cos^2\theta]}{\Delta(a^2+2r^2+a^2\cos 2\theta)^2}, \qquad (23)\\ \Gamma^1_{11} =&\frac{1}{2}g^{11}g_{11,1}\\ =&\frac{r(a+q^2+\phi^2-mr)+a^2(m-r)\cos^2\theta}{\Sigma\Delta}, \qquad (24)\\ \Gamma^2_{11} =& -\frac{1}{2}g^{22}g_{11,2}=\frac{1}{2}\frac{a^2\sin 2\theta}{\Sigma\Delta}, \qquad (25)\end{aligned}$$

and

$$\Gamma^3_{00}=\Gamma^1_{01}=\Gamma^2_{01}=\Gamma^3_{11}=0. \qquad (26)$$

3. Acceleration effect in the gravitational field

In Lorentz local inertia coordinates, we have [13,14]

$$g^i=-\Gamma^i_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}s}\frac{\mathrm{d}x^\nu}{\mathrm{d}s}=-\frac{\mathrm{d}}{\mathrm{d}s}\frac{v^i}{\sqrt{1-\beta^2}}. \qquad (27)$$

Expanding Eq.(27), we obtain

$$\begin{aligned}g^i =&\frac{1}{1-\beta^2}\Bigg[-\frac{\Gamma^i_{00}}{g_{00}}-\frac{2}{\sqrt{g_{00}}}\left(\Gamma^i_{0k}-\frac{g_{0k}}{g_{00}}\Gamma^i_{00}\right)v^k\\ &-\left(\Gamma^i_{kj}-\frac{g_{0j}}{g_{00}}\Gamma^i_{0k}-\frac{g_{0k}}{g_{00}}\Gamma^i_{oj}\right.\\ &\left.+\frac{g_{0j}g_{0k}}{g^2_{00}}\Gamma^i_{00}\right)v^jv^k\Bigg]. \qquad (28)\end{aligned}$$

Assuming that a neutral test particle moves along the radial direction, i.e. $v^1\neq 0$, $v^2=v^3=0$, we have

$$g^i=\frac{1}{1-\beta^2}\left(-\frac{\Gamma^i_{00}}{g_{00}}-\frac{2}{\sqrt{g_{00}}}\Gamma^i_{01}v^1-\Gamma^i_{11}v^1v^1\right). \qquad (29)$$

Substituting Eqs.(21)–(26) into Eq.(29), we obtain

$$\begin{aligned}g^1 =&\frac{\Delta}{2(1-\beta^2)\Sigma}\\ &\times\left\{-\frac{2v^{1^2}[r(\Delta-r^2+mr)+a^2(m-r)\cos^2\theta]}{\Delta^2}\right.\\ &\left.+\frac{2[r(q^2+\phi^2-mr)+a^2m\cos^2\theta]}{\Sigma(q^2+\phi^2-2mr+\Sigma)}\right\}, \qquad (30)\\ g^2 =&\frac{-a^2\sin 2\theta}{2(1-\beta^2)\Sigma^2}\left(\frac{v^{1^2}\Sigma}{\Delta}+\frac{q^2+\phi^2-2mr}{q^2+\phi^2-2mr+\Sigma}\right), \qquad (31)\\ g^3 =&\frac{8av^1[r(q^2+\phi^2-mr)+a^2m\cos^2\theta]}{(1-\beta^2)\Delta(a^2+2r^2+a^2\cos 2\theta)^2}\\ &\times\left(1+\frac{q^2+\phi^2-2mr}{\Sigma}\right)^{-\frac{1}{2}}. \qquad (32)\end{aligned}$$

The magnitude of acceleration is given by the pure space metric

$$g_{m,a,q,\phi}=\sqrt{\gamma_{ij}g^ig^j}, \qquad (33)$$

where

$$\gamma_{ij}=\frac{g_{oi}g_{oj}}{g_{00}}-g_{ij}.$$

Substituting Eqs.(30)–(32) into Eq.(33) and assuming a=0, we obtain

$$\begin{aligned}g_{m,a,q,\phi} =&\frac{1}{(1-\beta^2)^2r^6(q^2+\phi^2+r^2-2mr)^2}\\ &\times\sqrt{\frac{r^2}{q^2+\phi^2+r^2-2mr}}\\ &\times\{q^2+\phi^4+\phi^2r(-3m+r-rv^{1^2})\\ &+mr^2[2m+r(v^{1^2}-1)]\\ &+q^2[2\phi^2+r(-3m+r-rv^{1^2})]\}^2. \qquad (34)\end{aligned}$$

4. Discussion

In comoving coordinates, $v^i=0$. So we have

$$g^i(v^i=0)=-\frac{\Gamma^i_{00}}{g_{00}}. \qquad (35)$$

Substituting Eqs.(21), (22) and (26) into Eq.(35), we have

$$\begin{aligned}g^1_{m,a,q,\phi} =& -\frac{\Gamma^1_{00}}{g_{00}}\\ =&\frac{\Delta[r(q^2+\phi^2-mr)+a^2m\cos^2\theta]}{\Sigma^2(q^2+\phi^2-2mr+\Sigma)}, \qquad (36)\\ g^2_{m,a,q,\phi} =& -\frac{\Gamma^2_{00}}{g_{00}}\\ =& -\frac{a^2(q^2+\phi^2-2mr)\cos\theta\sin\theta}{\Sigma^2(q^2+\phi^2-2mr+\Sigma)}, \qquad (37)\\ g^3_{m,a,q,\phi} =& -\frac{\Gamma^3_{00}}{g_{00}}=0. \qquad (38)\end{aligned}$$

Besides $g^1 \neq 0$, we can see that the transverse component $g^2 \neq 0$ in the Kerr–Newman–Kasuya field which includes mass m, angular momentum a, electric charge q and magnetic charge ϕ. g^1, g^2 and g are functions of radius r and angle θ, which means that they are related to the position of the test particle.

In the case of $\theta = 0$, we have

$$g^1_{m,a,q,\phi} = -\frac{m}{r^2} + \frac{q^2+\phi^2}{r^3} + 3a^2\frac{m}{r^4} + O\left(\frac{m^5}{r^5}\right), \quad (39)$$

and in the case of $\theta = \pi/2$, we have

$$g^2_{m,a,q,\phi} = -\frac{m}{r^2} + \frac{q^2+\phi^2}{r^3} - a^2\frac{m}{r^4} + O\left(\frac{m^5}{r^5}\right). \quad (40)$$

From Eqs.(39) and (40), we can see:

(a) When θ is very small, the existence of the angular momentum of gravitational source will reduce Newton acceleration, and when $\theta \approx \pi/2$, it will increase Newton acceleration. So, from Eqs.(36)–(40) we can find the position (r, θ) at which the contribution of the angular momentum of the gravitational source is zero.

(b) The effects of electric charge q and magnetic charge ϕ both reduce Newton acceleration. At $r = \dfrac{q^2+\phi^2}{m}$, they will completely counteract Newton acceleration.

(c) We know that the mass m of celestial bodies corresponds to gravitational radius $r = 2m$. So when $q^2 + \phi^2 > 2m$, the celestial bodies will not collapse to black holes. However, it is known that electric charges of celestial bodies would not be very abundant. So this case will not occur unless the celestial bodies contain a large number of magnetic monopoles.[1,3,4,14–21]

References

[1] Rubakov V 1981 *JETP. Lett.* **33** 644

[2] Wilczek F 1982 *Phys. Rev. Lett.* **48** 1146

[3] Dimopolos S *et al* 1982 *Catalyzed Nucleon in Neutron Stars* (New York: Springer) p637

[4] Kolb E *et al* 1982 *Monople Catalysis of Nucleon Decey in Neutron Stars* (New York: Springer) p174

[5] Wang Y J and Peng Q H *Scient. Sin.* A **422**

[6] Parker E 1970 *Ap. J.* **160** 383

[7] Cabrera B 1979 *Phys. Rev. Lett.* **48** 1378

[8] Dirac P A M 1948 *Phys. Rev.* **74** 817

[9] Schwinger J 1966 *Phys. Rev.* **144** 1087

[10] Wang Y J and Tang Z M 2001 *Sci. China* A **44** 801

[11] Kasuya M K 1982 *Phys. Rev.* D **25** 995

[12] Wang Y J and Tang Z M 1990 *The Theory and Effects of Gravitation* (Changsha: Hunan Science and Technology Press) pp745–750

[13] Peca C S and Lemos J P S 1999 *Phys. Rev.* D **59** 124007

[14] Wang Y J and Tang Z M 2002 *Commun. Theor. Phys.* **37** 667

[15] Wang Y J and Tang Z M 2002 *Astrophys. Space Sci.* **281** 689–697

[16] Wang Y J and Tang Z M 2002 *Astrophys. Space Sci.* **281** 1–17

[17] Wang Y J and Tang Z M 2002 *Chin. Phys. Lett.* **19** 166

[18] Wang Y J and Tang Z M 2002 *Sci. China* A **45** 528

[19] Chen J H and Wang Y J 2003 *Class. Quantum Grav.* **20** 3897

[20] Chen J H and Wang Y J 2003 *Chin. Phys.* **12** 836

[21] Tang Z M 2000 *Chin. Phys.* **9** 881